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Estimation of adjusted relative risks in log-binomial regression

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List of Abbreviations

Abbreviation	Definition
α	Significance level
$\hat{\beta}$	Point estimate of a model parameter from the original sample
$B(n, p)$	Binomial distribution with parameters n (number of trials) and p (success probability in each trial)
CI	Confidence interval
exp	Natural exponential function
$E(X)$	Expected value (mean) of a random variable X
ε	Residual(s)
F	Unknown distribution
GLM	Generalized linear model
$\in$	Membership
IRLS	Iteratively reweighted least squares
L	Likelihood
$\mathcal{L}$	Log-likelihood
ln	Natural logarithm
MLE	Maximum-likelihood estimation
MSE	Mean squared error
μ	Unknown expected value (mean) of a random variable X
$\mathbb{N}$	Real number(s)
n	Number of realizations of a random variable
$N(\mu, \sigma^2)$	Normal distribution with parameters μ (expected value) and σ^2 (variance)
OR	Odds ratio
$P(M)$	Probability of an event M
p	Number of model parameters
Π	Product
R	Statistical programming language
RCT	Randomized controlled trial
RR	relative risk
SE	Standard error
Σ	Sum
σ^2	see $Var(X)$
θ	Unknown model parameter
$Var(X)$	Variance of a random variable X
$\bar{x}$	Estimated arithmetic mean
X	Random variable
$\mathbf{X}$	Design matrix
x	Realization of a random variable
x	Independent variable
y	Dependent variable
$\mathbb{Z}$	Integer(s)

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1 Abstract

Problem statement: For binary outcome data, the relative risk (RR) is an essential measure of association, which can be estimated directly for prospective studies. Calculating the odds ratio (OR) can overestimate and magnify the risk heavily, especially if dealing with a disease outcome of high incidence [3]. In such cases OR should be avoided and RR can be used. The log-binomial model is a straightforward statistical approach in case of risk adjustment and estimation, also it is much easier to interpret [3]. However, the log-binomial model might fail or have difficulties maximizing log-likelihood function due to numerical instability, implicit parameter constraints, or naïve starting value, that leads to a dramatic increase of the required number of iterations therefore convergence failure.

Approach: In this study, a modified Newton-type algorithm was developed for solving the maximum likelihood estimation problem under linear constraints. Moreover, a new system of linear inequality constraints on the number of covariates was imposed. In this approach, log-likelihood function of the log-binomial regression model is maximized sequentially. The modified-newton-type algorithm proceeds iteratively by generating a sequence of estimates which solves the quadratic sub-problems obtained from a second-order Taylor approximation and converges under the linear inequality constraints [3].

Monte Carlo Simulation design: For validation and evaluation purposes, a large full-factorial simulation study was conducted in order to study the behavior of the method "squadP" compared with other methods investigated in this research such as Fisher scoring, EM-type, BFGS, and Nelder-Mead. Assessment of coverage probability, model accuracy, and model bias were the primary objectives, while at the same time allowing for many different scenarios (varying number of events, sample size, and number of covariates). The 12 underlying covariates were generated via copula package in R with a specified correlation structure between all variables [3]. In total, 104800 data sets were generated and analyzed for 108 scenarios. The complete simulation study including pseudo-random number generator using L'Ecuyer algorithm was conducted using parallel processing in R 3.5. Additionally, the running time of the six methods was computed.

Results: The proposed modified Newton-type algorithm SquadP generally showed significantly higher convergence rate than EM-type, Fisher-scoring, BFGS, and Nelder-Mead particularly in scenarios with small samples and small event probabilities. SquadP and Poisson(log) showed similar convergence rate in most scenarios with significant excel of SquadP error measurements. EM-type and SquadP showed similar bias, coverage probabilities, and error measurements in most cases with insignificant difference however EM-type had in most scenarios the smallest convergence rate. Furthermore, SquadP converged always faster than EM-type algorithm with a highly significant difference.

Problemstellung: Für binäre Ergebnisdaten ist das relative Risiko (RR) ein wesentliches Assoziationsmaß, das für prospektive Studien direkt geschätzt werden kann. Die Berechnung des Odds Ratio (OR) kann das Risiko stark überschätzen und vergrößern, insbesondere wenn es sich um ein Krankheitsergebnis mit hoher Inzidenz handelt. In solchen Fällen sollte OR vermieden werden und RR kann verwendet werden. Das Log-Binomial-Modell ist ein einfacher statistischer Ansatz für die Risikoanpassung und -schätzung und auch viel einfacher zu interpretieren. Das Log-Binomial-Modell kann jedoch Schwierigkeiten haben und die Log-Likelihood-Funktion aufgrund numerischer Instabilität, impliziter Parameterbeschränkungen oder naiven Startwerts nicht maximieren, was zu einer dramatischen Zunahme der Anzahl der erforderlichen Iterationen führt, weshalb die Konvergenz fehlschlägt.

Ansatz: In dieser Studie wurde ein modifizierter Newton-Algorithmus entwickelt, um das Problem der Schätzung der maximalen Wahrscheinlichkeit unter linearen Bedingungen zu lösen. Wir haben auch ein neues System linearer Ungleichheitsbeschränkungen für die Anzahl der Kovariaten eingeführt. Bei diesem Ansatz maximieren wir die Log-Wahrscheinlichkeit des log-binomialen Regressionsmodells nacheinander. Der Algorithmus vom modifizierten Newton-Typ geht iterativ vor, indem eine Folge von Schätzungen erzeugt wird, die die quadratischen Unterprobleme löst, die aus einer Taylor-Näherung zweiter Ordnung erhalten werden, und unter den linearen Ungleichungsbeschränkungen konvergiert.

Monte-Carlo-Simulationsdesign: Zu Validierungs- und Bewertungszwecken wurde eine große vollfaktorielle Simulationsstudie durchgeführt, um das Verhalten der Methode (squadP) im Vergleich zu anderen in dieser Studie untersuchten Methoden wie Fisher Scoring, EM-type, BFGS und Nelder-Mead. Die Bewertung der Abdeckungswahrscheinlichkeit, der Modellgenauigkeit und der Modellverzerrung waren die Hauptziele, wobei gleichzeitig viele verschiedene Szenarien berücksichtigt wurden (unterschiedliche Anzahl von Ereignissen, Stichprobengröße und Anzahl von Kovariaten). Die 12 zugrunde liegenden Kovariaten wurden über ein Copula-Paket in R mit einer festgelegten Korrelationsstruktur zwischen allen Variablen generiert. Insgesamt wurden 104800 Datensätze generiert und für 108 Szenarien analysiert. Die vollständige Simulationsstudie einschließlich Zufallszahlengenerator unter Verwendung des L'Ecuyer-Algorithmus wurde unter Verwendung der Parallelverarbeitung in R 3.5 durchgeführt. Zusätzlich wurde die Laufzeit der sechs Methoden berechnet.

Ergebnisse: Das vorgeschlagene modifizierte Newton-Algorithmus SquadP zeigte im Allgemeinen eine signifikant höhere Konvergenzrate als EM-Typ, Fisher-Scoring, BFGS und Nelder-Mead, insbesondere in Szenarien mit kleinen Stichproben und kleinen Ereigniswahrscheinlichkeiten. SquadP und Poisson(log) zeigten in den meisten Szenarien eine ähnliche Konvergenzrate mit einer signifikanten Überlegenheit der SquadP-Fehlermessungen. EM-Typ und SquadP zeigten in den meisten Fällen ähnliche

Verzerrungen, Abdeckungswahrscheinlichkeiten und Fehlermessungen mit unbedeutendem Unterschied, jedoch hatte EM-Typ in den meisten Szenarien die geringste Konvergenzrate. Darüber hinaus konvergierte SquadP immer schneller als ein EM-Algorithmus mit einem sehr signifikanten Unterschied.

2 Background

2.1 Introduction

Occurrence of an event with a probability is the fraction of times that specific event is expected to see in many trials. In other words, how likely that event to occur. Probability has a range between $[0, 1]$, therefore, if we have a probability, fully values is expected to lie in this rage and not beyond it's limits. 0 in the range denotes that the probability of an event to occur is impossible, and the higher the probability of an event than 0, the greater are the chances of the event to occur till it reaches 1 which indicates that the occurrence of an event is certain.

Probability is intuitive, however, the odds, in contrast, is not intuitive. Odds is defined as the probability which a specific event occurs (can be called p) divided by the probability which that event does not occur $(1 - p)$, moreover, odds usually range from $[0, \infty]$. This could be presented as the following:

$$\text{Odds of event} = p/(1 - p)$$

Suppose we have a probability of the event to occur = 0.70, then odds will be expressed as follows:

$$\text{Odds} = 0.70/(1 - 0.70) = 0.70/0.30 = 2.33$$

If the probability of occurrence is very low, suppose = 0.05, then:

$$\text{Odds} = 0.05/0.95 = 0.05$$

We notice that when the probability of an event to occur is low (rare event), the odds and the probability share similar results. On the other hand, The similarities between the probabilities dissipate fastly when the probability becomes high or common, because the odds is not bounded above ($[0, \infty]$), and the approximation becomes increasingly poor.

Suppose we have another example from registry of a prospective study as shown in table 1, and the aim is to assess the effect of smoking on Lung cancer.

	Lung cancer	No lung cancer	Total
Smokers	80	20	100
Nonsmokers	50	50	100

Table 1: Table shows the exposure (smoking) and the outcome (cancer).

In order to study the association between the outcome (which is binary) and the exposure, odds ratio (OR) or relative risk (RR) required to be calculated.

$$\text{Odds ratio (OR)} = \frac{p_1/(1-p_1)}{p_0/(1-p_)} = \frac{80}{20}/\frac{50}{50} \approx 4$$
$$\text{Relative risk (RR)} = \frac{p_1}{p_0} = \frac{80}{80+20}/\frac{50}{50+50} \approx 1.6$$

OR (odds ratio) is a ratio of two odds, p_0 which is interpreted as the probability of the outcome in the unexposed group, while p_1 is the probability of the outcome in the exposed group [57]. RR (relative risk) is a ratio of the two probabilities, which is in the above example and can be simply interpreted as the exposed are likely (1.6 times) to have lung cancer disease.

For binary outcome data, as mentioned above in the example, the relative risk is an essential measure of association, which can be estimated directly for prospective studies using logistic regression model [3]. The logistic model approximates the odds ratio and it can be represented as relative risk in case of rare events. However, when events are common, calculating the odds ratio can overestimate and magnify the risk heavily, particularly if dealing with a disease outcome of high incidence [23] [58]. Furthermore, A correction for odds ratio to give a relative risk in the studies of common or high outcomes was suggested by Zhang and Yu. Moreover, it occurs very often that the disease is not rare, thus, in such cases OR should be avoided and RR can be directly computed.

The log-binomial regression model is a straightforward statistical approach to be used in case of estimating the relative risk for binary response variables. Moreover, RR can be directly interpreted and understand easier than OR. log-binomial model is commonly and widely used in medical and public health research for RR estimation for both common and rare outcomes, and most researchers and physicians are interested more in RR. However, with case-control studies we cannot compute the probability of an event or a disease in each of the exposure groups, therefore, we cannot compute RR. However, OR should be computed.

The log-binomial model can be defined as a generalized linear model (GLM) with binomial probability and a log link function, which is discussed later in this section.

2.2 Generalized linear model (GLM)

Normal, binomial, and Poisson responses are common modeling problems used, and there are many forms of regression models can be used as approaches to a broad range of response modeling problems. The term generalized linear model (GLMs) goes back to Nelder and Wedderburn (1972), and McCullagh and Nelder (1982, 2nd edition 1989), who refer to a larger class of flexible generalization of ordinary linear models which allow the response distributions rather than the normal distribution. Response variable y_i in a GLM follows an exponential family distribution with mean μ_i, which is the function of ${x_i}^T\beta$.

GLMs have many models classified into categories which include logistic regression, log-binomial regression, Poisson regression, etc. These class of GLM models have three components:

- Random component: is the probability distribution of the dependent variable (also known as response variable) (e.g. binomial, Poisson, ...).
- Systematic component: refers to the linear combination of the explanatory variables $X_1, X_2, ...X_k$ to create the linear predictor in the model $\beta_0 + \beta_1 x_1 + \beta_2 x_2, ...\beta_k x_k$
- Link function $g(\mu)$: which identifies the link between the random and systematic components. In other words, the link function describes how the expected value of the response variable relates to the linear predictor of explanatory variables, and the "most common link function for binary outcomes is the logit $log(\mu/(1-\mu)$ in a logistic regression model, the $log(\mu)$ in a Poisson model, or a log-binomial model" [10] [41] [36] [1].

GLM models have general assumptions:

- The explanatory variables $X_1, X_2, ...X_k$ are independently distributed.
- The response variable Y_i follows a distribution from an exponential family (e.g. binomial, Poisson, ...)
- GLMs assuming linear association between the dependent variable (response) in terms of the link function and the independent variables. For instance, Log-binomial regression is:
$$log(\pi) = \beta_0 + \beta X$$
- The explanatory (independent) variables are linear or nonlinear transformations of the original independent variables.
- GLMs rely on large-sample approximations and using MLE (maximum likelihood estimation) for parameters estimation

For risk ratio estimation and analysis, Generalized linear models (GLMs) provide a standard form of regression models. Fisher based log-binomial model is the standard regression form included in GLM.

One of the generalized linear models, as logistic regression, is log-binomial regression model that uses a link function and binomial outcome distribution. As mentioned by Williamson et al. (2013), "Everything is common between the two models except for the link function. Log-binomial models use a log link function, rather than a logit link, to connect the dichotomous outcome to the linear predictor" [57].

$$log(p_i) = x_i\beta$$

the response variable in log-binomial model depends on a set of independent (explanatory) variables x_i, which models the log of the probability of "success" as a function of explanatory variables. By assuming that the distribution of the response variable follows to the exponential family it is possible to derive maximum likelihood estimates for the coefficients of the log-binomial model. Log-binomial model uses MLE (maximum likelihood estimation) for estimating the parameters, therefore, an iterative method for estimation is required. Fisher's scoring method (which is additionally called "Iteratively Reweighted Least Squares estimates (IRLS)") solves it iteratively. The iterative method maximizes the likelihood by getting closer to the solution (point estimation) by taking another step (an iteration) till it finds the maxima which is specified by the convergence criteria and it generally converges to a solution [26] [31].

2.3 Challenges of GLM

Relative risk can be modeled and calculated directly using the standard log-binomial regression form of the generalized linear model. As a GLM model form, log-binomial regression is fitting by maximizing the log-likelihood function (it is explained later in greater detail) yielding a global maximum that is called the maximum likelihood estimate (MLE). During the maximization process, the iterative algorithm can fail to converge therefore fails to find the MLE. Failed convergence in GLM models is presented and discussed in greater detail by Williamson et al. (2013) [57]. However, here few causes of convergence failure are briefly mentioned.

The probability in the log-likelihood function $p = \exp(x_i'\beta)$ must satisfy $0 \leq p \leq 1$, which implies

$$\Rightarrow x_i'\beta \leq 0$$

Failed convergence occurs often whenever these natural parameter constraints (parameter space) are violated, therefore, causing predicted probabilities > 1 which leads to numerical instability therefore convergence problem.

One of the obvious causes of failed convergence is choosing a starting value that are outside the feasible region. Naïve starting values can lead to a dramatic increase of the number of iterations required and in most cases convergence failure [3]. The starting value problem is discussed in the Methods section with a solution in greater detail.

Algorithmic maximum likelihood estimation failure was observed for small data sets using popular and standard software like IBM-SPSS or SAS [57].

2.4 Alternative solutions

Here we discuss some common alternative solutions for estimating the relative risk. 4 statistical methods are introduced in this chapter.

Poisson regression model: which is form of the generalized linear models for regression analysis used to model count data. Furthermore, it can be used with log link function to directly estimate RR (relative risk). In Poisson regression, the outcome/dependent variable Y_i follows a distribution, which is the Poisson distribution from an exponential family with the link function which is the logarithm. “The logarithm of its expected value can be modeled by a linear combination of unknown parameters” [53] [4]. Because Poisson regression method uses a log link function, it shares similar form from log-binomial model however a Poisson distribution is applied to the data rather than a binomial distribution. If x_i is a set of explanatory variables, then the model can be expressed by the following form:

$$\log(\mathrm{E}(Y \mid \mathbf{x})) = \alpha + \beta' \mathbf{x}$$

EM-based model: in order to analyze binary outcome data, RR regression using a log-link binomial model, which is a form of generalized linear models, is generally used. Several algorithms are implemented in the statistical software and tools to fit the relative risk (RR) regression models and allow the estimation of the maximum likelihood. However, fisher scoring, which is the standard method for maximum likelihood estimation and fitting GLMs in the statistical software, may encounter numerical problems as provided by Marschner (2015) [34], such as the required constraints of the parameter space are not obeyed. There is no simple solution in the Fisher scoring to impose parameter constraints so that the fitted event probabilities do not exceed 1.

Donoghoe (2018) presented `logbin` package which is implemented in R programming language [45], which fits the generalized linear model with a binomial error distribution and log link function [13]. `logbin` package performs maximum likelihood estimation for log-binomial regression models, using a different computational approach to find the maximum likelihood estimate such as EM-type algorithm. EM-type algorithm is straightforward to maximize the log-likelihood $L(\theta)$ and imposes the natural constraints [12].

Nelde-Mead algorithm is the default method used for minimization is implementation of Nelder and Mead (1965) which is a heuristic search technique that can converge to non-stationary points of a function. Nelder-Mead is an applied numerical method that uses only the log-likelihood function values to find the maximum in a multidimensional space [40] [39].

Nelde-Mead is implemented in R programming language as the default iterative method in `optim` package that is implemented for general purpose optimization. `optim` package performs minimization by default, but it will maximize if the objective function to be maximized is negative. In this case, the objective function being maximized is the log-likelihood function of log-binomial regression model, furthermore the optimization process is done iterativly based on methods such as quasi-Newton, and Nelder–Mead algorithm.

BFGS algorithm (Broyden–Fletcher–Goldfarb–Shanno) can be defined as a quasi-Newton method which solve optimization problems that is nonlinear unconstrained iteratively. BFGS algorithm implements the objective function values (log-likelihood of log-binomial regression model) and gradients (first derivative) for building up an image of the surface to be optimized [17] [6] [19] [47]. BFGS is a method implemented in R and can be used in `optim` package as well.

2.5 Aim and main idea

As mentioned above, GLM standard model for estimating RR doesn't respect the natural constrained, therefore a constrained model and problem reparameterization are needed.

Donoghoe et al. (2018) [13] introduced an R package called `logbin`, that implements the stable Expectation-Maximization (EM) algorithm to tackle these issues but it is extremely slow as shown and discussed later in the Results section. Furthermore, default implemented accelerated version of EM, which is known as "cem" for combinatorial EM algorithm, gives different results for the same data set in case of changing the type of a variable from numeric to be a factor. However, in this study, we used the stable EM version "em" that is based on a single EM which gives identical results if the type of a variable changed from numeric to a factor.

In this study, A modified Newton-type algorithm (called squadP) is developed to tackle the maximum likelihood estimation problems, such as the constrained parameter space. Moreover, optimized self-generated starting values.

Furthermore, a large Monte Carlo simulation study was conducted for comparing and evaluating the properties of the six statistical methods being discussed here (EM-type algorithm, Fisher scoring algorithm, Poisson, BFGS, Nelder-Mead, and squadP). The execution time (speed of the algorithm) is calculated as well and compared with the other underlying algorithms.

3 ML estimation of RR using constrained optimization

The goal of this chapter is to present the fundamental technique of the proposed approach, which is the constrained optimization of the log-binomial model using analytical and numerical tools.

3.1 Log-likelihood function of log-binomial model

The log-likelihood function of the log-binomial model represents the combination of model parameter values at the maximum in a procedure known as MLE (maximum likelihood estimation). Moreover, log-likelihood function is used to determine the standard error associated with the point estimate at the maximum, which provides information about all of the estimations for the log-binomial model.

Log-binomial model is a straightforward alternative for estimation of the risk ratio and it belongs to the class of Generalized Linear Models (GLM). However, the iterative method of log-binomial models sometimes fails to converge to MLE (maximum likelihood estimate) and therefore does not find a solution. The convergence problem is related to the constrained space of the linear predictors using the log link. The problem is exacerbated when multiple variables are used in the model [44] [33].

To obtain the MLE, an iterative numerical optimization technique is used. The iterative method is known as the iteratively re-weighted least squares (IRLS) estimating algorithm and it's equivalent to Fisher's scoring method [37] [24].

In the log-binomial model we have a binary response (Bernoulli distributed), y_i, $i = 1, ..., n$ which means that, $Pr(y_i = 1|x_i) = p_i$. The outcome variable Y follows binomial distribution and the mean μ of that distribution is assumed to depend on a vector of variables, X, by

$$E(Y) = \mu = g^{-1}(X\beta)$$

$E(Y)$ is the expected value of Y , $X\beta$ or η is the linear predictor, where g is the link function that relates the linear predictor to the mean of the distribution [37] [18]. Where $E(Y) = p_i$, the linear predictor is sat as:

$$\eta = \beta_0, \beta_1 x_1, \beta_2 x_2, ..., \beta_m x_n$$

Log-link $\eta = log(p_i)$ can be written likewise as $log(p_i) = X\beta$. The row vector X representing the set of covariates and the vector of coefficients β are directly related to relative risk estimation. Therefore the log-likelihood function of $\beta = (\beta_0, \beta_1, \beta_2, ..., \beta_m)$ is given by the following equation:

$$l(\beta) = \sum_i y_i \ log(p_i(\beta)) + \sum_i (1 - y_i) \ log(1 - p_i(\beta)) \quad (1)$$

Examining the log-likelihood function for the log-binomial model shows causes of the estimation issues which need to be solved [37].

Suppose that the log-binomial regression model and its parameter space are given by:

$$log(p_i) = x_i'\beta \quad (2)$$
$$0 \leq p_i = e^{X\beta} \leq 1$$

However, the probability $p_i = E(Y) = e^{X\beta}$ in the log-likelihood function, as shown in equation (1), may theoretically exceed the parameter space between [0, 1] during the iterative steps. For the estimation of β parameters using the log-likelihood in practice, the probability p has to satisfy this condition $0 < p < 1$, otherwise, if the probability $p = 0$, then $log(p) = log(0)$ is not defined. This issue is tackled as well in the proposed modified Newton-type method as presented next.

3.2 Log-likelihood maximization using a constrained optimization approach

In this modified Newton-type approach called squadP, the log-likelihood function of the log-binomial regression model is maximized sequentially. The linear inequality restricted estimator is computed iteratively, and in each iteration (step) a quadratic optimization problems is solved under restrictions [14].

As shown in figure 1, the process of maximizing the log-likelihood and finding a solution using the proposed modified-Newton algorithm called squadP is presented and explained as follows.

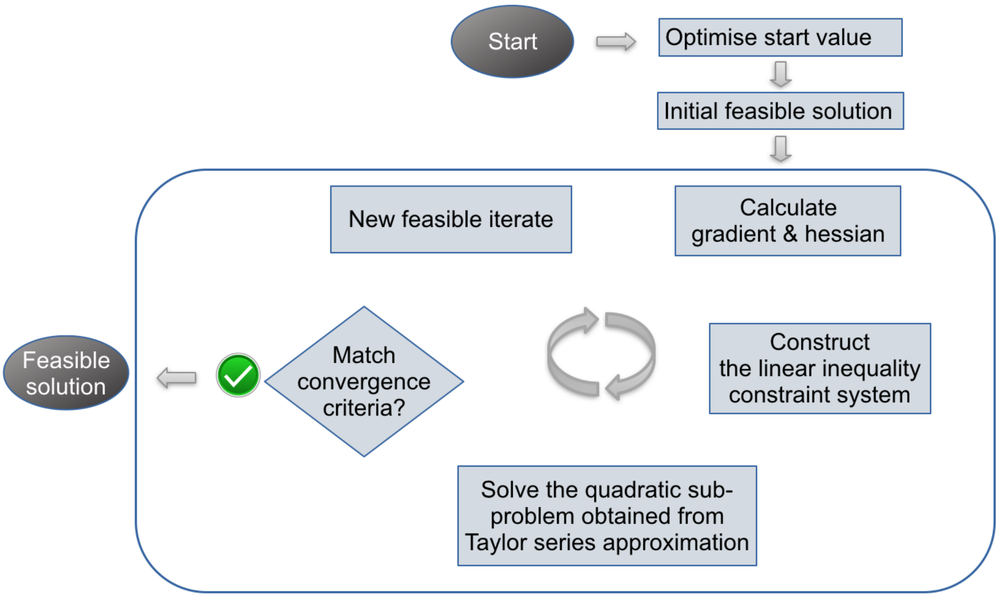

Figure 1: Graphical representation of squadP model steps from generating the initial values automatically until convergence and finding a feasible solution.

3.2.1 Optimization of the initial values

Optimization of the initial values, also know as starting values, is the first step in the proposed method in this study. To estimate the relative risk, the estimator often requires that the user provide the initial values as a necessary step. Entering the initial values closer to the solution which is the point estimate (MLE) is considered more efficent than entering the initial values far from the solution (the maxima) because the number of iterations required to find the solution could be significantly smaller, which saves time and computational cost. The starting values further from the solution might be out of the feasible region, and therefore the algorithm will not start the processes. The problem becomes more complex with multiple covariates in the model. Guessing the combination of the starting values inside the feasible region is a difficult task, particularly, when the number of covariates to be fitted in the model is larg.

Approximating the starting values is a self-generated step in the algorithm "sqaudP" and optimized using Newton's method. The log-likelihood function for initial value estimation β_0 is given by the following equation:

$$f(\beta_0) = -(n\ \beta_0 + m\ log(1 - \exp(\beta_0)) \tag{3}$$

β_0 is the intercept and the only parameter being optimized assuming that $\beta_1, \beta_2, ..., \beta_m = 0$. Theoretically, 0 is always inside the feasible region. n and m are the number of cases and non-cases respectively in the outcome variable.

Calculating the first derivative and the hessian matrix of the log-likelihood function respectively as follows.

$$f\prime(\beta_0) = \frac{n_0 \ \exp(\beta_0)}{(1 - \exp(\beta_0)) - n_1} \tag{4}$$

$$f\prime\prime(\beta_0) = \frac{n_0 \ \exp(\beta_0)}{(\exp(\beta_0) - 1)^2} \tag{5}$$

The approximation of the starting value for β_0 is processed by Newton's method. The method named after Isaac Newton and Joseph Raphson as an algorithm to find the root (or zero) of a real-valued function. To approximate the root, calculating recursively is done using:

$$x_{n+1} = x_n - \frac{f'(x_n)}{f''(x_n)} \tag{6}$$

As discussed by Gould and Pitblado (2006) [22], The iterations of the method can be summarized by the following steps:

Algorithm

1. Start with an initial guess x_0 ;
2. Calculate x_{n+1}, $n = 0, 1, ...$;
3. Repeat step 2 until the method converges.

This version of Newton's method is with a single variable. f' is the function's derivative, and x_n is the initial guess. Calculating recursively, the x_{n+1} often becomes increasingly better approximation of the function's root.

Numerically calculating the log of the mean of the outcome data variable gives a smaller approximation of the intercept β_0 as in the Newton's method.

Using squadP method, the user is not required to enter the starting values or struggle to find a combination that is inside the feasible area regardless of the number of covariates in the model.

3.2.2 Computing the score function and Hessian matrix (observed information matrix)

After self-initiating the starting values by the proposed method, computing the derivatives of the log-likelihood function is the next step. The first derivative, also known as Fisher's score function $s(\beta)$, of the log-likelihood function is calculated and is denoted by the following equation,

$$s(\beta) = \frac{\partial l(\beta)}{\partial \beta} = \sum_i x_i \, \frac{y_i - p_i(\beta)}{1 - p_i(\beta)} \tag{7}$$

If $\hat{\beta}$ maximizes the log-likelihood function, it likewise maximizes the likelihood function. The score is a vector for all β elements. Given that the log-likelihood function is concave, the maximum likelihood estimate can be found by setting the score to zero as shown in the following.

$$s(\hat{\beta}) = 0$$

and solving the likelihood equation for $\hat{\beta}$ which leads to the maximum likelihood estimator. The score is evaluated at the true unknown parameter value β with a mean of zero

$$E[s(\beta)] = 0$$

and variance-covariance matrix given by the hessian matrix, and is also known as the observed information matrix $I(\beta)$, which is the second derivative of log-likelihood function that is negative [46].

$$I(\beta) = \frac{\partial^2 l(\beta)}{\partial^2 \beta} = -\sum_i x_i x_i' \, \frac{p_i(\beta)(y_i - 1)}{1 - p_i(\beta)^2} \tag{8}$$

The score function and the Hessian matrix (observed information matrix) of the log-likelihood function are calculated to determine all points of maxima, furthermore, to indicate that the log-likelihood function is concave and has a peak and not flat. The score function and Hessian matrix are used in the modified Newton-type method squadP as well because they are the coefficient of the quadratic term of the Taylor expansion of the function as explained later in this chapter.

3.2.3 Constructing the linear inequality constraint system

One of the reasons that the standard log-binomial regression model is often subject to numerical instability is that it does not respect the natural constraints of the parameter space in the process of estimating the relative risk [16]. The parameter space for the log-binomial model is restricted causing difficulties for the maximum likelihood estimators (numerical algorithms) to converge and calculate the standard errors [59]. Wherefore, a flexible and reliable linear inequality constrained system was developed to ensure the stability of the modified Newton-type method squadP. Furthermore, respecting the natural constraints of the parameter space.

As discussed by McCullagh and Nelder [36], the link function, such as *logit*, for binomial data maps $[0, 1]$ onto $(-\infty, \infty)$. For this reason, a logistic model generally will not fail to find the estimates of β in the parameter space: $-\infty \leq X\beta \leq -\infty$

In case of log-link function the closed interval $[0, 1]$ is mapped onto $(-\infty, 0]$, that is the parameter space:

$$-\infty \leq X\beta \leq 0$$

X must include the observed covariate vectors [36].

The linear inequality constraints solution (2^r) that is developed in this study induces constraints on the parameter space to guarantee that the fitted values are in the feasible region and the fitted probabilities p_i are within the closed interval [0, 1]. Therefore, $log(p_i)$ is defined.

The linear inequality constraints 2^r are dependent on two factors: 1. The number of variables in the model which is r. 2. The possible values of each variable which can have generally three main categories:

- Variables with binary data
- Variables with quantitative data
- Mixed variables

I. **A model with binary variables**

Suppose the model is given by

$$\eta = log(\pi) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + ... + \beta_m x_n$$

Then, a model with a single binary variable is given as

$$\eta = \beta_0 + \beta_1 x_1, \qquad x \in \{0, 1\}$$

Based on the constraints matrix 2^r, this model will have two constraints $2^1 = 2$ as follows

$$X = 0: \qquad -\infty \leq \beta_0 \leq 0$$

$$X = 1: \qquad -\infty \leq \beta_0 + \beta_1 \leq 0$$

If the model has two binary variables, then it is given as

$$\eta = \beta_0 + \beta_1 x_1 + \beta_2 x_2, \qquad x_1, x_2 \in \{0, 1\}$$

In this case a matrix of four different constraints is constructed $2^2 = 4$.

$$x_1 = x_2 = 0: \qquad -\infty \leq \beta_0 \leq 0$$

$$x_1 = 1, x_2 = 0: \qquad -\infty \leq \beta_0 + \beta_1 \leq 0$$

$$x_1 = 0, x_2 = 1: \qquad -\infty \leq \beta_0 + \beta_2 \leq 0$$

$$x_1 = x_2 = 1: \qquad -\infty \leq \beta_0 + \beta_1 + \beta_2 \leq 0$$

II. **A model with quantitative variables**

A model with a single quantitative variable is given by

$$\eta = \beta_0 + \beta_1 x_1, \qquad x \in \{x_{min}, x_{max}\}$$

Two constraints for this model $2^1 = 2$ are generated as follows:

$$x = x_{min}: \qquad -\infty \leq \beta_0 + x_{min}\ \beta_1 \leq 0$$

$$x = x_{max}: \qquad -\infty \leq \beta_0 + x_{max}\ \beta_1 \leq 0$$

If the model has two quantitative variables, then it is given by

$$\eta = \beta_0 + \beta_1 x_1 + \beta_2 x_2 \qquad x_1 \in \{x_{min}, x_{max}\},\ x_2 \in \{x_{min}, x_{max}\}$$

Four constraints for this model are constructed ($2^2 = 4$)

$$-\infty \leq \beta_0 + x_{1\ min}\ \beta_1 + x_{2\ min}\ \beta_2 \leq 0$$

$$-\infty \leq \beta_0 + x_{1\ min}\ \beta_1 + x_{2\ max}\ \beta_2 \leq 0$$

$$-\infty \leq \beta_0 + x_{1\ max}\ \beta_1 + x_{2\ min}\ \beta_2 \leq 0$$

$$-\infty \leq \beta_0 + x_{1\ max}\ \beta_1 + x_{2\ max}\ \beta_2 \leq 0$$

III. **A model with mixed variables**

A model with two variables x_1, which is binary, and x_2, which is quantitative, is given by

$$\eta = \beta_0 + \beta_1 x_1 + \beta_2 x_2, \qquad x_1 \in \{0, 1\},\ x_2 \in \{x_{min}, x_{max}\}$$

Based on 2^r, four constraints for this model $2^2 = 4$ are generated.

$$-\infty \leq \beta_0 + x_{2\ min}\ \beta_2 \leq 0$$

$$-\infty \leq \beta_0 + x_{2\ max}\ \beta_2 \leq 0$$

$$-\infty \leq \beta_0 + \beta_1 + x_{2\ min}\ \beta_2 \leq 0$$

$$-\infty \leq \beta_0 + \beta_1 + x_{2\ max}\ \beta_2 \leq 0$$

3.2.4 Taylor approximation of the log-likelihood function

An approximation of the log-likelihood function of the log-binomial model is based on a quadratic function. This is done by applying a Taylor approximation of the second order around the maximum likelihood estimate $\hat{\beta}$ [51].

The score function $S(\beta)$, which is the first derivative of the log-likelihood as explained previously, and the Hessian matrix $I(\beta)$ (also interpreted as the observed information matrix), that is the negative second derivative of the log-likelihood, are used for solving such an optimization problem because they are the coefficient of the quadratic term of Taylor expansion of log-likelihood function. Expanding $logL(\beta)$ which is the log-likelihood function into a Taylor series of second-order about $\hat{\beta}$ is given by:

$$logL(\beta) = logL(\hat{\beta}) + S(\hat{\beta})(\beta - \hat{\beta}) - \frac{1}{2}I(\hat{\beta})(\beta - \hat{\beta})^2 \tag{9}$$

To solve the quadratic sub-problem obtained from the second-order Taylor expansion of $logL(\beta)$ and converging to $\hat{\beta}$ under the linear inequality constraints solution 2^r, as shown earlier in this chapter, a numerical method is required.

3.2.5 Solving the quadratic sub-problem

A numerical approximation is required to solve the restricted optimization sub-problem quadratically by proceeding iteratively and generating a sequence of estimates $\hat{\beta}$ till convergence. The algorithm or the quadratic solver being used in this study is called 'quadprog.'

'quadprog' is a package implemented in R that is based on the dual active set method of Goldfarb and Idnani (1982, 1983) [20] [21] to solve the quadratic objective functions with linear constraints.

In the quadratic solver, first, the quadratic objective function $F(x)$, which is a convex function, is considerd of the form:

$$F(x) = \frac{1}{2}x^T Hx - d^T x$$

Note that x is a vector of β in $\mathbb{R}^n$, and x^T denotes the transpose of x. The solver has no explicit knowledge of x itself because it is the internal variable being optimized, and everything else is defined by the supplied parameters. H is the $n \times n$ positive definite matrix (observed information matrix) and must be positive definite for the problem to have a finite minimum. d is a constant vector in $\mathbb{R}^n$.

As imposed earlier in this chapter, a new linear inequality constraints solution on the vector $x \in \mathbb{R}^n$ is generated and written in the form $A^T x \leq b$. However, the quadratic programming formulation of the constraints must be transformed to match $A^T x \geq b$. For this reason, the formulation was manipulated to conform to the above form and rewritten as:

$$-A^T x \leq -b$$

Then, the quadratic problem for solving compactly is expressed as follows:

$$\begin{cases} \min_{x \in \mathbb{R}^n} : & F(x) = \frac{1}{2}x^T Hx - d^T x \\ \text{subject to} : & -Ax \leq -b \end{cases}$$

3.2.6 Iterations and the termination criteria

The log-likelihood $l(\beta)$ with negative observed Fisher information is convex, which means that it has no more than one minimum, even in infinite-dimensional spaces. Therefore, the quadratic approximation to $l(\beta)$ will as a result be well-understood and interpreted.

F is convex and differentiable, therefore each stationary point $\hat{x}$ is a global minimum.

Algorithm squadP

1. Start with the optimized initial guess (feasible x_0)
2. Approximate $F(x)$ by its second order Taylor expansion
3. Solve the sub-quadratic problem obtained from the second-order Taylor approximation under the linear inequality restrictions $-A^T x \leq -b$ by the quadratic programming solver.
4. If $||x_{r+1} - x_r|| \leq eps$ stop.
   Set $\hat{x} = x_{r+1}$.
   Else set $r = r + 1$ and go to step 2.

Step 4 in the algorithm above is used to repeat the process until the differences between successive estimates are close to zero with an adequate degree, therefore obtaining an improved estimate. squadP method generally tends to converge quickly after small number of iterations, particularly, if the log-likelihood is close to quadratic and in a neighborhood of the maximum, or if the optimized initial values are reasonably close to the MLE. Furthermore, the slandered error of the estimates and the confidence interval are calculated by squadP algorithm.

Visualization of the iterative procedure

In order to visualize the iterative procedure of squadP till convergence, a simulated example is generated. Furthermore, the log-likelihood function of the log-binomial model is visualized using contour plot. The simulated dataset is conducted using the pseudo-random number generator RNGkind of L'Ecuyer (1999) [28] that has a single uniform random variable X (independent variable) and a response binary variable Y with an incidence rate of 21.8% as shown in table 2. The true values are formerly predefined during the data generating process equal to: $\beta_0 \approx -1.5$, and $\beta_1 \approx 0.2$.

y (response)	x (predictor)
0:391	Min. :0.0035
1:109(21.8%)	SD :0.2902
	Mean :0.4928
	Max. :0.9950

Table 2: Simulated example with a single predictor variable and a binary response variable.

squadP estimates β_0 and β_2 by an iterative procedure till convergence if the criteria are matched. The results are then matched to the predefined true values of β_0 and β_2.

Certainly, if the iterative method continues to iterate resulting in no solution, then convergence is not declared. In this case, the method reaches the maximum number of iterations defined even though the convergence criteria are still not matched.

The log-likelihood function of the log-binomial model, as shown in figure 2, is represented on a contour plot of a three-dimensional surface, $\beta_0, \beta_1, -logL(\beta)$ on a two-dimensional plane.

$$l(\beta) = \sum_i y_i \; log(p_i(\beta)) + \sum_i (1 - y_i) \; log(1 - p_i(\beta))$$

[37]. The negative $l(\beta_0, \beta_1)$ is maximized using the iterative algorithm squadP as explained later. Furthermore, the iterative steps starting from the initial value till convergence are presented in figure 3.

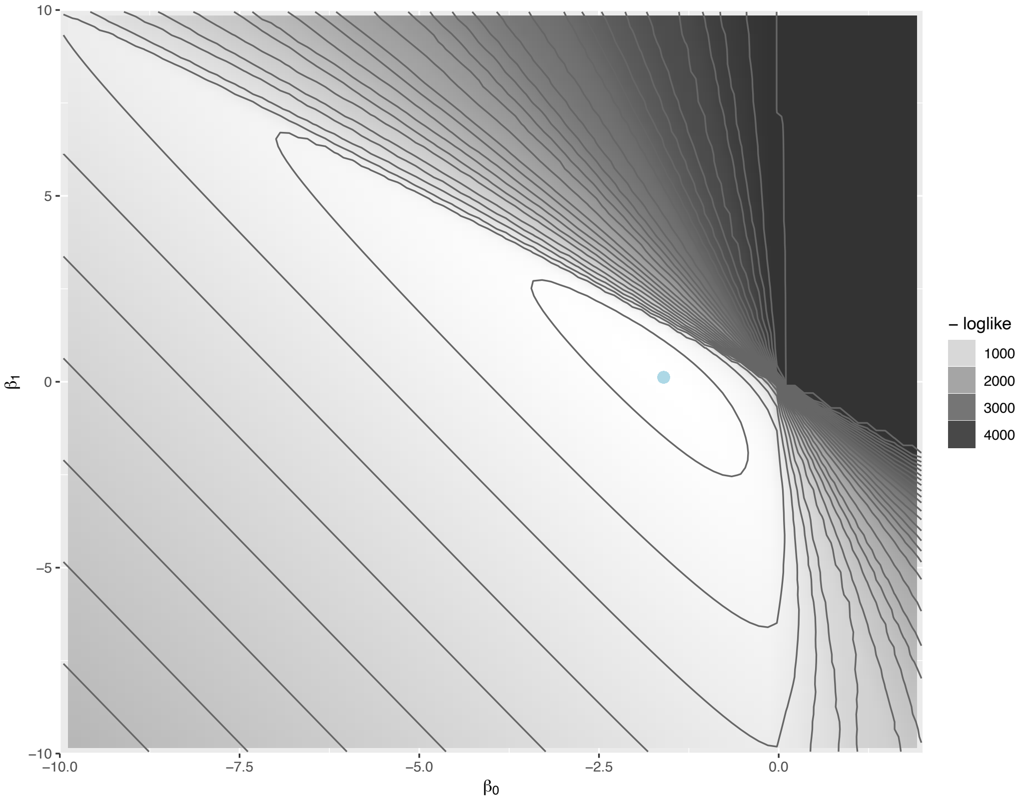

Figure 2: Countourplot of the negative log-likelihood function of the log-Binomial model. It is a representation of the example in table 2 with a single independent variable. Lines in the plot presents the negative log-likelihood function. Nearer the line to the point in the center, closer to the minimum.

In the example shown in table 2, the objective function is $l(\beta_0, \beta_1)$, and the aim is to minimize this function over the restricted region with two constraints $2^r = 2^1 = 2$:

$$x_{min} = 0.0035: \qquad -\infty \leq \beta_0 + \beta_1 * 0.0035 \leq 0$$

$$x_{max} = 0.9950: \qquad -\infty \leq \beta_0 + \beta_1 * 0.9950 \leq 0$$

To solve this quadratic problem under the two linear inequality constraints, a transformation of both the objective function $l(\beta_0, \beta_1)$ and the constraints into the form $min(-d^T b + \frac{1}{2} b^T D b)$ subjected to $-A^T x \leq -b$ that is required by squadP as explained earlier in greater detail.

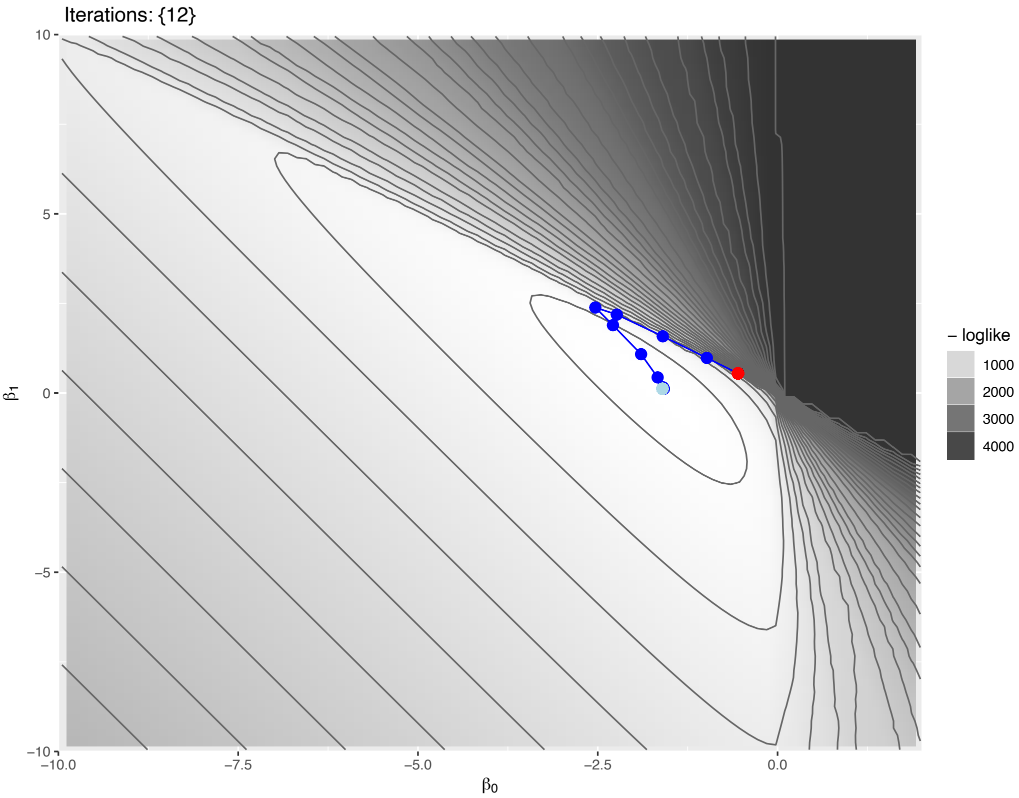

Figure 3: Visualization of the number of iterations of squadP algorithm. The iterative process from the starting point untill convergence contains 12 iterations. The point in the center of the oval shape is the solution and the line with data points represents the number of iterations.

In figure 3, the optimized initial value started near the solution, which is in the center of the oval shape, and it required 12 iterations until the squadP reached the minimum. Each point on the line represents one iteration and the line represents the direction towards the solution.

4 Monte Carlo simulation

It is very common that the performance of a statistical method varies when number of events, sample size, or number of covariates varies. Here comes the power of Monte Carlo simulation, which uses repeated sampling from a probability distribution for each variable to produce thousands of possible outcomes [7]. Therefore, results of Monte Carlo simulation, statistical properties of different outcomes occurring, are used for further examination and comparison of the methods being studied [38].

In this section, a structured approach (protocol) for planning and reporting Monte Carlo simulation study is presented, which is giving us the ability to study, analyse, and understand the behavior of the underlying statistical methods, because of the parameters of interest (true values) are already predefined and known from the data-generating process. therefore the estimates of different methods can be compared with those true values.

4.1 Objectives

The primary objective for conducting this larg Monte Carlo simulation was to evaluate the properties of the six statistical methods being studied (EM-type algorithm, Fisher scoring algorithm, Poisson , BFGS, Nelder-Mead, and squadP). These six estimators were examined to analyse the probable behavior of each estimator and which one performs best with regard to estimating the correct risk ratio and standard confidence interval.

The statistical methods were compared in terms of convergence rate, coverage probability, bias, mean squared error (MSE), and empirical standard error (EmpSE). Designing and conducting the Monte Carlo simulation study was inspired by the articles of Morris et al. (2019) and Burton et al. (2006) [38] [7].

Procedures, execution, analysis, and reporting of the Monte Carlo simulation are described in the following points.

4.2 Simulation procedures

4.2.1 Dependencies between the simulated data sets

In total, 108 different scenarios were designed and executed, and 1000 simulated independent datasets for each scenario were carried out using Monte Carlo method. Each simulated dataset was analysed and used for evaluating the six statistical methods being studied. In other words, each generated dataset was used by the six underlying statistical methods to compare and study their probable behavior and performance.

4.2.2 Allowness of failure and error

There are two types of failures that can occur during the data generation process, which are explained in the following two points.

1. The required samples generated by the random number generator during the data generating process were tested for errors to be certain that desired samples are generated, otherwise, the whole process is repeated.

2. During the process of generating the outcome variable (also called the response variable) with a prespecified event rate, particularly in processes with too low event probabilities, It could happen that the event rate is 0, therefore such datasets were omitted and not used for evaluating the statistical methods.

4.2.3 Software used for implementation

The complete simulation study including random number generation were conducted using the statistical software R, version 3.5.2 (Eggshell Igloo) on Darwin 15.6.0 (x86-64). R packages used in the study were `copula` for multivariate dependence modeling, and `rsimsum` for the analysis of simulation study including Monte Carlo error.

4.2.4 The random number generator used

The simulated datasets in the study were generated by pseudo-random sampling for each scenario using a pseudo-random generator (pseudo-RNG) of L'Ecuyer [28] [29].

This pseudo-RNG being used has certain characteristics and produces values that pass tests for randomness. It has an ability of multiple streams for parallel processing, and it can generate thousands of different independent datasets using a thousand different starting seeds for each scenario for ensuring accurate results. Seeds were generated in order to give the random number generator the ability to reproduce the identical set of random numbers when the same seed is specified [35]. It is an essential process when performing the simulation study to be able to regenerate the same datasets and hence results to be reproduced and monitored.

4.2.5 Multi-core computing

Multi-core computing was done using the local Linux server with 12 cores for the purpose of running multiple threads simultaneously for reducing the computational time required and fasten data generating process, analysis, and the visual representation of the output.

In the data generating process, a parallel function was created using "Parallel-package" in R to allow multi-core and carry out multiple calculations and execution of processes at the same time including random-number generation for multiple RNG streams with the "L'Ecuyer-CMRG" [28] [29].

4.3 Methods for generating simulated data sets.

4.3.1 Covariate correlation structure

In practice, multivariate data is presumed to have a certain degree of correlations between covariates. Therefore, an additional specification was added in the data generating process to be used as a diagnostic and input into a more advanced analysis, which is the correlations matrix. Correlation matrix includes the parameters which specify the correlation coefficients between each pair of random variables. The correlation degree between each pair of variables could be positive or negative or equal to zero.

There are in total 12 random variables $X_1, X_2, ..., X_{12}$ in the simulation study. As shown in Figure 4 the matrix describes the correlation between 12 underlying variables. For instance, between the random variable X_1 and X_2 the specified correlation coefficient is equal to positive 0.8. All given values specify the corresponding correlation coefficients are shown in figure 5.

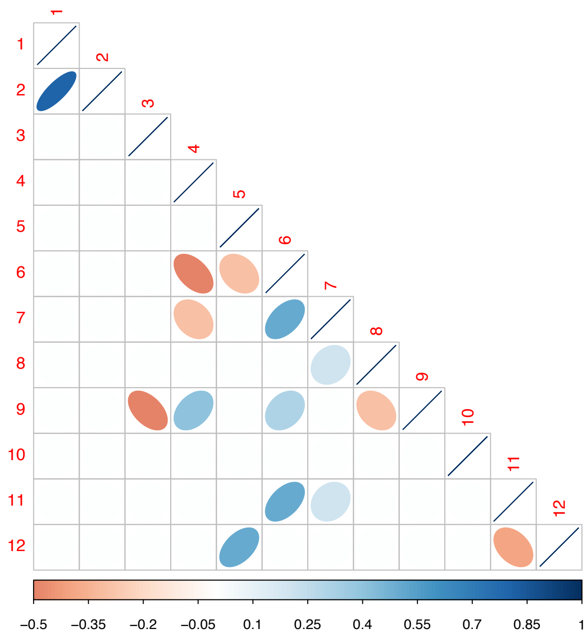

Figure 4: Correlation matrix of the 12 random variables. Shapes on the graph represent the degree of correlation between two variables. Blue shapes representing a positive correlation and the orange shapes represent a negative correlation

In figure 4, the correlation matrix of all variables used in the simulation study is shown. Vertically are the 12 random variables listed and the horizontal color scale from orange to dark blue presents the parameters that specify the degree of correlation between each pair of these random variables.

4.3.2 Probability distribution of the random variables

Data can be distributed in different ways. In Monte Carlo simulation that was created to perform quantitative analysis in this study, uncertain inputs were presented using possible range of values which is recognized as the probability distributions. These types of probability distributions were implemented in the study to yield different probabilities of different outcomes occurring. For describing uncertainty in variables, this is more realistic way [42]. the following types of distributions were used.

- **Normal distribution**
  Sometimes called "bell curve," is one type of probability distributions that occurs in many circumstances naturally. Bell curve is symmetrical, therefore the half of the data Falls on the left side of the mean μ of the distribution, and the other half falls to the right [15] [5]. The parameter σ is the scale parameter and also know as the standard deviation of the normal distribution and σ^2 is its variance. Notation of normal distribution $N(\mu, \sigma^2)$ with specified μ and σ^2 as shown in figure 5.

- **Binomial distribution**
  Binomial distribution is another probability distribution type that has two possible outcomes (probability of a success or failure in an experiment that is repeated multiple times). Parameter $n \in \{0, 1, 2, \ldots\}$ in a binomial distribution is the number of trials in the binomial experiment with probability of success on each individual trial $p \in [0, 1]$ and probability of failure $1 - p$. Notation of binomial distribution $B(n, p)$ with specified probability p [15].

- **Poisson distribution**
  Poisson distribution is a general statistical distribution type that is defined as "the probability of acquiring exactly n successes in N trials which is given by the limit of a binomial distribution" [43]. Variables with Poisson distribution $P(\lambda)$ are shown in figure 5 with specified value for lambda λ which is a rate parameter.

- **Gamma distribution**
  Is defined as general type of probability distributions that is skewed and used for modeling continuous variables that are always positive. This distribution type has two free parameters, labeled, alpha, and theta [25]. The notation of *Gamma* probability distribution is expressed as $\Gamma(\alpha, \beta)$. α is a shape parameter and β is an inverse scale parameter which is > 0.

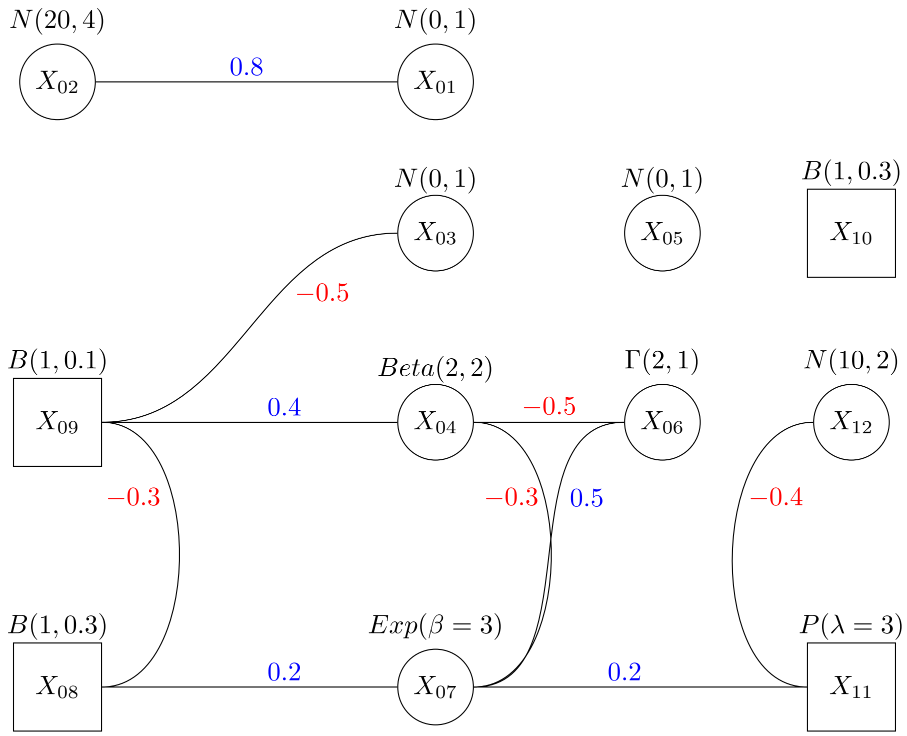

Figure 5: The random variables being used in the study $X_1, X_2, ..., X_{12}$ with predefined probability distribution and corresponding correlation coefficients. Each circle or square represents a single random variable with a predefined probability distribution. Circles indicate that the variable is continuous and squares are for the count or binary variables. Connections between the random variables indicate the predefined underlying stochastic dependency structure, where the given values specify the corresponding correlation coefficients. Positive values indicate positive association and negative values presents a negative association between variables.

- **Beta distribution**
  Is general type of continuous probability distributions which is related to gamma distribution and is a versatile way to represent outcomes for percentages or proportions [5]. $Beta(\alpha, \beta)$ is the notation of *Beta* probability distribution. α and β are the shape parameters which are > 0.

- **Exponential distribution**
  Is defined as a continuous probability distribution which is used for modeling time elapsed before a given event occurs. [55]. $Exp(\lambda)$ is the notation of the probability distribution and λ is an inverse scale parameter > 0.

4.3.3 Multivariate dependencies modeling

Generating random samples from a multivariate normal distribution in R are smooth processes. However, it is not so easy to be done without using "copula." Copula as a function was used to model the correlation structure and generate the desired multivariate distribution for the random variables.

Suppose we have the random vector $X_1, X_2, \ldots, X_{12}$, and its marginals are continuous functions $F_i(x) = \Pr[X_i \leq x]$, then the copula of this random vector can be described as the joint cumulative distribution function of $(U_1, U_2, \ldots, U_{12})$.

$$C(u_1, u_2, \ldots, u_{12}) = \Pr[U_1 \leq u_1, U_2 \leq u_2, \ldots, U_{12} \leq u_{12}].$$

On the right side of the formula F_i is "the marginal cumulative distribution functions, which contain all information on the marginal distributions" [32], whereas C is the copula, which contains all information on the correlation structure between the variables [32].

Sklar's theorem [49] states that for a random vector $(X_1, X_2, \ldots, X_d)$, every multivariate cumulative distribution function can be presented with regard to its marginals $F_i(x_i) = \Pr[X_i \leq x_i]$ and a copula C

$$H(x_1, \ldots, x_d) = C\left(F_1(x_1), \ldots, F_d(x_d)\right).$$

Gaussian copula was used which is derived from the multivariate Gaussian distribution, and there are two parameters that were specified to go through copula.

- The marginals, which are the distribution of each of the underlying random variables as explained in the last point
- The correlation structure, which is the matrix of correlation coefficients used to determine the relationship between the random variables.

In this simulation study modeling the correlation structure and the distribution for each of the 12 underlying random variables were carried out. Using `copula` package, the random samples with the specified distribution and correlations were generated.

4.3.4 Generating the outcome variable

After the process of generating the random variables from marginal probability distributions was carried out using copula, the outcome variable is generated. The outcome variable (has binary values (0 or 1)), also called dependent or response variable, was generated randomly from a binomial probability distribution for each dataset. In order to pre-specify an individual probabilities for the outcome variable Y, the linear combinations of parameters

$\beta_0, \beta_1, \ldots, \beta_m$ were predefined in log-binomial model. Therefore, the exponential of the output is the probability that is used to generate the random outcome variable Y from a binomial probability distribution.

In table 3, we see for instance the scenarios from 1 to 45 have 2 covariates with varieties of event probability and sample size. In this case, β_0, β_1, and β_2 are predefined to get the probability that is used to generate the outcome variable Y.

Scenarios	Cov.	Event	β_0	β_1	β_2	β_3	β_4	β_5	β_6	β_7	β_8
1→6	2	3%	-3	-0.30	-0.02	x	x	x	x	x	x
7→12	2	6%	-2.4	-0.30	-0.02	x	x	x	x	x	x
13→18	2	12%	-1.5	-0.2	-0.03	x	x	x	x	x	x
19→24	2	24%	-0.5	-0.08	-0.05	x	x	x	x	x	x
25→30	2	48%	-0.35	-0.05	-0.02	x	x	x	x	x	x
31→36	4	6%	-2	0.04	-0.08	0	-0.03	x	x	x	x
37→42	4	12%	-1.5	0.02	-0.05	0.03	-0.01	x	x	x	x
43→48	4	24%	-1.7	-0.03	0.02	-0.06	0.05	x	x	x	x
49→54	4	48%	-0.9	-0.02	0.01	-0.03	0.04	x	x	x	x
55→57	8	12%	-2.1	-0.2	-0.15	0.05	0	0.08	-0.15	0	0
58→60	8	24%	-1.4	-0.1	-0.05	0	0	0.1	-0.03	0	0
61→63	8	48%	-0.85	-0.05	0.07	0	0	0.2	-0.03	0	0
64	8	12%	-1.5	-0.1	-0.1	-0.06	-0.1	-0.2	-0.1	-0.2	-0.1
65	8	24%	-1	-0.05	-0.06	-0.01	-0.04	-0.06	-0.03	-0.01	-0.02
66	8	48%	-0.5	-0.02	-0.03	-0.01	-0.05	-0.04	-0.03	-0.01	-0.02

Table 3: The table shows the predefined true values for each scenario. the first column in the table represents number and order of scenarios. Column "Cov." represents the number of covariates (variables), and "Event" is the incidence rate in each scenario. Betas from β_0 to β_8 are the coefficient for each scenario (predefined true values). Scenarios 64, 65, and 66 are for sample size one million with eight covariates and 12%, 24%, and 48% incidence rate.

4.4 Scenarios under correct model specifications

Monte Carlo simulation is conducted using varying sample sizes, number of events, and number of covariates which are called scenarios. There are in total 108 scenarios classified into tow categories, First category includes 66 scenarios under correct model specifications, and the second category includes 42 scenarios under model misspecifications which is explained later in the next section.

Scenario	Cov.	Event	N_1	N_2	N_3	N_4	N_5	N_6
1→6	2	3%	60	80	100	500	1000	50000
7→12	2	6%	60	80	100	500	1000	50000
13→18	2	12%	60	80	100	500	1000	50000
19→24	2	24%	60	80	100	500	1000	50000
25→30	2	48%	60	80	100	500	1000	50000
31→36	4	6%	250	300	400	500	1000	10000
37→42	4	12%	250	300	400	500	1000	10000
43→48	4	24%	250	300	400	500	1000	10000
49→54	4	48%	250	300	400	500	1000	10000
55→57	8	12%	500	1000	10000	x	x	x
58→60	8	24%	500	1000	10000	x	x	x
61→63	8	48%	500	1000	10000	x	x	x
64	8	12%	1000000	x	x	x	x	x
65	8	24%	1000000	x	x	x	x	x
66	8	48%	1000000	x	x	x	x	x

Table 4: The table represents scenarios from 1 to 66 with variety of covariates, incidence rates, and sample sizes. Scenarios from 1 to 30 are examined with 2 covariates and variety of incidence rates (3%, 6%, 9%, 12%, 24%, and 48%), and sample sizes as shown above. Scenarios 64, 65, and 65 are with sample size one million.

As specified in table 4 the scenarios with correct model specification are shown. The first column in the table is showing the number of scenarios. In the first row presents the first six scenarios (1-6) with 2 covariates and 3% incidence rate. Columns from N_1 to N_6 are presenting the sample sizes for the first six scenarios in ascending order from left to right.

There are three subcategories of scenarios based on the number of covariates we see in table 4, scenarios with 2 covariates, scenarios with 4 covariates, and scenarios with 8 covariates.

4.4.1 Scenarios from 1 to 30 with 2 covariates

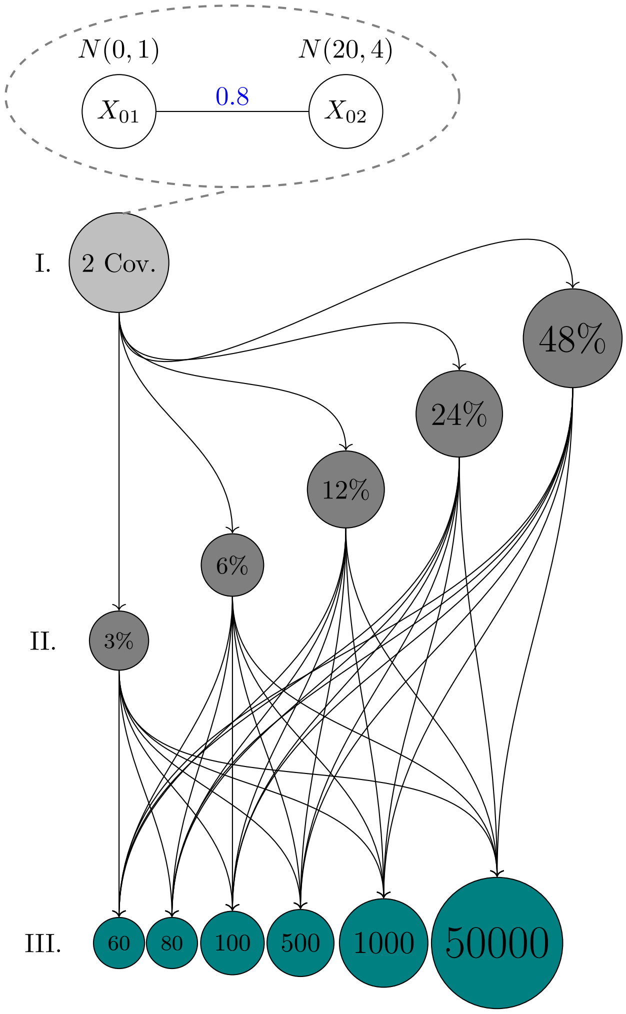

Figure 6: The figure represents 30 scenarios (from 1 to 30) with two normally distributed covariates and variety of incidence rates and sample sizes. I. The dashed oval shape contains variables X_1 and X_2 with positive association 0.8. II. Variety of indecent rates connected with variety of sample sizes as shown in "III".

As shown in figure 6, the dashed oval shape contains two covariates, X_{01} and X_{02} are random variables normally distributed with mean μ and σ^2.

$$X_{01} \sim N(\mu = 0, \sigma^2 = 1)$$

$$X_{02} \sim N(\mu = 20, \sigma^2 = 4)$$

The relationship between the two covariates is positively correlated 0.8, in which one variable increases as the other increases as well. A perfect positive correlation is represented by the value +1, a 0 indicates no correlation, and a −1 indicates a perfect negative correlation between, which shows us that the two covariates are positively strongly correlated and collinear (a high linear association between the two explanatory variables).

$$corr(X_{01}, X_{02}) = 0.8$$

In figure 6 "II" the scenarios have variety of incidence rates 3%, 6%, 12%, 24%, and 48%. Each incidence rate is involved in six scenarios with six different sample sizes. "III" shows six different sample sizes for each incidence rate.

4.4.2 Scenarios from 31 to 54 with 4 covariates

In figure 7, we see four covariates in the dashed oval shape X_{05}, X_{06}, X_{10}, and X_{12} with no connection between any of the mentioned variables which indicates no correlation between the covariates. The random variables X_{05} and X_{12} are normally distributed with mean and variance values as shown in figure 7.

$$X_{05} \sim N(\mu = 0, \sigma^2 = 1)$$

$$X_{12} \sim N(\mu = 10, \sigma^2 = 2)$$

The random variable X_{06} has a gamma distribution that is parameterized in terms of a shape parameter $\alpha = 2$ and an rate parameter $\beta = 1$ (also known as an inverse scale parameter).

$$X_{06} \sim \Gamma(\alpha = 2, \beta = 1)$$

$$X_{10} \sim B(n = 1, p = 0.3)$$

X_{10} is a Bernoulli distributed random variable with probability $p = 0.3$.

As shown in figure 7 "II" the scenarios have variety of incidence rates 6%, 12%, 24%, and 48%. Each incidence rate is involved in six scenarios with six different sample sizes. "III" shows six different sample sizes for each incidence rate.

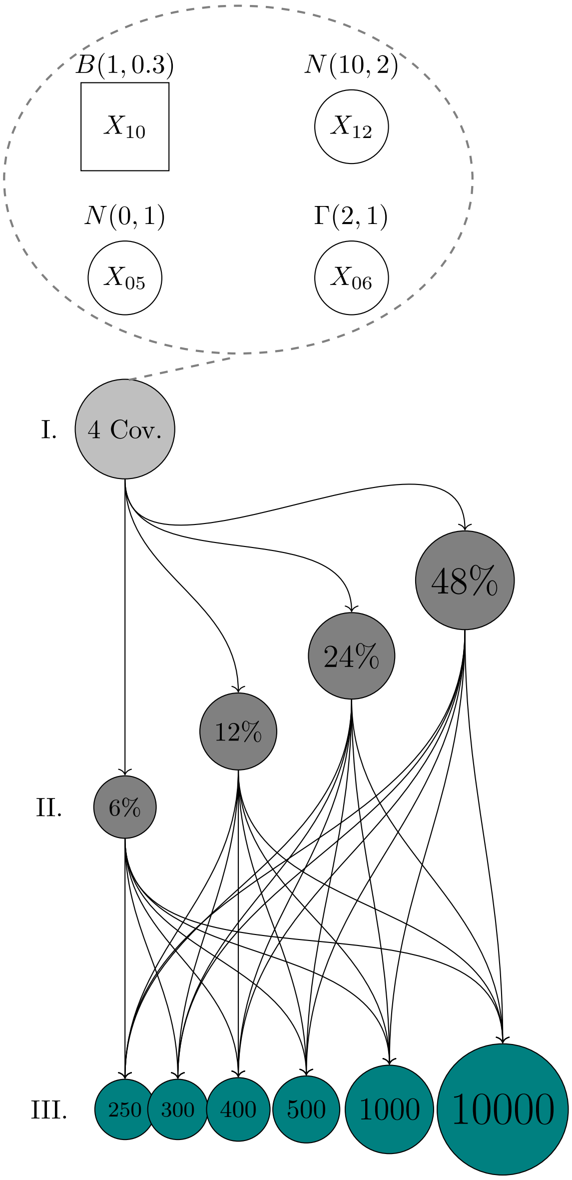

Figure 7: Scenarios from 31 to 54 with four covariates (no correlation between variables) which have variety of distributions, incidence rates and sample sizes

4.4.3 Scenarios from 55 to 66 with 8 covariates

In figure 8, we see eight random variables in the dashed oval shape with variety of positive and negative correlations between them.

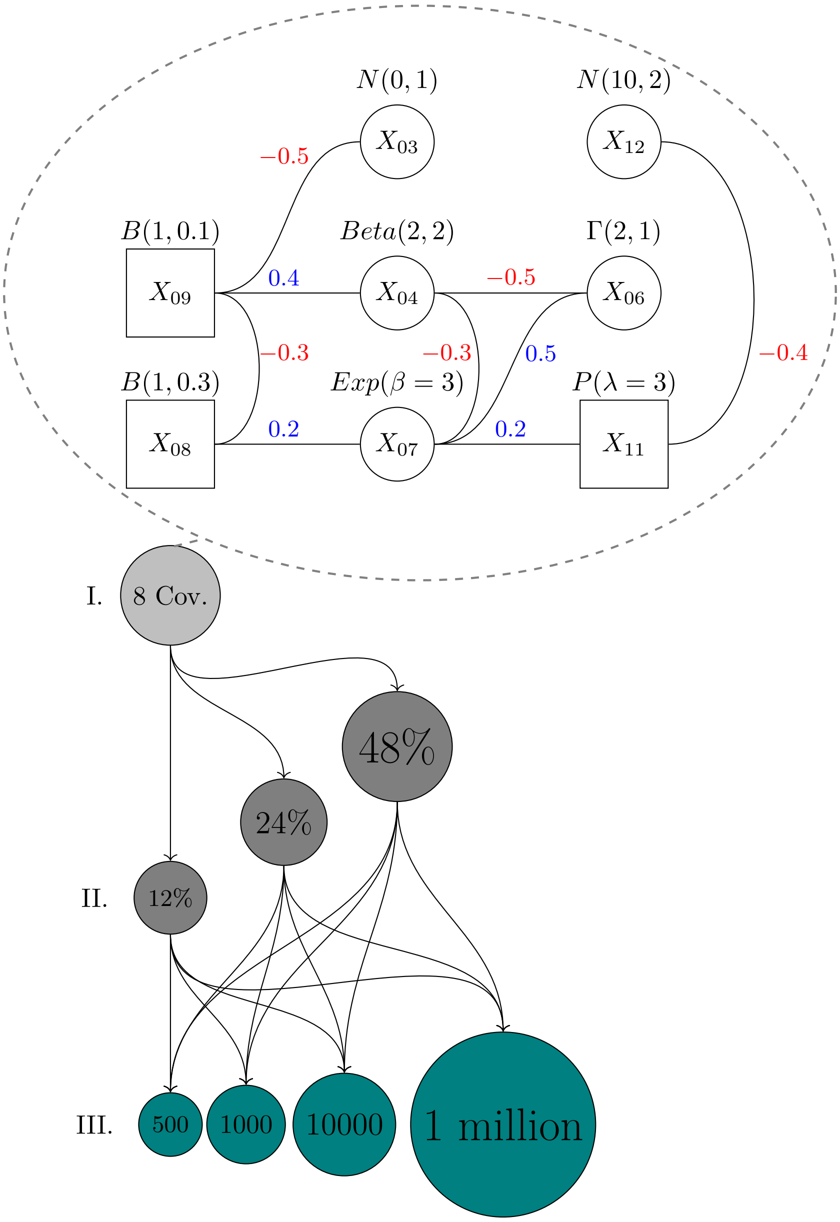

Figure 8: Scenarios with 8 covariates which have variate of probability distribution as shown in "I". "II" represents different incidence rate that are connected to variety of sample sizes as shown in "III"

The probability distribution of each variable in figure 8 is described as follows.

$$X_{03} \sim N(\mu = 0, \sigma^2 = 1)$$

$$X_{12} \sim N(\mu = 10, \sigma^2 = 2)$$

The random variables X_{03} and X_{12} are normally distributed with mean and variance parameters as shown above.

$$X_{04} \sim Beta(\alpha = 2, \beta = 2)$$

$$X_{06} \sim \Gamma(\alpha = 2, \beta = 1)$$

X_{04} and X_{06} are random variables following Beta and Gamma distributions with the shape and scale parameters as described.

$$X_{08} \sim B(n = 1, p = 0.3)$$

$$X_{09} \sim B(n = 1, p = 0.1)$$

As we see the random variables X_{08} and X_{09} are Bernoulli distributed with probabilities of 0.3 and 0.1.

$$X_{07} \sim Exp(\beta = 3)$$

$$X_{11} \sim P(\lambda = 3)$$

X_{07} is a random variable follows the exponential distribution with an inverse scale parameter of 3, and X_{11} is Poisson distributed with rate parameter of 3.

As shown in figure 8 "II" the scenarios have variety of incidence rates 12%, 24%, and 48%. Each incidence rate is involved in six scenarios with four different sample sizes. "III" shows four different sample sizes for each incidence rate.

We see in figure 8, the last three scenarios were executed with a large sample size (1 million). The aim was to study the behavior and the computational cost for the underlying statistical methods.

4.5 Scenarios under model misspecification

The underlying regression methods being studied require a particular functional form for identifying the relationship between the response variable and the explanatory variable.

Adding non-linear components or interaction terms to the model can cause model misspecification. In addition, if the true data generating process is subject to non-linear effects and the statistical model does not account for it, this can lead to model misspecification and thus biased estimates of model parameters [10]. Regression methods in scenarios from 1 to 66 in the former section were shown to yield comparable point estimates and standard errors under correct model specifications. In this section, scenarios from 67 to 108 are shown under model misspecifications to study the uncertainty and robustness of each regression method under model misspecification.

In order to specify the required event rate for the outcome variable, the linear combinations of parameters were predefined in log-binomial model as shown in table 5 for scenarios from 67 to 108.

Scenarios	Cov.	Event	β_0	β_1	β_2	β_3	β_4	β_5	β_6	β_7	β_8
67→72	2	3%	-3	-0.2	-0.02	x	x	x	x	x	x
73→78	2	6%	-2.5	-0.15	-0.01	x	x	x	x	x	x
79→84	2	12%	-2	-0.1	-0.02	x	x	x	x	x	x
85→90	2	24%	-1.1	-0.2	-0.01	x	x	x	x	x	x
91→96	2	48%	-0.6	-0.06	-0.01	x	x	x	x	x	x
97→99	8	12%	-2.1	-0.2	-0.15	0.05	0	0.08	-0.15	0	0
100→102	8	24%	-1.4	-0.1	-0.05	0	0	0.1	-0.03	0	0
103→105	8	48%	-0.85	-0.05	0.07	0	0	0.2	-0.03	0	0
106	8	12%	-1.5	-0.1	-0.1	-0.06	-0.1	-0.2	-0.1	-0.2	-0.1
107	8	24%	-1	-0.05	-0.06	-0.01	-0.04	-0.06	-0.03	-0.01	-0.02
108	8	48%	-0.5	-0.02	-0.03	-0.01	-0.05	-0.04	-0.03	-0.01	-0.02

Table 5: The coefficients in the table shows the predefined true values for each scenario

As we see in table 5, β_0, β_1, and β_2 are pre-specified for scenarios from 67 to scenarios 96, and β_0 to β_8 are pre-specified for scenarios from 97 to scenarios 108.

In table 6, all scenarios under model misspecifications are shown. Scenarios with 2 and 8 covariates with varieties of incidence rate and sample sizes.

Scenario	Covar	Event	N_1	N_2	N_3	N_4	N_5	N_6
67→72	2	3%	60	80	100	500	1000	10000
73→78	2	6%	60	80	100	500	1000	10000
79→84	2	12%	60	80	100	500	1000	10000
85→90	2	24%	60	80	100	500	1000	10000
91→96	2	48%	60	80	100	500	1000	10000
97→99	8	12%	500	1000	10000	x	x	x
100→102	8	24%	500	1000	10000	x	x	x
103→105	8	48%	500	1000	10000	x	x	x
106	8	12%	1000000	x	x	x	x	x
107	8	24%	1000000	x	x	x	x	x
108	8	48%	1000000	x	x	x	x	x

Table 6: Scenarios from 67 to 108 with variety of covariates, incidence rates, and sample sizes under model misspecificatins. Scenarios from 67 to 96 are examined with 2 covariates and variety of incidence rates (3%, 6%, 9%, 12%, 24%, and 48%), and sample sizes as shown above. Scenarios 97 to 105, are with 8 covariates, variety of incidence rates and sample sizes. Scenarios 106, 107, and 108 are for sample size one million.

4.5.1 Scenarios from 67 to 96 with 2 covariates

As we see table 6, the scenarios from 67 to 96 have only 2 covariates with varieties of incidence rate and sample size. X_{01} and X_{02} in this model are random normally distributed covariates which follow the same specifications (distribution and correlation between the two variables, figure 6) as described in scenarios from 1 to 45 with minor difference which is the model misspecification.

After the data generating process of the two random variables from marginal probability distributions was carried out, the model is required to be linearly specified to generate the outcome variable. However, during the process of predefining the linear combinations the model was non-linearly specified, then the outcome variable was generated randomly from a binomial probability distribution for each dataset. The probability of the outcome variable in this case is calculated as shown in the following equation with the predefined true values in table 5.

$$y = \beta_0 + \beta_1 * ln(x_1) + \beta_2 * exp(x_2)$$

4.5.2 Scenarios from 97 to 108 with 8 covariates

Table 6 shows the scenarios from 97 to 108 as well, which have 8 covariates with varieties of incidence rate and sample size. $X_{01}, X_{02}, ..., X_{08}$ follow the same distribution specifications (distribution and correlation between variables, figure 8) as described before with minor difference which is the model misspecification.

$$y = \beta_0 + \beta_1 x_{01} + \beta_2 x_{02} + ... + \beta_8 x_{08} + \beta_9 (x_{02} * x_{03}) + \beta_{10} (x_{07} * x_{08})$$

In this case, the model is misspecified with adding interaction terms between variables. Interaction between X_{01} and X_{02}, and between variable X_{07} and X_{08} as shown in the formula.

4.6 Statistical methods to be evaluated and compared

In total, six different statistical methods were used in the simulation study to be evaluated and compared, including the novel approach presented "squadP."

1. Fisher based log-binomial regression model
2. Poisson regression model which is a form of generalized linear model for regression analysis used in this study to estimate the relative risk.
3. EM-type based model which is implemented in R in `logbin` package.
4. Nelde-Mead based algorithm. The heuristic search technique is used for minimization and is implemented in R as the default choice for the package `optim`.
5. BFGS (Broyden–Fletcher–Goldfarb–Shanno) quasi-Newton method that solves optimization problems iteratively, and is also implemented in R package `optim`.
6. squadP is the novel modified Newton-type approach presented in this study, which is the constrained optimization of log-binomial model and explained in the previous chapter.

4.7 Storing estimates for each simulation

Each simulated dataset was analysed by the underlying statistical models. Models coefficients and the standard errors for each simulation were calculated. The estimates, SE, event probability, sample size, and the statistical method used of each simulation were structured and stored as separate objects to be able to perform a consistency check, identify any errors or outlying values, explore any patterns within the individual simulations, and calculate summary measures overall simulation as well.

4.8 Number of simulations

In total, 108 scenarios were carried out. The simulation process was repeated 1000 times for each simulated scenario. The log risk ratio for each simulated dataset in the 1000 simulations was estimated from the six statistical methods being studied. Only the scenarios with one million as sample size (6 scenarios) were repeated 350 times for each, alternatively, the simulation process with such a large sample size could be computationally expensive.

Coverage probability, bias, mean squared error, empirical standard error, and the convergence rate were calculated for each scenario by summarizing the results from the 1000 datasets for each regression model. Furthermore, the Monte Carlo standard error was calculated for each of the measurements.

In this large Monte Carlo simulation study, **104100** different datasets in total were generated, processed, analysed, and summarized.

4.9 Evaluation criteria for the performance of statistical methods for different scenarios

After storing the estimates for each simulation and summary measures were calculated, the criteria for evaluating the performance of the statistical approaches being studied were considered [7]. The true value used in the data generating process was essential to define the criteria and compare it with the simulated results of the underlying statistical approaches. What often used as performance measures is the assessment of bias, coverage, and accuracy [11] [7].

The performance measures that were used are as following:

Assessment of bias: “Bias quantifies whether the estimator targets the true value θ on average” [38] [56], and it was computed as shown in the following equation.

$$\text{Bias} = \frac{1}{n_{\text{sim}}} \sum_{i=1}^{n_{\text{sim}}} \hat{\theta}_i - \theta \tag{10}$$

- θ is the true value
- $\hat{\theta}_i$ is the estimated value of θ for the i^{th} replication
- n_{sim} is number of simulations

The Monte Carlo standard error of bias was computed using the following equation:

$$\text{MCSE(Bias)} = \sqrt{\frac{\frac{1}{n_{\text{sim}}-1}\sum_{i=1}^{n_{\text{sim}}}(\hat{\theta}_i - \bar{\theta})^2}{n_{\text{sim}}}} \tag{11}$$

Assessment of coverage: "Coverage is the probability that a confidence interval contains the true value θ" [38] [56], and it was computed using the following equation:

$$\text{Coverage} = \frac{1}{n_{\text{sim}}} \sum_{i=1}^{n_{\text{sim}}} I(\hat{\theta}_{i,\text{low}} \leq \theta \leq \hat{\theta}_{i,\text{upp}}) \tag{12}$$

$I(\cdot)$ is the indicator function, and the Monte Carlo standard error of coverage was calculated as following:

$$\text{MCSE(Coverage)} = \sqrt{\frac{\text{Coverage} \times (1 - \text{Coverage})}{n_{\text{sim}}}} \tag{13}$$

Mean Squared Error (MSE):, "MSE is the sum of the squared bias and variance of $\hat{\theta}$" [38]. MSE takes into account both accuracy and precision of each statistical approach being studied and was computed as shown in the following equation:

$$\text{MSE} = \frac{1}{n_{\text{sim}}} = \sum_{i=1}^{n_{\text{sim}}} (\hat{\theta}_i - \theta)^2 \tag{14}$$

Here we see how the Monte Carlo standard error of MSE was computed:

$$\text{MCSE(MSE)} = \sqrt{\frac{\sum_{i=1}^{n_{\text{sim}}} \left[(\hat{\theta}_i - \theta)^2 - \text{MSE}\right]^2}{n_{\text{sim}}(n_{\text{sim}} - 1)}} \tag{15}$$

Empirical Standard Error (EmpSE) EmpSE estimates the standard deviation of $\hat{\theta}$ over the n_{sim} replications [38] [56], as we see in the following equation:

$$\text{Empirical SE} = \sqrt{\frac{1}{n_{\text{sim}} - 1} \sum_{i=1}^{n_{\text{sim}}} (\hat{\theta}_i - \bar{\theta})^2} \tag{16}$$

One key difference between empSE and MSE is that empSE of θ depends only on $\hat{\theta}$ and does not require any knowledge of θ. The Monte Carlo standard error of empSE is computed here.

$$\text{MCSE(Emp. SE)} = \frac{\widehat{\text{Emp. SE}}}{\sqrt{2(n_{\text{sim}} - 1)}} \tag{17}$$

Sampling variance and the amount of bias are two important components of sampling error therefore for judging the performance of the statistical methods being studied. Sampling variance is crucial because we have simulated samples in this study rather than the real data of the entire population, but some argue that bias is more crucial than sampling variance [30]. This means that if the estimands are relatively unbiased with narrow confidence intervals, that implies more accuracy, efficiency, and power [9]. However, methods that result a biased estimate may be considered of little practical use which makes unbiasedness to be a key property [7] [38].

The formulas for summary statistics with Monte Carlo standard errors from the Monte Carlo simulation studies which were presented in this section were computed using `rsimsum` package in R [56].

5 Results

5.1 Initial values

As explained in the method section, the initial guesses for β are self-estimated by squadP approach. Therefore, the user is not required to search for initial values inside the feasible region. Furthermore, the self-estimated initial values not only solving the starting value problem but also reduced the required number of iterations by the iterative method in a regression model to converge, therefore reducing the computational cost.

Results of the simulated dataset in table 7 shows that the optimized initial value for β_0, β_1 reduces the number of iterations significantly in the log-binomial GLM from 6 to 2 iteration, and in squadP from 10 to 3 iteration. With EM-type algorithm, optimization of the initial values has no significant difference in the number of iterations that are required by the algorithm.

<table>
<tr><td>Method</td><td>GLM(log)</td><td>EM-type</td><td>squadP</td></tr>
<tr><td>Variables
(simulated example)</td><td colspan="3">
<table>
<tr><th>Predictor</th><th>Response = 0</th><th>Response = 1</th><th>Total</th></tr>
<tr><td>N.</td><td>7957</td><td>2043</td><td>10000</td></tr>
<tr><td>Min.</td><td>67.06</td><td>68</td><td></td></tr>
<tr><td>Median</td><td>75.03</td><td>75.03</td><td></td></tr>
<tr><td>Mean</td><td>75.02</td><td>74.99</td><td></td></tr>
<tr><td>Max.</td><td>82.71</td><td>82.3</td><td></td></tr>
<tr><td>SD.</td><td>1.99</td><td>1.97</td><td></td></tr>
</table>
</td></tr>
<tr><td>The log-likelihood</td><td colspan="3">$l(\beta) = \sum_i y_i \; log(p_i(\beta)) + \sum_i (1 - y_i) \; log(1 - p_i(\beta))$</td></tr>
<tr><td>Initial guess</td><td colspan="3">$\beta_0 = -0.05$
$\beta_1 = 0$</td></tr>
<tr><td>Optimized initial guess</td><td colspan="3">$\hat{\beta_0} = -1.58$
$\hat{\beta_1} = 0$</td></tr>
<tr><td>N. of iterations using initial guess</td><td>6</td><td>429</td><td>10</td></tr>
<tr><td>N. of iterations using optimized initial guess</td><td>2</td><td>431</td><td>3</td></tr>
<tr><td>Solution</td><td>$\hat{\beta_0} = -1.0894$
$\hat{\beta_1} = -0.0066$</td><td>-1.0894
-0.0066</td><td>-1.0894
-0.0066</td></tr>
</table>

Table 7: Comparing methods without and with the estimated initial guess. The same simulated dataset is analysed twice using the underlying statistical methods, once with an initial guess for β, and once with the optimized initial value. Solution is the output of the regression analysis.

The reason for lower number of iterations when using $\hat{\beta}_0 = -1.58$ and $\hat{\beta}_1 = 0$ is that the optimized starting values are near to the minimum or solution, which is $\hat{\beta}_0 = -1.089$ and $\hat{\beta}_1 = -0.006$. Therefore, the iterative methods are not requiring large number of iterations for finding the solution in parameter space.

5.2 Convergence of squadP compared with other estimators

In this chapter, the six statistical methods being studied including the provided approach squadP are examined using two types of examples, a simulated dataset, and a real data example as following.

5.2.1 Simulated example

The dataset is simulated using the pseudo-random number generator of L'Ecuyer (1999) [28]. The dataset contains two random variables that are normally and binomially distributed with a strong positive correlation of 0.7 between them, which makes it a problematic dataset for estimators to analyse, as shown in figure 9.

In table 8, there are in the simulated example two independent random variables $Var._1, Var._2$, and a response binary variable Y with event probability of 12%.

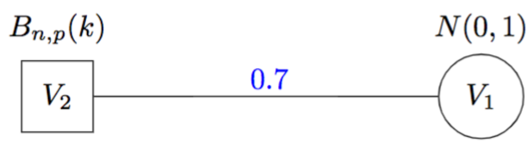

Figure 9: Correlation between the normally distributed random variable V_1 and the binomial random variable V_2

V_1		V_2	Y (response)
Min.	:-1.4722	0:34	0:44
Max.	: 3.5007	1:16	1: 6
Mean	:-0.1183		
SD	:0.9172		

Table 8: The simulated dataset consists of two independent variables (V_1, V_2) and binary outcome/response variable Y. The measures of central tendency summarize information about the average values, minimum, maximum, and standard deviation of a each variable in the example.

The above mentioned simulated dataset was generated with predefined true values, which means that a prior knowledge of the true estimation is known. The dataset was analysed using the underlying statistical methods including squadP and the results were collected and a comparison between the methods was carried out.

As shown in table 9, The predefined true values, which are $\beta_0 = -2.50, \beta_1 = -0.70, \beta_2 = 1.40$, were used to examine and compare the results of each statistical method for identifying potential bias. The results of the six methods are plotted on figure 10. The three vertical dotted lines on the graph represent the true values of $intercept$, $Var._1$, and $Var._2$. The small shapes representing the estimators and how far the solution of each estimator from the true value is.

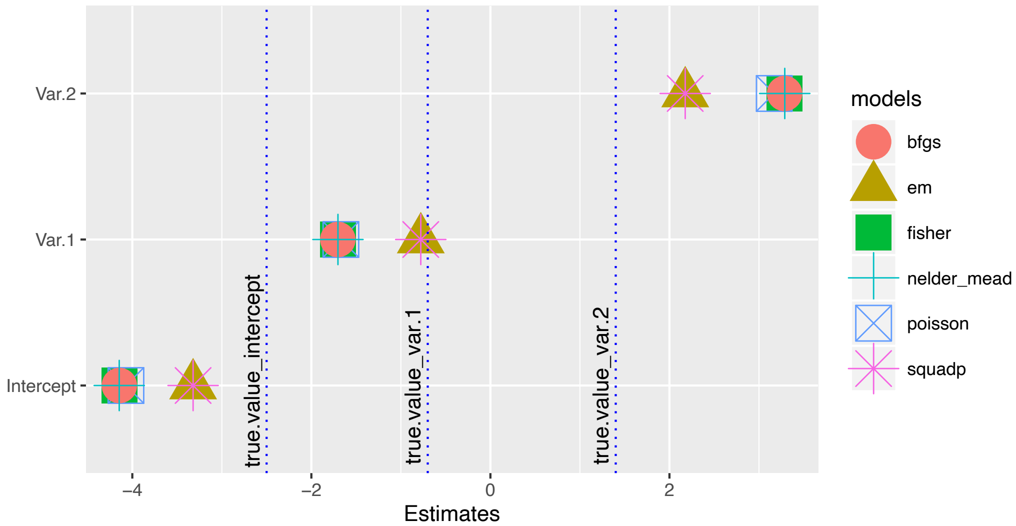

Figure 10: Bias comparison of the underlying estimators. Dotted line represents the true values of β_0, β_1 , and β_2. y-axis has the variables and x-axis has the estimated values. Shapes represent the six estimators

intercept	Var.1	Var.2	models
-2.50	-0.70	1.40	True values
-3.32	-0.78	2.18	EM-type
-3.32	-0.78	2.18	squadp
-4.15	-1.70	3.29	Fisher
-4.07	-1.67	3.17	Poisson
-4.14	-1.70	3.29	BFGS
-4.15	-1.70	3.29	Nelder-Mead

Table 9: Results of the simulated data example with predefined true values

As a result of the analysis of this example and the evaluation of the statistical methods, EM-type method and squadP have identical results estimating the three values which are the nearest to the predefined true values therefore less bias. The other methods (Fisher, Poisson, BFGS, and Nelder-Mead) have similar results which are significantly far from the true value therefore more biased results.

5.2.2 Real data analysis

The data set is taken from the published Veterans' Administration Lung Cancer study by Kalbfleisch and Prentice (1980), in the book "The Statistical Analysis of Failure Time Data" [27].

The data set is standard survival analysis. Randomized trial of two treatment regimens for lung cancer [27]. Dataset has a treatment variable (trt: 1=standard, 2=test), time from diagnosis to randomization variable (diagtime), prior therapy variable (prior: 0=no, 10=yes), and age. The data set is used for comparing and evaluating the estimated relative risk (RR) in log scale from the six statistical methods being studied. The binary variable censoring status (status) is used as an outcome variable Y in the regression models.

As presented in figure 11, the five covariates including the intercept are on y-axis (the treatment as factor variable, prior as factor, diagtime, and age). On x-axis are the estimated relative risk (RR) in log scale ($\beta_0, \beta_1, \beta_2, \beta_3, \beta_4$). Variables in the data set are fitted to the model as follow:

$$log(\pi) = \beta_0 + \beta_1 * trt + \beta_2 * prior + \beta_3 * digtime + \beta_4 * age$$

The estimations of all statistical methods share similar results with a similar confidence interval, except Poisson regression model has a slightly wider range of confidence interval, therefore a higher standard error compared with other methods as shown in table 10.

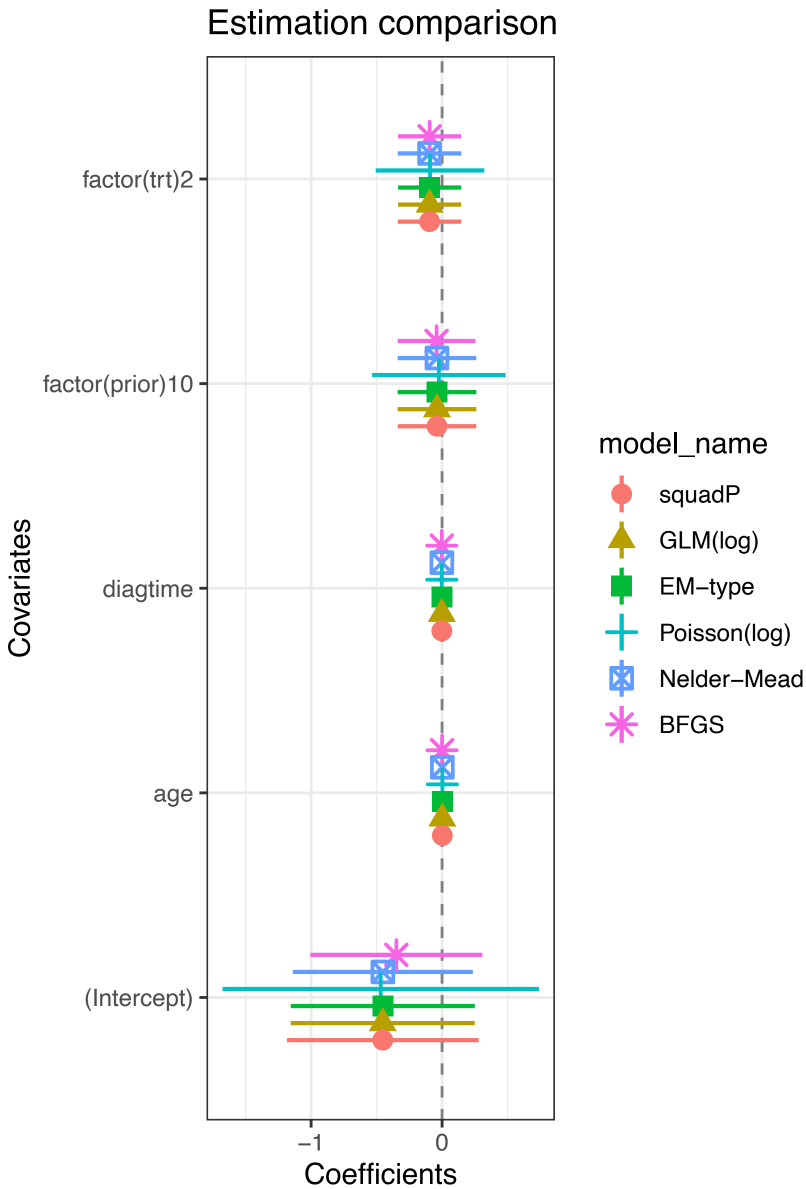

Figure 11: Data of Veterans' Administration Lung Cancer study. x-axis has the coefficients of the variables treatment (trt), prior, diagnostics time (daigtime), and age which are on y-axis. Each shape represents a different statistical method with a horizontal line that indicates the confidence interval. the methods are squadP, GLM(log), EM-type, Poisson(log), Nelder-Mead, and BFGS.

Model	Covariates	Coefficients	Standard error
squadP	(Intercept)	-0.453	0.374
	trt	-0.095	0.124
	age	0.002	0.006
	diagtime	-0.002	0.007
	prior	-0.039	0.153
GLM(log)	(Intercept)	-0.453	0.359
	trt	-0.095	0.123
	age	0.002	0.006
	diagtime	-0.002	0.007
	prior	-0.039	0.154
EM-type	(Intercept)	-0.453	0.359
	trt	-0.095	0.123
	age	0.002	0.006
	diagtime	-0.002	0.007
	prior	-0.039	0.154
Poisson(log)	(Intercept)	-0.469	0.616
	trt	-0.092	0.212
	age	0.002	0.010
	diagtime	-0.002	0.011
	prior	-0.025	0.260
Nelder-Mead	(Intercept)	-0.452	0.351
	trt	-0.095	0.124
	age	0.002	0.006
	diagtime	-0.002	0.007
	prior	-0.039	0.153
BFGS	(Intercept)	-0.349	0.335
	trt	-0.095	0.123
	age	0.000	0.005
	diagtime	-0.002	0.007
	prior	-0.042	0.152

Table 10: Results of the real data example using the underlying statistical methods. Covariates are the variables in the study being estimated, and coefficients are the estimates (log(RR)) with the standard error.

For studying the behavior of the estimators, two examples are not sufficient to analyse. Therefore, the results of the conducted large simulation study are presented next.

5.3 Monte Carlo simulation results: scenarios with 2 covariates

The first simulation study consists of 30 scenarios with 2 independent variables and variety of event probabilities $3\%, 6\%, 12\%, 24\%$, and 48%. Which means 6 different scenarios for each event probability. The two independent variables are strongly correlated as explained in *Monte Carlo Simulation* section. In this section, the main findings of this Monte Carlo simulation study are summarized in the following points.

5.3.1 Scenarios with event probability 3%

The probability of the event occurring in these 6 scenarios is 3%. The results of scenarios from $1 \rightarrow 6$ using the six statistical methods being studied are presented with regard to the performance measures, which are:

The assessment of bias
Figure 12 represents a comparison of the estimators with regard to the absolute bias with approximated Monte Carlo standard error and additionally confidence interval. As shown in figure 12, 6 different sample sizes $(60, 80, 100, 500, 1000, 50000)$ indicating 6 different scenarios with the same event probability. The estimates of the intercept (Int.), coefficient C.1 and C.2, shown on y-axis, for the six methods that are shown on x-axis. As observed, the range of confidence interval is getting relatively narrower with the increase of sample size, which indicating that the effect size is known precisely with larger sample size. If the interval is wider the uncertainty is greater with small sample sizes such as 60 and 80.

It is obvious in figure 12 that squadP and EM-type methods have similar results with narrow confidence interval and lowest biased coefficient estimates ≈ 0. The estimates of the intercept with a small sample size of 60 show that Poisson(log) and Fisher (GLM) have a wider confidence interval with the highest bias. Looking at the coefficients of C.1 and C.1 with a small sample size of 60, we notice that Poisson(log) has likewise the highest biased estimates and widest confidence interval. With bigger sample sizes such as $100, 1000$ and 50000, all methods have relatively similar biases close to 0

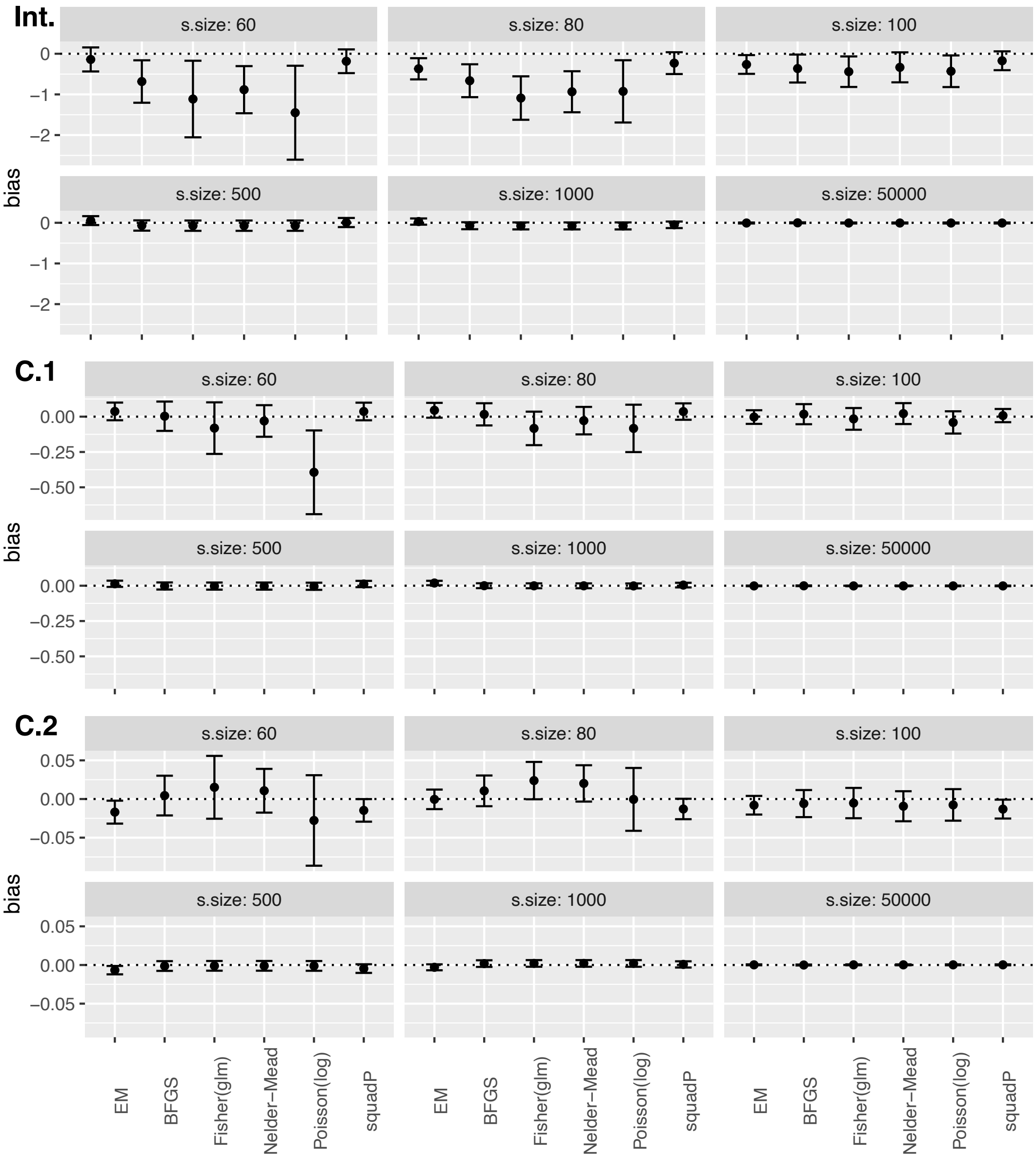

Figure 12: Absolute bias of scenarios 1→6 with event probability 3% and sample sizes 60, 80, 100, 500, 1000, 50000. y-axis: bias for intercept (Int), coefficients C.1, and C.2. x-axis: the six statistical methods being tested for each scenario.

Convergence rate

The common pattern between the statistical models being evaluated is lower convergence rate with small sample sizes $(60, 80)$ compared with larger sample sizes $(500, 1000, 50000)$ as shown in figure 13. That is because the independent variables in the regression models are designed to be not only with a small sample size but also highly correlated, which causes a multicollinearity problem, therefore difficulties with fitting the models and interpreting the results. Such design is purposed to examine the six statistical methods under extreme circumstances.

As shown in figure 13, scenario with sample size 60, squadP and Poisson(log) methods have the highest convergence rate (nearly 85%) out of 1000 simulations for each scenario. Fisher GLM has converged with a rate of 82%, while Nelder-Mead, BFGS, and EM-type have the lowest convergence rate $(82\%, 78\%$, and $77\%)$. Convergence rate is increasing with the increase of the sample sizes to reach 100% starting from sample sizes 500. EM-type method has the lowest convergence rate and reached 100% only when the sample size was 50000.

The assessment of coverage

squadP and EM-type methods have approximately 100% coverage probability in scenarios with small sample sizes, and the probability started decreasing with the increase of the sample size to reach 95% when the sample size was 50000. The rest of the methods have coverage probability between 95% and 98%.

MSE and empSE

In figure 13, the mean squared error (MSE) and empirical standard error (empSE) estimated using squadP and EM-type algorithms are the lowest among the statistical methods being examined. As a general pattern, MSE and empSE decrease in scenarios with a larger sample size to reach approximately 0 with a sample size of 50000 in all methods. In addition, figure 13 shows that Poisson(log) and Fisher GLM have the highest MSE and empSE in scenarios with small sample size $(60, 80, 100)$.

The performance measurements (with Monte Carlo standard error) such as bias, coverage probability, MSE, empSE and convergence for the intercept, independent variable coef.1, and independent variable coef.2 are shown in table 11 for the intercept, Table 12 for coef.1, and Table 13 for coef.2.

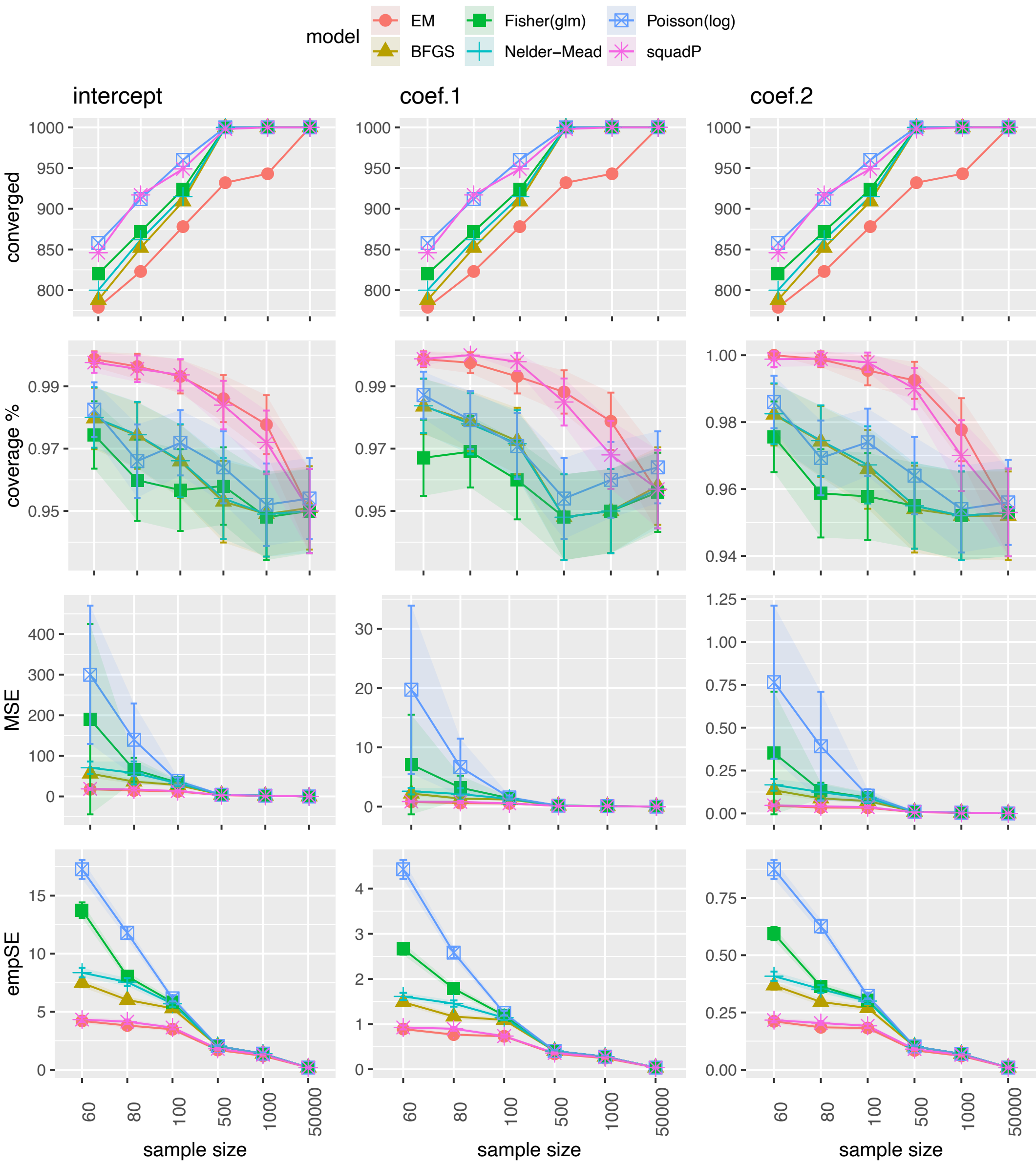

Figure 13: Performance measurements of scenarios (1→6) with event probability 3%. Measurements are convergence rate, coverage probability, MSE, and empSE (y-axis) for the intercept, variable coef.1, and variable coef.2 using the underlying statistical methods. Each type of shapes with shaded area (confidence interval) is representing a different method. The measurements are done for 6 scenarios that are derived from six different sample sizes (60,80,100,500,1000,50000) on x-axis.

Table 11: Performance measurements of scenarios 1→6 for intercept.

Coverage		Bias		MSE		EmpSE				
cover.	mcse	bias	mcse	mse	mcse	empse	mcse	model	s.size	converged
1.00	0.00	-0.14	0.15	17.68	1.16	4.20	0.11	EM	60	779
0.98	0.00	-0.68	0.27	56.29	5.04	7.48	0.19	BFGS	60	788
0.97	0.01	-1.11	0.48	190.17	119.58	13.75	0.34	Fisher(glm)	60	820
0.98	0.00	-0.88	0.30	70.68	7.87	8.37	0.21	Nelder-Mead	60	800
0.98	0.00	-1.45	0.59	299.85	86.90	17.26	0.42	Poisson(log)	60	858
1.00	0.00	-0.18	0.15	18.72	1.25	4.33	0.10	squadP	60	846
1.00	0.00	-0.37	0.13	14.67	1.21	3.81	0.09	EM	80	823
0.97	0.00	-0.66	0.21	36.64	2.96	6.02	0.15	BFGS	80	852
0.96	0.01	-1.09	0.27	65.88	14.91	8.05	0.19	Fisher(glm)	80	872
0.97	0.00	-0.94	0.26	57.88	14.45	7.56	0.18	Nelder-Mead	80	862
0.97	0.01	-0.93	0.39	139.82	45.43	11.80	0.28	Poisson(log)	80	912
1.00	0.00	-0.23	0.14	17.22	1.42	4.15	0.10	squadP	80	917
0.99	0.00	-0.26	0.12	12.24	0.79	3.49	0.08	EM	100	878
0.97	0.01	-0.36	0.18	28.11	2.06	5.29	0.12	BFGS	100	909
0.96	0.01	-0.44	0.19	34.10	3.70	5.83	0.14	Fisher(glm)	100	924
0.97	0.01	-0.34	0.19	32.42	3.59	5.69	0.13	Nelder-Mead	100	915
0.97	0.00	-0.43	0.20	37.99	4.48	6.15	0.14	Poisson(log)	100	960
0.99	0.00	-0.17	0.12	13.27	0.87	3.64	0.08	squadP	100	949
0.99	0.00	0.05	0.06	2.90	0.11	1.70	0.04	EM	500	932
0.95	0.01	-0.07	0.06	4.17	0.19	2.04	0.05	BFGS	500	1000
0.96	0.01	-0.07	0.06	4.17	0.19	2.04	0.05	Fisher(glm)	500	1000
0.95	0.01	-0.07	0.06	4.17	0.19	2.04	0.05	Nelder-Mead	500	1000
0.96	0.01	-0.07	0.06	4.20	0.20	2.05	0.05	Poisson(log)	500	1000
0.98	0.00	0.01	0.06	3.27	0.12	1.81	0.04	squadP	500	998
0.98	0.00	0.03	0.04	1.44	0.07	1.20	0.03	EM	1000	943
0.95	0.01	-0.07	0.04	1.92	0.10	1.38	0.03	BFGS	1000	1000
0.95	0.01	-0.08	0.04	1.92	0.10	1.38	0.03	Fisher(glm)	1000	1000
0.95	0.01	-0.08	0.04	1.92	0.10	1.38	0.03	Nelder-Mead	1000	1000
0.95	0.01	-0.08	0.04	1.93	0.10	1.39	0.03	Poisson(log)	1000	1000
0.97	0.00	-0.05	0.04	1.68	0.07	1.30	0.03	squadP	1000	1000
0.95	0.01	-0.01	0.01	0.04	0.00	0.19	0.00	EM	50000	1000
0.95	0.01	0.00	0.01	0.04	0.00	0.19	0.00	BFGS	50000	1000
0.95	0.01	-0.01	0.01	0.04	0.00	0.19	0.00	Fisher(glm)	50000	1000
0.95	0.01	0.00	0.01	0.04	0.00	0.19	0.00	Nelder-Mead	50000	1000
0.95	0.01	-0.01	0.01	0.04	0.00	0.19	0.00	Poisson(log)	50000	1000
0.95	0.01	-0.01	0.01	0.04	0.00	0.19	0.00	squadP	50000	1000

Table 12: Performance measurements of scenarios 1→6 for coef.1.

Coverage		Bias		MSE		EmpSE				
cover.	mcse	bias	mcse	mse	mcse	empse	mcse	model	s.size	converged
1.00	0.00	0.04	0.03	0.79	0.07	0.89	0.02	EM	60	779
0.98	0.00	0.00	0.05	2.20	0.22	1.49	0.04	BFGS	60	788
0.97	0.01	-0.08	0.09	7.11	4.29	2.67	0.07	Fisher(glm)	60	820
0.98	0.00	-0.03	0.06	2.60	0.28	1.61	0.04	Nelder-Mead	60	800
0.99	0.00	-0.39	0.15	19.74	7.24	4.43	0.11	Poisson(log)	60	858
1.00	0.00	0.04	0.03	0.86	0.08	0.93	0.02	squadP	60	846
1.00	0.00	0.04	0.03	0.59	0.06	0.77	0.02	EM	80	823
0.98	0.00	0.02	0.04	1.36	0.11	1.17	0.03	BFGS	80	852
0.97	0.01	-0.08	0.06	3.19	1.04	1.79	0.04	Fisher(glm)	80	872
0.98	0.00	-0.03	0.05	2.12	0.52	1.46	0.04	Nelder-Mead	80	862
0.98	0.00	-0.08	0.09	6.67	2.45	2.58	0.06	Poisson(log)	80	912
1.00	0.00	0.04	0.03	0.81	0.11	0.90	0.02	squadP	80	917
0.99	0.00	0.00	0.03	0.53	0.04	0.73	0.02	EM	100	878
0.97	0.00	0.02	0.04	1.19	0.09	1.09	0.03	BFGS	100	909
0.96	0.01	-0.02	0.04	1.42	0.15	1.19	0.03	Fisher(glm)	100	924
0.97	0.00	0.02	0.04	1.30	0.12	1.14	0.03	Nelder-Mead	100	915
0.97	0.00	-0.04	0.04	1.55	0.16	1.25	0.03	Poisson(log)	100	960
1.00	0.00	0.01	0.02	0.54	0.04	0.74	0.02	squadP	100	949
0.99	0.00	0.01	0.01	0.12	0.00	0.34	0.01	EM	500	932
0.95	0.01	0.00	0.01	0.16	0.01	0.41	0.01	BFGS	500	1000
0.95	0.01	0.00	0.01	0.16	0.01	0.41	0.01	Fisher(glm)	500	1000
0.95	0.01	0.00	0.01	0.16	0.01	0.41	0.01	Nelder-Mead	500	1000
0.95	0.01	0.00	0.01	0.17	0.01	0.41	0.01	Poisson(log)	500	1000
0.98	0.00	0.01	0.01	0.13	0.00	0.36	0.01	squadP	500	998
0.98	0.00	0.02	0.01	0.06	0.00	0.25	0.01	EM	1000	943
0.95	0.01	0.00	0.01	0.08	0.00	0.28	0.01	BFGS	1000	1000
0.95	0.01	0.00	0.01	0.08	0.00	0.28	0.01	Fisher(glm)	1000	1000
0.95	0.01	0.00	0.01	0.08	0.00	0.28	0.01	Nelder-Mead	1000	1000
0.96	0.01	0.00	0.01	0.08	0.00	0.28	0.01	Poisson(log)	1000	1000
0.97	0.01	0.00	0.01	0.07	0.00	0.26	0.01	squadP	1000	1000
0.96	0.01	0.00	0.00	0.00	0.00	0.04	0.00	EM	50000	1000
0.96	0.01	0.00	0.00	0.00	0.00	0.04	0.00	BFGS	50000	1000
0.96	0.01	0.00	0.00	0.00	0.00	0.04	0.00	Fisher(glm)	50000	1000
0.96	0.01	0.00	0.00	0.00	0.00	0.04	0.00	Nelder-Mead	50000	1000
0.96	0.01	0.00	0.00	0.00	0.00	0.04	0.00	Poisson(log)	50000	1000
0.96	0.01	0.00	0.00	0.00	0.00	0.04	0.00	squadP	50000	1000

Table 13: Performance measurements of scenarios 1→6 for coef.2.

Coverage		Bias		MSE		EmpSE				
cover.	mcse	bias	mcse	mse	mcse	empse	mcse	model	s.size	converged
1.00	0.00	-0.02	0.01	0.04	0.00	0.89	0.02	EM	60	779
0.98	0.00	0.00	0.01	0.14	0.01	1.49	0.04	BFGS	60	788
0.98	0.00	0.01	0.02	0.35	0.18	2.67	0.07	Fisher(glm)	60	820
0.98	0.00	0.01	0.01	0.17	0.02	1.61	0.04	Nelder-Mead	60	800
0.99	0.00	-0.03	0.03	0.76	0.23	4.43	0.11	Poisson(log)	60	858
1.00	0.00	-0.01	0.01	0.05	0.00	0.93	0.02	squadP	60	846
1.00	0.00	0.00	0.01	0.03	0.00	0.77	0.02	EM	80	823
0.97	0.00	0.01	0.01	0.09	0.01	1.17	0.03	BFGS	80	852
0.96	0.01	0.02	0.01	0.13	0.02	1.79	0.04	Fisher(glm)	80	872
0.97	0.00	0.02	0.01	0.12	0.02	1.46	0.04	Nelder-Mead	80	862
0.97	0.01	0.00	0.02	0.39	0.16	2.58	0.06	Poisson(log)	80	912
1.00	0.00	-0.01	0.01	0.04	0.00	0.90	0.02	squadP	80	917
1.00	0.00	-0.01	0.01	0.03	0.00	0.73	0.02	EM	100	878
0.97	0.01	-0.01	0.01	0.07	0.01	1.09	0.03	BFGS	100	909
0.96	0.01	0.00	0.01	0.09	0.01	1.19	0.03	Fisher(glm)	100	924
0.97	0.01	-0.01	0.01	0.09	0.01	1.14	0.03	Nelder-Mead	100	915
0.97	0.00	-0.01	0.01	0.10	0.02	1.25	0.03	Poisson(log)	100	960
1.00	0.00	-0.01	0.01	0.04	0.00	0.74	0.02	squadP	100	949
0.99	0.00	-0.01	0.00	0.01	0.00	0.34	0.01	EM	500	932
0.95	0.01	0.00	0.00	0.01	0.00	0.41	0.01	BFGS	500	1000
0.96	0.01	0.00	0.00	0.01	0.00	0.41	0.01	Fisher(glm)	500	1000
0.96	0.01	0.00	0.00	0.01	0.00	0.41	0.01	Nelder-Mead	500	1000
0.96	0.01	0.00	0.00	0.01	0.00	0.41	0.01	Poisson(log)	500	1000
0.99	0.00	0.00	0.00	0.01	0.00	0.36	0.01	squadP	500	998
0.98	0.00	0.00	0.00	0.00	0.00	0.25	0.01	EM	1000	943
0.95	0.01	0.00	0.00	0.00	0.00	0.28	0.01	BFGS	1000	1000
0.95	0.01	0.00	0.00	0.00	0.00	0.28	0.01	Fisher(glm)	1000	1000
0.95	0.01	0.00	0.00	0.00	0.00	0.28	0.01	Nelder-Mead	1000	1000
0.95	0.01	0.00	0.00	0.00	0.00	0.28	0.01	Poisson(log)	1000	1000
0.97	0.00	0.00	0.00	0.00	0.00	0.26	0.01	squadP	1000	1000
0.95	0.01	0.00	0.00	0.00	0.00	0.04	0.00	EM	50000	1000
0.95	0.01	0.00	0.00	0.00	0.00	0.04	0.00	BFGS	50000	1000
0.95	0.01	0.00	0.00	0.00	0.00	0.04	0.00	Fisher(glm)	50000	1000
0.95	0.01	0.00	0.00	0.00	0.00	0.04	0.00	Nelder-Mead	50000	1000
0.96	0.01	0.00	0.00	0.00	0.00	0.04	0.00	Poisson(log)	50000	1000
0.95	0.01	0.00	0.00	0.00	0.00	0.04	0.00	squadP	50000	1000

5.3.2 Scenarios with event probability 6%

The main findings of scenarios from $7 \rightarrow 12$ (6 scenarios) with event probability 6% and different sample sizes $(60, 80, 100, 500, 1000, 50000)$ are discussed here.

Estimates in figure 14 follow relatively similar patterns in regard to the range of confidence interval which is getting narrower with the increase of sample size indicating a high degree of certainty with a larger sample size. In addition, The estimates are less biased with the increase of the sample size to reach approximately 0 bias when the sample size is 50000. As shown in figure 14, estimated values of intercept (Int.) vary from a method to another, particularly with small samples of $60, 80$, and 100. squadP and EM-type methods have the lowest bias of approximately 0, while Poisson(log), Nelder-Mead, BFGS, and Fisher GLM are significantly biased. Data is shown in table 14. Regarding the independent variables $C.1$ and $C.2$, all methods are relatively less biased (near 0) as shown in table 15 for variable $C.1$, and table 16 for $C.2$.

As shown in figure 15, the six statistical methods, in case of scenario with sample sizes of 50000, have relatively similar convergence rate and coverage probability. In the other 5 scenarios, Poisson(log) and squadP methods have the highest convergence rate, while EM-type method has the lowest convergence rate. Regarding coverage probability, squadP and EM-type methods have the highest percentage near to 100% in case of scenarios with small sample sizes such as $60, 80$, and 100

MSE and empSE measurements are shown in figure 15 with the lowest error values for squadP and EM-type methods. Highest MSE and empSE values are from Poisson(log) and Fisher GLM estimates, in case of scenarios with small sample sizes such as $60, 80$, and 100. It is noticeable in figure 15 that MSE and empSE measurements are showing association with sample size. The error measurements decrease with the increase of sample size until they reach the lowest error values in case of the scenario with 50000 sample size.

Table 17, table 18, and table 19 show the performance measurements coverage probability, bias, MSE, and empSE (with Monte Carlo standard error) for the intercept, and coefficients coef.1 and coef.2.

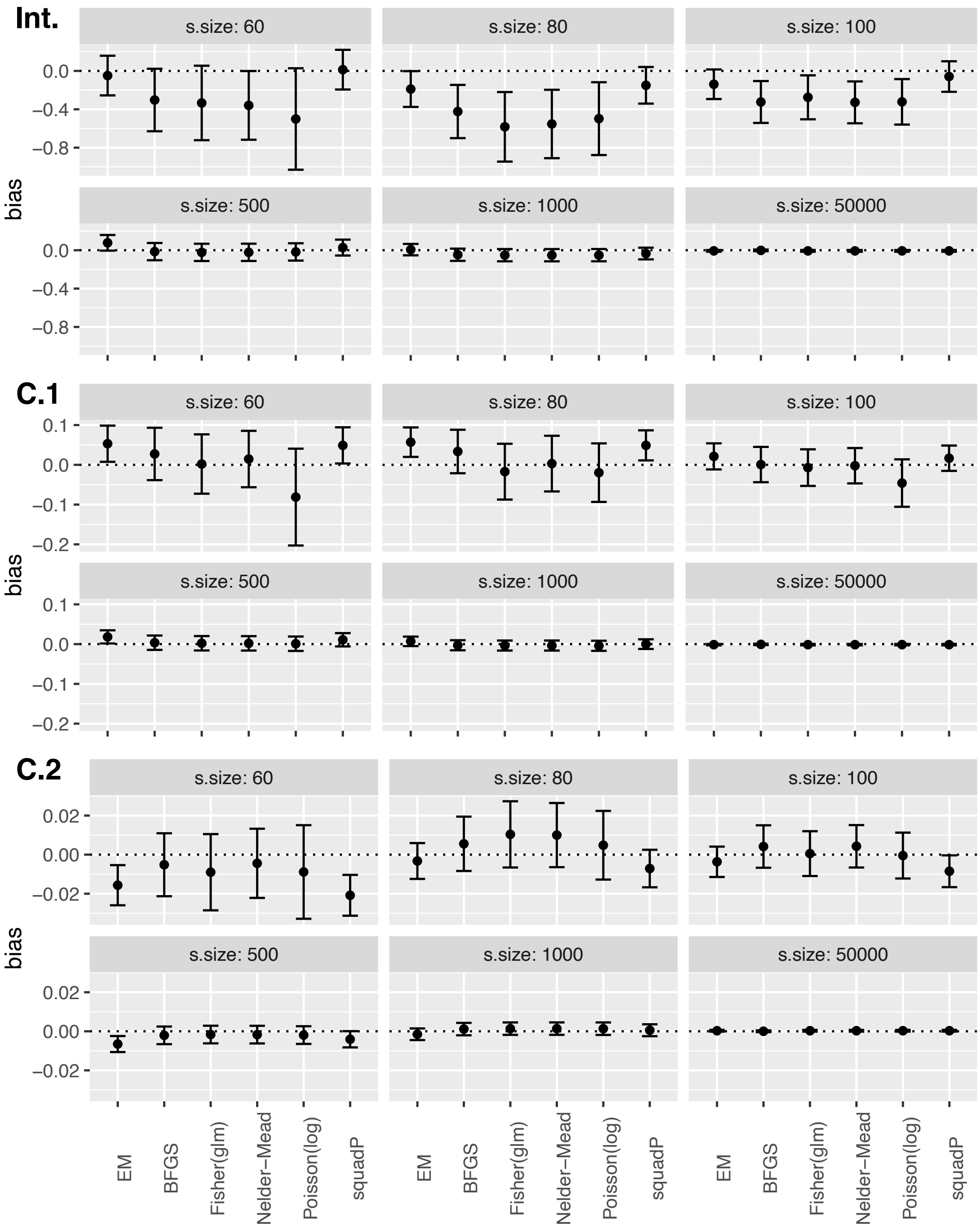

Figure 14: Absolute bias centered on 0 with confidence interval at 95% level. The bias estimates are for 6 scenarios with event probability 6% and sample (sizes 60, 80, 100, 500, 1000, 50000). y-axis: bias for intercept (Int) and coefficients C.1 and C.2. x-axis: the six statistical methods used for each scenario.

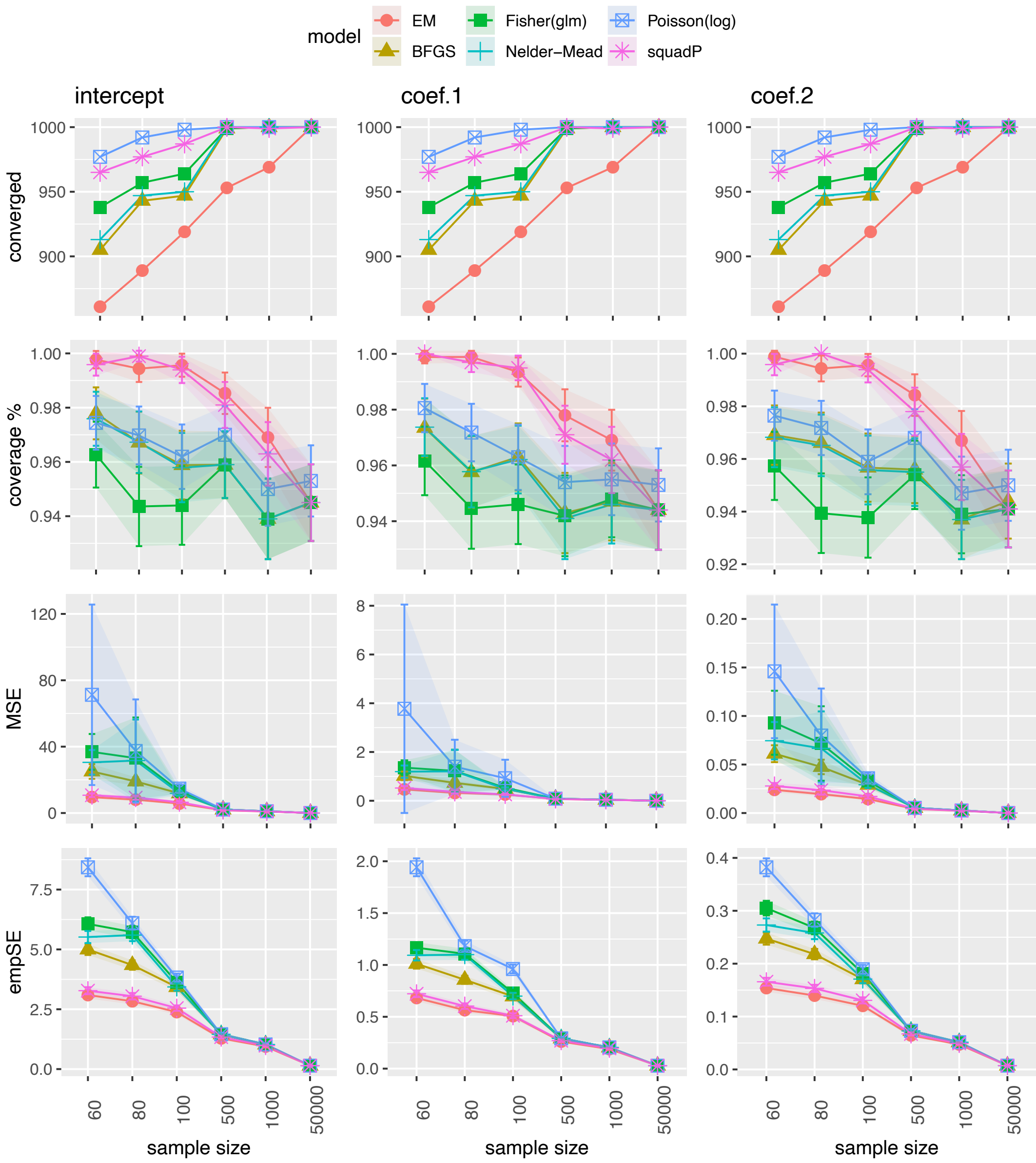

Figure 15: Performance measurements of scenarios with event probability 6%. Measurements are convergence rate at the top, coverage probability, MSE, and empSE (y-axis) for the intercept, independent variable coef.1, and independent variable coef.2 using the underlying statistical methods fro each scenario. Each type of shapes with shaded area (confidence interval) is representing a different method. The measurements are done for 6 scenarios that are derived from six different sample sizes (60,80,100,500,1000,50000) on x-axis.

Table 14: Performance measurements of scenarios 7→12 for intercept.

Coverage		Bias		MSE		EmpSE				
cover.	mcse	bias	mcse	mse	mcse	empse	mcse	model	s.size	converged
1.00	0.00	-0.05	0.10	9.57	0.52	3.10	0.07	EM	60	861
0.98	0.00	-0.30	0.17	25.04	2.26	5.00	0.12	BFGS	60	905
0.96	0.01	-0.33	0.20	36.86	5.49	6.07	0.14	Fisher(glm)	60	938
0.98	0.00	-0.36	0.18	30.54	3.79	5.52	0.13	Nelder-Mead	60	913
0.97	0.00	-0.50	0.27	71.24	27.75	8.43	0.19	Poisson(log)	60	977
1.00	0.00	0.01	0.10	10.72	0.55	3.28	0.07	squadP	60	965
0.99	0.00	-0.19	0.10	8.07	0.45	2.84	0.07	EM	80	889
0.97	0.01	-0.42	0.14	19.05	1.27	4.35	0.10	BFGS	80	943
0.94	0.01	-0.58	0.18	33.07	12.51	5.72	0.13	Fisher(glm)	80	957
0.97	0.01	-0.55	0.18	31.61	12.62	5.60	0.13	Nelder-Mead	80	947
0.97	0.00	-0.50	0.19	37.39	15.86	6.10	0.14	Poisson(log)	80	992
1.00	0.00	-0.15	0.10	9.28	0.62	3.04	0.07	squadP	80	977
1.00	0.00	-0.14	0.08	5.69	0.25	2.38	0.06	EM	100	919
0.96	0.01	-0.32	0.11	11.83	0.71	3.43	0.08	BFGS	100	947
0.94	0.01	-0.28	0.12	13.23	0.89	3.63	0.08	Fisher(glm)	100	964
0.96	0.01	-0.33	0.11	11.88	0.71	3.43	0.08	Nelder-Mead	100	950
0.96	0.01	-0.32	0.12	14.71	1.36	3.82	0.09	Poisson(log)	100	998
0.99	0.00	-0.06	0.08	6.49	0.33	2.55	0.06	squadP	100	987
0.98	0.00	0.08	0.04	1.65	0.06	1.28	0.03	EM	500	953
0.96	0.01	-0.01	0.05	2.10	0.09	1.45	0.03	BFGS	500	999
0.96	0.01	-0.02	0.05	2.11	0.09	1.45	0.03	Fisher(glm)	500	999
0.96	0.01	-0.02	0.05	2.11	0.09	1.45	0.03	Nelder-Mead	500	1000
0.97	0.00	-0.02	0.05	2.12	0.09	1.46	0.03	Poisson(log)	500	1000
0.98	0.00	0.03	0.04	1.79	0.07	1.34	0.03	squadP	500	1000
0.97	0.01	0.01	0.03	0.89	0.04	0.94	0.02	EM	1000	969
0.94	0.01	-0.05	0.03	1.06	0.05	1.03	0.02	BFGS	1000	1000
0.94	0.01	-0.05	0.03	1.06	0.05	1.03	0.02	Fisher(glm)	1000	1000
0.94	0.01	-0.05	0.03	1.06	0.05	1.03	0.02	Nelder-Mead	1000	1000
0.95	0.01	-0.05	0.03	1.06	0.05	1.03	0.02	Poisson(log)	1000	1000
0.96	0.01	-0.04	0.03	0.96	0.04	0.98	0.02	squadP	1000	999
0.94	0.01	-0.01	0.00	0.02	0.00	0.14	0.00	EM	50000	1000
0.94	0.01	0.00	0.00	0.02	0.00	0.14	0.00	BFGS	50000	1000
0.94	0.01	-0.01	0.00	0.02	0.00	0.14	0.00	Fisher(glm)	50000	1000
0.94	0.01	-0.01	0.00	0.02	0.00	0.14	0.00	Nelder-Mead	50000	1000
0.95	0.01	-0.01	0.00	0.02	0.00	0.14	0.00	Poisson(log)	50000	1000
0.94	0.01	-0.01	0.00	0.02	0.00	0.14	0.00	squadP	50000	1000

Table 15: Performance measurements of scenarios 7→12 for coef.1.

Coverage		Bias		MSE		EmpSE				
cover.	mcse	bias	mcse	mse	mcse	empse	mcse	model	s.size	converged
1.00	0.00	0.05	0.02	0.47	0.03	0.68	0.02	EM	60	861
0.97	0.00	0.03	0.03	1.02	0.09	1.01	0.02	BFGS	60	905
0.96	0.01	0.00	0.04	1.36	0.15	1.17	0.03	Fisher(glm)	60	938
0.97	0.00	0.01	0.04	1.20	0.13	1.09	0.03	Nelder-Mead	60	913
0.98	0.00	-0.08	0.06	3.77	2.18	1.94	0.04	Poisson(log)	60	977
1.00	0.00	0.05	0.02	0.52	0.04	0.72	0.02	squadP	60	965
1.00	0.00	0.06	0.02	0.32	0.02	0.56	0.01	EM	80	889
0.96	0.01	0.03	0.03	0.74	0.05	0.86	0.02	BFGS	80	943
0.94	0.01	-0.02	0.04	1.23	0.44	1.11	0.03	Fisher(glm)	80	957
0.96	0.01	0.00	0.04	1.21	0.45	1.10	0.03	Nelder-Mead	80	947
0.97	0.00	-0.02	0.04	1.40	0.56	1.18	0.03	Poisson(log)	80	992
1.00	0.00	0.05	0.02	0.36	0.03	0.60	0.01	squadP	80	977
0.99	0.00	0.02	0.02	0.26	0.01	0.51	0.01	EM	100	919
0.96	0.01	0.00	0.02	0.48	0.03	0.70	0.02	BFGS	100	947
0.95	0.01	-0.01	0.02	0.53	0.04	0.73	0.02	Fisher(glm)	100	964
0.96	0.01	0.00	0.02	0.49	0.03	0.70	0.02	Nelder-Mead	100	950
0.96	0.01	-0.05	0.03	0.92	0.39	0.96	0.02	Poisson(log)	100	998
1.00	0.00	0.02	0.02	0.26	0.01	0.51	0.01	squadP	100	987
0.98	0.00	0.02	0.01	0.07	0.00	0.26	0.01	EM	500	953
0.94	0.01	0.00	0.01	0.09	0.00	0.29	0.01	BFGS	500	999
0.94	0.01	0.00	0.01	0.09	0.00	0.29	0.01	Fisher(glm)	500	999
0.94	0.01	0.00	0.01	0.09	0.00	0.29	0.01	Nelder-Mead	500	1000
0.95	0.01	0.00	0.01	0.09	0.00	0.29	0.01	Poisson(log)	500	1000
0.97	0.00	0.01	0.01	0.07	0.00	0.27	0.01	squadP	500	1000
0.97	0.01	0.01	0.01	0.04	0.00	0.19	0.00	EM	1000	969
0.95	0.01	0.00	0.01	0.04	0.00	0.20	0.00	BFGS	1000	1000
0.95	0.01	0.00	0.01	0.04	0.00	0.20	0.00	Fisher(glm)	1000	1000
0.95	0.01	0.00	0.01	0.04	0.00	0.20	0.00	Nelder-Mead	1000	1000
0.96	0.01	0.00	0.01	0.04	0.00	0.20	0.00	Poisson(log)	1000	1000
0.96	0.01	0.00	0.01	0.04	0.00	0.20	0.00	squadP	1000	999
0.94	0.01	0.00	0.00	0.00	0.00	0.03	0.00	EM	50000	1000
0.94	0.01	0.00	0.00	0.00	0.00	0.03	0.00	BFGS	50000	1000
0.94	0.01	0.00	0.00	0.00	0.00	0.03	0.00	Fisher(glm)	50000	1000
0.94	0.01	0.00	0.00	0.00	0.00	0.03	0.00	Nelder-Mead	50000	1000
0.95	0.01	0.00	0.00	0.00	0.00	0.03	0.00	Poisson(log)	50000	1000
0.94	0.01	0.00	0.00	0.00	0.00	0.03	0.00	squadP	50000	1000

Table 16: Performance measurements of scenarios 7→12 for coef.2.

Coverage		Bias		MSE		EmpSE				
cover.	mcse	bias	mcse	mse	mcse	empse	mcse	model	s.size	converged
1.00	0.00	-0.02	0.00	0.02	0.00	0.68	0.02	EM	60	861
0.97	0.01	0.00	0.01	0.06	0.00	1.01	0.02	BFGS	60	905
0.96	0.01	-0.01	0.01	0.09	0.02	1.17	0.03	Fisher(glm)	60	938
0.97	0.01	0.00	0.01	0.07	0.01	1.09	0.03	Nelder-Mead	60	913
0.98	0.00	-0.01	0.01	0.15	0.04	1.94	0.04	Poisson(log)	60	977
1.00	0.00	-0.02	0.00	0.03	0.00	0.72	0.02	squadP	60	965
0.99	0.00	0.00	0.00	0.02	0.00	0.56	0.01	EM	80	889
0.97	0.01	0.01	0.01	0.05	0.00	0.86	0.02	BFGS	80	943
0.94	0.01	0.01	0.01	0.07	0.02	1.11	0.03	Fisher(glm)	80	957
0.96	0.01	0.01	0.01	0.07	0.02	1.10	0.03	Nelder-Mead	80	947
0.97	0.00	0.00	0.01	0.08	0.03	1.18	0.03	Poisson(log)	80	992
1.00	0.00	-0.01	0.00	0.02	0.00	0.60	0.01	squadP	80	977
1.00	0.00	0.00	0.00	0.01	0.00	0.51	0.01	EM	100	919
0.96	0.01	0.00	0.01	0.03	0.00	0.70	0.02	BFGS	100	947
0.94	0.01	0.00	0.01	0.03	0.00	0.73	0.02	Fisher(glm)	100	964
0.96	0.01	0.00	0.01	0.03	0.00	0.70	0.02	Nelder-Mead	100	950
0.96	0.01	0.00	0.01	0.04	0.00	0.96	0.02	Poisson(log)	100	998
0.99	0.00	-0.01	0.00	0.02	0.00	0.51	0.01	squadP	100	987
0.98	0.00	-0.01	0.00	0.00	0.00	0.26	0.01	EM	500	953
0.96	0.01	0.00	0.00	0.00	0.00	0.29	0.01	BFGS	500	999
0.95	0.01	0.00	0.00	0.00	0.00	0.29	0.01	Fisher(glm)	500	999
0.96	0.01	0.00	0.00	0.00	0.00	0.29	0.01	Nelder-Mead	500	1000
0.97	0.01	0.00	0.00	0.00	0.00	0.29	0.01	Poisson(log)	500	1000
0.98	0.00	0.00	0.00	0.00	0.00	0.27	0.01	squadP	500	1000
0.97	0.01	0.00	0.00	0.00	0.00	0.19	0.00	EM	1000	969
0.94	0.01	0.00	0.00	0.00	0.00	0.20	0.00	BFGS	1000	1000
0.94	0.01	0.00	0.00	0.00	0.00	0.20	0.00	Fisher(glm)	1000	1000
0.94	0.01	0.00	0.00	0.00	0.00	0.20	0.00	Nelder-Mead	1000	1000
0.95	0.01	0.00	0.00	0.00	0.00	0.20	0.00	Poisson(log)	1000	1000
0.96	0.01	0.00	0.00	0.00	0.00	0.20	0.00	squadP	1000	999
0.94	0.01	0.00	0.00	0.00	0.00	0.03	0.00	EM	50000	1000
0.94	0.01	0.00	0.00	0.00	0.00	0.03	0.00	BFGS	50000	1000
0.94	0.01	0.00	0.00	0.00	0.00	0.03	0.00	Fisher(glm)	50000	1000
0.94	0.01	0.00	0.00	0.00	0.00	0.03	0.00	Nelder-Mead	50000	1000
0.95	0.01	0.00	0.00	0.00	0.00	0.03	0.00	Poisson(log)	50000	1000
0.94	0.01	0.00	0.00	0.00	0.00	0.03	0.00	squadP	50000	1000

5.3.3 Scenarios with event probability 12%

The results of scenarios from 13 $\rightarrow$ 18 using the six statistical methods being studied are presented with regard to the performance measures mentioned earlier. The probability of the event occurring in these 6 scenarios is 12%.

Figure 16 show a comparison between the estimators with regard to the absolute bias with confidence interval. Each separate cell represents a different scenario. Sample sizes $(60, 80, 100, 500, 1000, 50000)$ indicating 6 different scenarios with the same event probability. The estimates of the intercept $(Int.)$, variable $C.1$, and variable $C.2$ are shown on y-axis for the six methods shown on x-axis. The range of confidence interval is getting narrower with the increase of sample size, which is a common finding among whole scenarios as expected.

In figure 16 for the intercept (scenarios on the top of the figure), squadP method behaving differently in scenarios with small sample sizes of $60, 80$, and 100. squadP estimates are significantly less biased than the other estimators with narrow confidence interval. As shown in table 17 for scenarios with sample size 60, the bias estimate of squadP method is -0.11 with MCSE of 0.07 compared to $0.21, 0.36, 0.25, 0.36, 0.21$ for the other statistical methods with MCSE of $0.1, 0.1, 0.1, 0.1, 0.07$. Scenarios with sample sizes 80, and 100 follow similar pattern. For other scenarios of the independent variables C.1 and C.2, all estimator have less bias relatively very near to 0 as shown in table 18 for the variable C.1 and table 19 for variable C.2.

In case of scenarios with large sample sizes such as 1000, and 50000 (in figure 17), the six methods have relatively similar convergence rate, coverage probability, MSE, and empSE. In contrast, the results vary in scenarios with small sample sizes such as $60, 80$, and 100. For instance, Poisson(log) and squadP methods have the heights convergence rate for the intercept, variables coef.1 and coef.2, while EM-type algorithm has the lowest. Regarding the coverage probability, squadP and EM-type algorithm have the highest coverage probability with lowest MSE and empSE.

Table 17, table 18, and table 19 show the coverage probability, bias, MSE, and EmpSE for the intercept, in dependent variables coef.1, and coef.2.

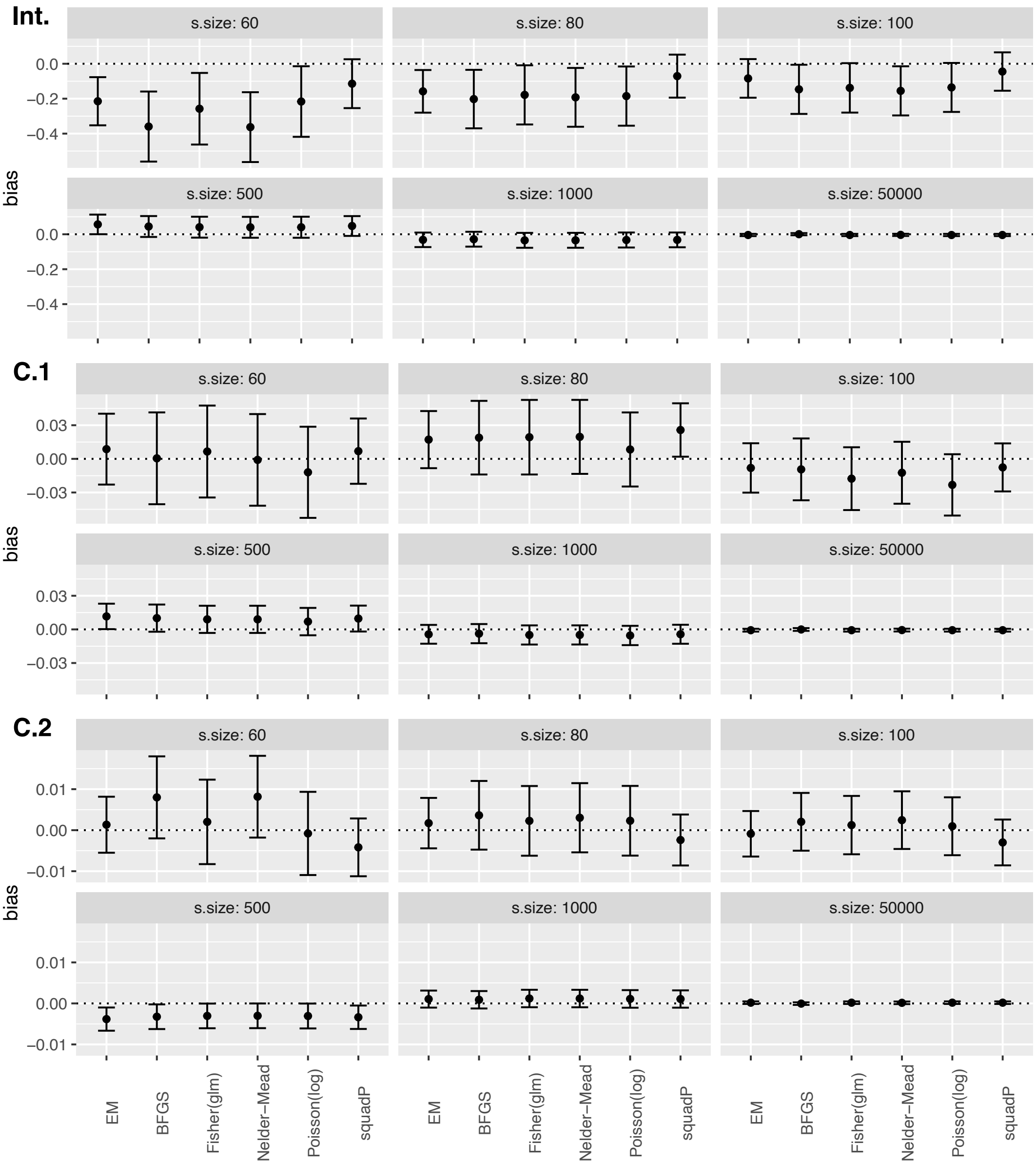

Figure 16: Absolute bias centered on 0 with confidence interval at 95% level. The bias estimates are for 6 scenarios with event probability 12%. Different sample sizes (60, 80, 100, 500, 1000, 50000) indicating different scenarios. y-axis: bias for intercept (Int), independent variables C.1, and C.2. x-axis: six statistical methods used for each scenario.

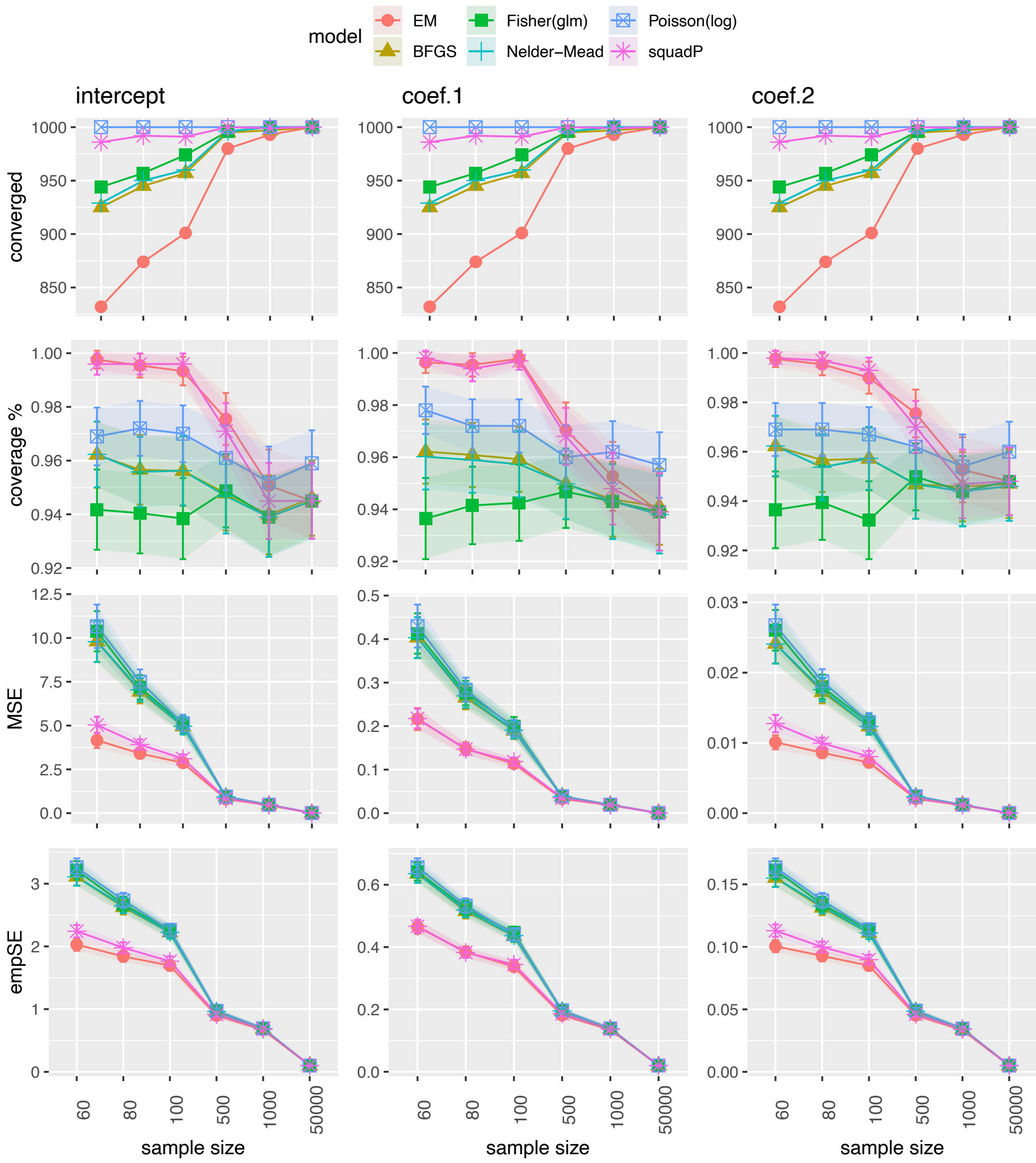

Figure 17: Performance measurements of 6 scenarios with event probability 12%. Convergence rate (at the top), coverage probability, MSE, and EmpSE (y-axis) for the intercept, independent variables coef.1, and coef.2 using the underlying statistical methods fro each scenario. Each type of shapes with shaded area (confidence interval) is representing a different method. The measurements are done for 6 scenarios that are derived from six different sample sizes (60,80,100,500,1000,50000) (x-axis).

Table 17: Performance measurements of scenarios 13→18 for intercept.

Coverage		Bias		MSE		EmpSE				
cover.	mcse	bias	mcse	mse	mcse	empse	mcse	model	s.size	converged
0.998	0.002	-0.215	0.070	4.157	0.223	2.029	0.050	EM	60	832
0.962	0.006	-0.360	0.102	9.801	0.594	3.112	0.072	BFGS	60	925
0.942	0.008	-0.258	0.105	10.382	0.584	3.214	0.074	Fisher(glm)	60	944
0.962	0.006	-0.363	0.102	9.787	0.593	3.109	0.072	Nelder-Mead	60	929
0.969	0.005	-0.216	0.103	10.668	0.631	3.261	0.073	Poisson(log)	60	1000
0.996	0.002	-0.114	0.071	5.034	0.240	2.242	0.051	squadP	60	986
0.995	0.002	-0.158	0.062	3.408	0.150	1.840	0.044	EM	80	874
0.957	0.007	-0.202	0.085	6.931	0.338	2.626	0.060	BFGS	80	945
0.940	0.008	-0.178	0.086	7.177	0.356	2.675	0.061	Fisher(glm)	80	957
0.956	0.007	-0.192	0.086	7.038	0.345	2.647	0.061	Nelder-Mead	80	950
0.972	0.005	-0.185	0.086	7.505	0.356	2.735	0.061	Poisson(log)	80	1000
0.996	0.002	-0.071	0.063	3.916	0.152	1.979	0.044	squadP	80	992
0.993	0.003	-0.084	0.056	2.874	0.124	1.694	0.040	EM	100	901
0.956	0.007	-0.146	0.072	4.954	0.238	2.222	0.051	BFGS	100	957
0.938	0.008	-0.138	0.072	5.091	0.249	2.253	0.051	Fisher(glm)	100	974
0.956	0.007	-0.155	0.072	4.965	0.238	2.224	0.051	Nelder-Mead	100	960
0.970	0.005	-0.136	0.072	5.131	0.248	2.262	0.051	Poisson(log)	100	1000
0.996	0.002	-0.045	0.056	3.110	0.126	1.764	0.040	squadP	100	991
0.976	0.005	0.057	0.029	0.807	0.031	0.897	0.020	EM	500	980
0.948	0.007	0.045	0.031	0.929	0.040	0.963	0.022	BFGS	500	995
0.949	0.007	0.041	0.031	0.928	0.040	0.963	0.022	Fisher(glm)	500	996
0.947	0.007	0.040	0.031	0.928	0.040	0.963	0.022	Nelder-Mead	500	996
0.961	0.006	0.040	0.031	0.946	0.041	0.972	0.022	Poisson(log)	500	1000
0.971	0.005	0.047	0.029	0.838	0.031	0.915	0.020	squadP	500	1000
0.951	0.007	-0.032	0.021	0.448	0.020	0.669	0.015	EM	1000	993
0.940	0.008	-0.028	0.022	0.470	0.022	0.685	0.015	BFGS	1000	997
0.939	0.008	-0.035	0.022	0.475	0.022	0.689	0.015	Fisher(glm)	1000	1000
0.939	0.008	-0.035	0.022	0.475	0.022	0.689	0.015	Nelder-Mead	1000	1000
0.952	0.007	-0.033	0.022	0.481	0.022	0.693	0.016	Poisson(log)	1000	1000
0.945	0.007	-0.032	0.022	0.465	0.021	0.681	0.015	squadP	1000	999
0.945	0.007	-0.004	0.003	0.010	0.000	0.101	0.002	EM	50000	1000
0.946	0.007	0.001	0.003	0.010	0.000	0.101	0.002	BFGS	50000	1000
0.945	0.007	-0.004	0.003	0.010	0.000	0.101	0.002	Fisher(glm)	50000	1000
0.945	0.007	-0.003	0.003	0.010	0.000	0.101	0.002	Nelder-Mead	50000	1000
0.959	0.006	-0.004	0.003	0.010	0.000	0.101	0.002	Poisson(log)	50000	1000
0.945	0.007	-0.004	0.003	0.010	0.000	0.101	0.002	squadP	50000	1000

Table 18: Performance measurements of scenarios 13→18 for coef.1

Coverage		Bias		MSE		EmpSE				
cover.	mcse	bias	mcse	mse	mcse	empse	mcse	model	s.size	converged
0.996	0.002	0.009	0.016	0.216	0.013	0.465	0.011	EM	60	832
0.962	0.006	0.000	0.021	0.403	0.024	0.635	0.015	BFGS	60	925
0.936	0.008	0.007	0.021	0.413	0.024	0.643	0.015	Fisher(glm)	60	944
0.960	0.006	-0.001	0.021	0.403	0.024	0.635	0.015	Nelder-Mead	60	929
0.978	0.005	-0.012	0.021	0.430	0.025	0.656	0.015	Poisson(log)	60	1000
0.998	0.001	0.007	0.015	0.218	0.011	0.467	0.011	squadP	60	986
0.995	0.002	0.017	0.013	0.148	0.008	0.384	0.009	EM	80	874
0.961	0.006	0.019	0.017	0.265	0.014	0.515	0.012	BFGS	80	945
0.941	0.008	0.019	0.017	0.276	0.014	0.525	0.012	Fisher(glm)	80	957
0.959	0.006	0.020	0.017	0.269	0.014	0.519	0.012	Nelder-Mead	80	950
0.972	0.005	0.008	0.017	0.284	0.014	0.533	0.012	Poisson(log)	80	1000
0.994	0.002	0.026	0.012	0.147	0.007	0.383	0.009	squadP	80	992
0.998	0.002	-0.008	0.011	0.114	0.005	0.337	0.008	EM	100	901
0.959	0.006	-0.009	0.014	0.189	0.010	0.435	0.010	BFGS	100	957
0.943	0.007	-0.018	0.014	0.199	0.011	0.446	0.010	Fisher(glm)	100	974
0.957	0.007	-0.012	0.014	0.191	0.010	0.437	0.010	Nelder-Mead	100	960
0.972	0.005	-0.023	0.014	0.195	0.010	0.441	0.010	Poisson(log)	100	1000
0.997	0.002	-0.008	0.011	0.118	0.005	0.344	0.008	squadP	100	991
0.970	0.005	0.012	0.006	0.033	0.001	0.181	0.004	EM	500	980
0.950	0.007	0.010	0.006	0.038	0.002	0.195	0.004	BFGS	500	995
0.947	0.007	0.009	0.006	0.038	0.002	0.195	0.004	Fisher(glm)	500	996
0.950	0.007	0.009	0.006	0.038	0.002	0.195	0.004	Nelder-Mead	500	996
0.960	0.006	0.007	0.006	0.039	0.002	0.197	0.004	Poisson(log)	500	1000
0.968	0.006	0.010	0.006	0.035	0.001	0.186	0.004	squadP	500	1000
0.953	0.007	-0.004	0.004	0.018	0.001	0.135	0.003	EM	1000	993
0.944	0.007	-0.004	0.004	0.019	0.001	0.138	0.003	BFGS	1000	997
0.943	0.007	-0.005	0.004	0.019	0.001	0.138	0.003	Fisher(glm)	1000	1000
0.943	0.007	-0.005	0.004	0.019	0.001	0.138	0.003	Nelder-Mead	1000	1000
0.962	0.006	-0.005	0.004	0.019	0.001	0.139	0.003	Poisson(log)	1000	1000
0.948	0.007	-0.004	0.004	0.019	0.001	0.137	0.003	squadP	1000	999
0.939	0.008	-0.001	0.001	0.000	0.000	0.020	0.000	EM	50000	1000
0.941	0.007	0.000	0.001	0.000	0.000	0.020	0.000	BFGS	50000	1000
0.939	0.008	-0.001	0.001	0.000	0.000	0.020	0.000	Fisher(glm)	50000	1000
0.938	0.008	-0.001	0.001	0.000	0.000	0.020	0.000	Nelder-Mead	50000	1000
0.957	0.006	-0.001	0.001	0.000	0.000	0.020	0.000	Poisson(log)	50000	1000
0.939	0.008	-0.001	0.001	0.000	0.000	0.020	0.000	squadP	50000	1000

Table 19: Performance measurements of scenarios 13→18 for coef.2.

Coverage		Bias		MSE		EmpSE				
cover.	mcse	bias	mcse	mse	mcse	empse	mcse	model	s.size	converged
0.998	0.002	0.001	0.003	0.010	0.001	0.465	0.011	EM	60	832
0.962	0.006	0.008	0.005	0.024	0.001	0.635	0.015	BFGS	60	925
0.936	0.008	0.002	0.005	0.026	0.001	0.643	0.015	Fisher(glm)	60	944
0.962	0.006	0.008	0.005	0.024	0.001	0.635	0.015	Nelder-Mead	60	929
0.969	0.005	-0.001	0.005	0.027	0.002	0.656	0.015	Poisson(log)	60	1000
0.998	0.001	-0.004	0.004	0.013	0.001	0.467	0.011	squadP	60	986
0.995	0.002	0.002	0.003	0.009	0.000	0.384	0.009	EM	80	874
0.957	0.007	0.004	0.004	0.017	0.001	0.515	0.012	BFGS	80	945
0.939	0.008	0.002	0.004	0.018	0.001	0.525	0.012	Fisher(glm)	80	957
0.954	0.007	0.003	0.004	0.018	0.001	0.519	0.012	Nelder-Mead	80	950
0.969	0.005	0.002	0.004	0.019	0.001	0.533	0.012	Poisson(log)	80	1000
0.997	0.002	-0.002	0.003	0.010	0.000	0.383	0.009	squadP	80	992
0.990	0.003	-0.001	0.003	0.007	0.000	0.337	0.008	EM	100	901
0.957	0.007	0.002	0.004	0.012	0.001	0.435	0.010	BFGS	100	957
0.932	0.008	0.001	0.004	0.013	0.001	0.446	0.010	Fisher(glm)	100	974
0.957	0.007	0.002	0.004	0.012	0.001	0.437	0.010	Nelder-Mead	100	960
0.967	0.006	0.001	0.004	0.013	0.001	0.441	0.010	Poisson(log)	100	1000
0.993	0.003	-0.003	0.003	0.008	0.000	0.344	0.008	squadP	100	991
0.976	0.005	-0.004	0.001	0.002	0.000	0.181	0.004	EM	500	980
0.947	0.007	-0.003	0.002	0.002	0.000	0.195	0.004	BFGS	500	995
0.950	0.007	-0.003	0.002	0.002	0.000	0.195	0.004	Fisher(glm)	500	996
0.947	0.007	-0.003	0.002	0.002	0.000	0.195	0.004	Nelder-Mead	500	996
0.962	0.006	-0.003	0.002	0.002	0.000	0.197	0.004	Poisson(log)	500	1000
0.970	0.005	-0.003	0.001	0.002	0.000	0.186	0.004	squadP	500	1000
0.953	0.007	0.001	0.001	0.001	0.000	0.135	0.003	EM	1000	993
0.946	0.007	0.001	0.001	0.001	0.000	0.138	0.003	BFGS	1000	997
0.944	0.007	0.001	0.001	0.001	0.000	0.138	0.003	Fisher(glm)	1000	1000
0.944	0.007	0.001	0.001	0.001	0.000	0.138	0.003	Nelder-Mead	1000	1000
0.954	0.007	0.001	0.001	0.001	0.000	0.139	0.003	Poisson(log)	1000	1000
0.947	0.007	0.001	0.001	0.001	0.000	0.137	0.003	squadP	1000	999
0.948	0.007	0.000	0.000	0.000	0.000	0.020	0.000	EM	50000	1000
0.947	0.007	0.000	0.000	0.000	0.000	0.020	0.000	BFGS	50000	1000
0.948	0.007	0.000	0.000	0.000	0.000	0.020	0.000	Fisher(glm)	50000	1000
0.946	0.007	0.000	0.000	0.000	0.000	0.020	0.000	Nelder-Mead	50000	1000
0.960	0.006	0.000	0.000	0.000	0.000	0.020	0.000	Poisson(log)	50000	1000
0.948	0.007	0.000	0.000	0.000	0.000	0.020	0.000	squadP	50000	1000

5.3.4 Scenarios with event probability 24%

Here the main findings of scenarios from 19 → 24 (6 scenarios) with event probability 24% and different sample sizes (60, 80, 100, 500, 1000, 50000) are discussed. The performance measurements such as absolute bias, convergence rate, coverage probability, mean squared error, and the empirical standard error are measured, compared, and presented as follows.

Figure 18 presents the absolute bias measurements for the intercept, and the independent variables C.1 and C.2. A comparison of bias measurements shows us a decrease in the range of confidence interval with the increase of sample size as expected. The six statistical methods being compared and evaluated show a relatively similar pattern in case of scenarios 500, 1000, and 5000. In scenarios with small sample sizes 60, 80 and 100 the EM-type method show a higher bias of 0.39, 0.35, and 0.31, which is significantly different compared with other methods. In addition, that EM-type method, as shown in figure 19, has the lowest convergence rate particularly in case of scenarios with small sample sizes of 60, 80, and 100. While squadP and Poisson(log) have the highest convergence rate near to 100% in all 6 scenarios.

Coverage probabilities from squadP and EM-type methods, as shown in figure 19, are approximately 100% in scenarios with small sample sizes such as 60, 80, 100. In addition, both methods have significantly less MSE and empSE compared with other methods. As expected, larger sample sizes (500, 1000, 50000) is associated with a decrease of coverage probabilities, MSE, and empSE.

Absolute bias, convergence rate, coverage probabilities, MSE, and empSE of the estimated relative risk (RR) in log scale from the six statistical methods in each of the 6 scenarios are shown in figure 19 and table 20 for the intercept (β_0), table 21 for coef.1 (β_1), and table 22 for coef.2 (β_2).

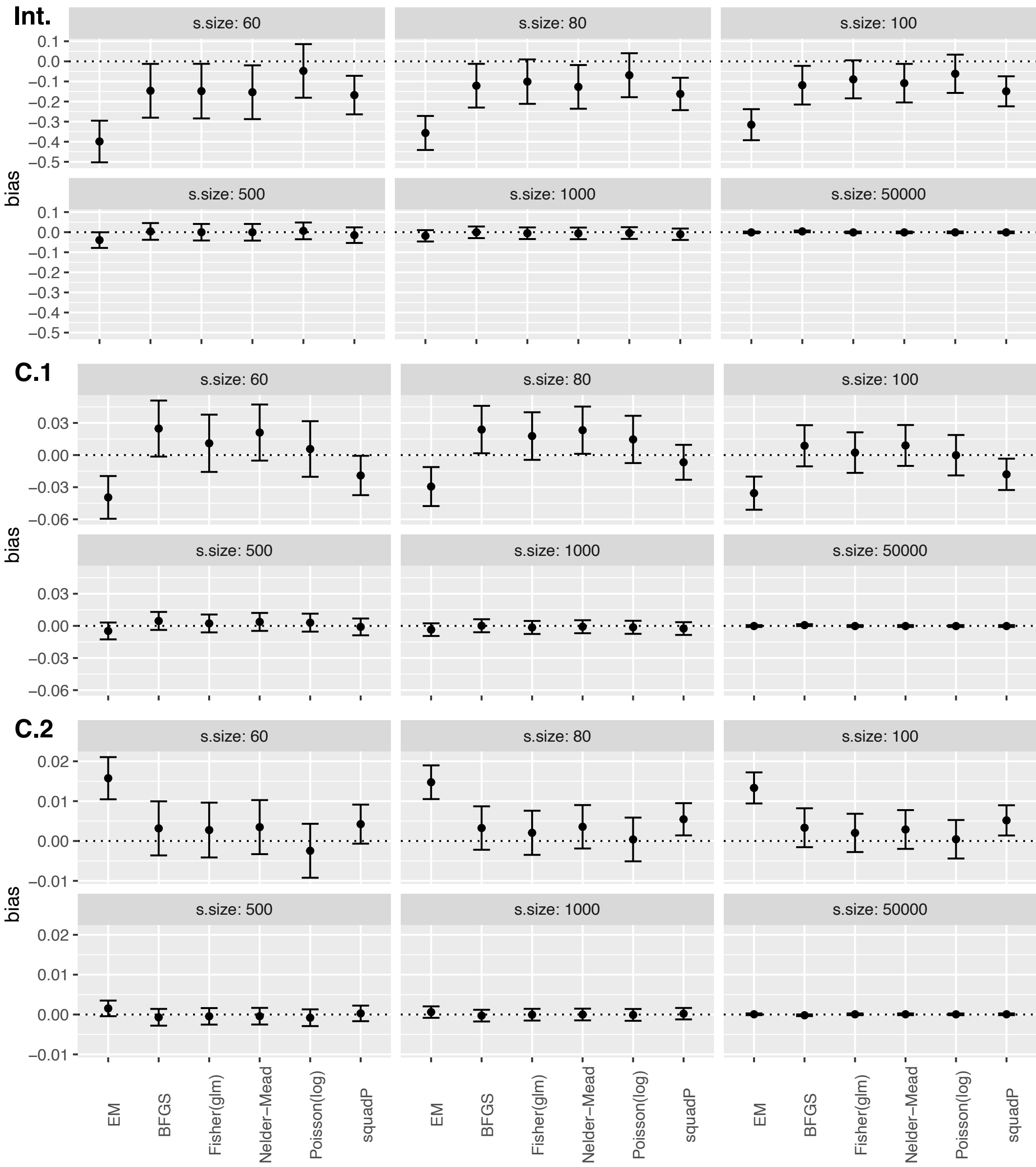

Figure 18: The absolute biases of the estimated log(RR) from the six methods in each of the 6 scenarios with event probability 24%. Different sample sizes (60, 80, 100, 500, 1000, 50000) indicating different scenarios. y-axis: bias for intercept (Int), independent variables C.1, and C.2. x-axis: six statistical methods used for each scenario.

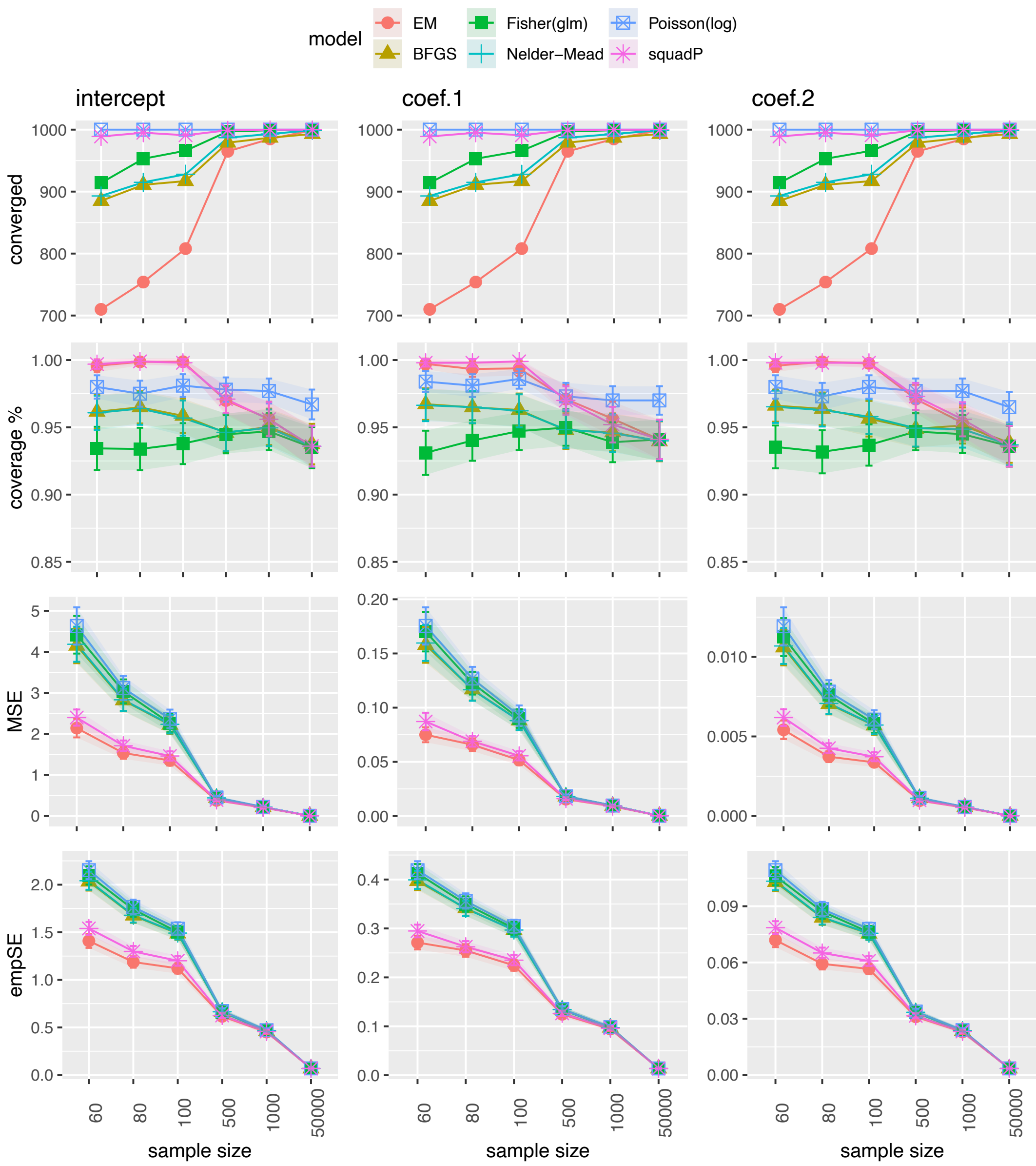

Figure 19: Performance measurements of 6 scenarios with event probability 24%. Convergence rate (at the top), coverage probability, MSE, and EmpSE (y-axis) for the intercept, independent variables coef.1, and coef.2 using the underlying statistical methods for each scenario. Each type of shapes with shaded area (confidence interval) is representing a different method. The measurements are done for 6 scenarios that are derived from six different sample sizes 60,80,100,500,1000, and 50000 (x-axis).

Table 20: Performance measurements of scenarios 19→24 for intercept.

Coverage		Bias		MSE		EmpSE				
cover.	mcse	bias	mcse	mse	mcse	empse	mcse	model	s.size	converged
0.996	0.002	-0.399	0.053	2.142	0.116	1.409	0.037	EM	60	710
0.962	0.006	-0.146	0.068	4.149	0.218	2.033	0.048	BFGS	60	885
0.934	0.008	-0.148	0.069	4.416	0.234	2.097	0.049	Fisher(glm)	60	914
0.961	0.006	-0.154	0.068	4.185	0.218	2.041	0.048	Nelder-Mead	60	893
0.980	0.004	-0.048	0.068	4.630	0.232	2.152	0.048	Poisson(log)	60	1000
0.997	0.002	-0.168	0.049	2.398	0.101	1.540	0.035	squadP	60	989
0.999	0.001	-0.357	0.043	1.535	0.072	1.187	0.031	EM	80	754
0.965	0.006	-0.121	0.056	2.819	0.137	1.675	0.039	BFGS	80	911
0.934	0.008	-0.101	0.056	3.037	0.148	1.741	0.040	Fisher(glm)	80	953
0.964	0.006	-0.127	0.056	2.832	0.137	1.679	0.039	Nelder-Mead	80	915
0.975	0.005	-0.069	0.056	3.118	0.150	1.765	0.039	Poisson(log)	80	1000
0.999	0.001	-0.162	0.041	1.712	0.065	1.299	0.029	squadP	80	995
0.999	0.001	-0.315	0.039	1.354	0.064	1.121	0.028	EM	100	808
0.959	0.007	-0.119	0.049	2.223	0.115	1.487	0.035	BFGS	100	917
0.938	0.008	-0.090	0.048	2.261	0.113	1.502	0.034	Fisher(glm)	100	966
0.957	0.007	-0.109	0.049	2.234	0.114	1.492	0.035	Nelder-Mead	100	928
0.981	0.004	-0.062	0.049	2.366	0.116	1.538	0.034	Poisson(log)	100	1000
0.998	0.001	-0.149	0.038	1.462	0.058	1.200	0.027	squadP	100	991
0.970	0.005	-0.039	0.020	0.381	0.017	0.616	0.014	EM	500	965
0.946	0.007	0.004	0.021	0.442	0.020	0.665	0.015	BFGS	500	979
0.945	0.007	0.000	0.021	0.440	0.020	0.663	0.015	Fisher(glm)	500	997
0.946	0.007	0.000	0.021	0.441	0.020	0.665	0.015	Nelder-Mead	500	987
0.978	0.005	0.007	0.021	0.451	0.020	0.672	0.015	Poisson(log)	500	1000
0.971	0.005	-0.015	0.020	0.391	0.016	0.626	0.014	squadP	500	999
0.956	0.007	-0.018	0.014	0.204	0.009	0.452	0.010	EM	1000	985
0.950	0.007	-0.001	0.015	0.213	0.010	0.462	0.010	BFGS	1000	987
0.947	0.007	-0.005	0.015	0.218	0.010	0.467	0.010	Fisher(glm)	1000	999
0.950	0.007	-0.006	0.015	0.217	0.010	0.466	0.010	Nelder-Mead	1000	993
0.977	0.005	-0.004	0.015	0.221	0.010	0.471	0.011	Poisson(log)	1000	1000
0.956	0.006	-0.010	0.014	0.209	0.009	0.457	0.010	squadP	1000	1000
0.935	0.008	-0.001	0.002	0.005	0.000	0.069	0.002	EM	50000	999
0.938	0.008	0.004	0.002	0.005	0.000	0.069	0.002	BFGS	50000	993
0.935	0.008	-0.001	0.002	0.005	0.000	0.069	0.002	Fisher(glm)	50000	1000
0.936	0.008	-0.001	0.002	0.005	0.000	0.069	0.002	Nelder-Mead	50000	999
0.967	0.006	-0.001	0.002	0.005	0.000	0.070	0.002	Poisson(log)	50000	1000
0.936	0.008	-0.001	0.002	0.005	0.000	0.069	0.002	squadP	50000	1000

Table 21: Performance measurements of scenarios 19→24 for coef.1.

Coverage		Bias		MSE		EmpSE				
cover.	mcse	bias	mcse	mse	mcse	empse	mcse	model	s.size	converged
0.997	0.002	-0.040	0.010	0.075	0.004	0.271	0.007	EM	60	710
0.967	0.006	0.025	0.013	0.158	0.008	0.397	0.009	BFGS	60	885
0.931	0.008	0.011	0.014	0.170	0.009	0.413	0.010	Fisher(glm)	60	914
0.966	0.006	0.021	0.013	0.160	0.008	0.399	0.009	Nelder-Mead	60	893
0.984	0.004	0.006	0.013	0.175	0.009	0.419	0.009	Poisson(log)	60	1000
0.998	0.001	-0.019	0.009	0.087	0.004	0.295	0.007	squadP	60	989
0.993	0.003	-0.029	0.009	0.066	0.003	0.255	0.007	EM	80	754
0.965	0.006	0.024	0.011	0.117	0.005	0.341	0.008	BFGS	80	911
0.940	0.008	0.018	0.011	0.123	0.005	0.350	0.008	Fisher(glm)	80	953
0.965	0.006	0.023	0.011	0.117	0.005	0.341	0.008	Nelder-Mead	80	915
0.981	0.004	0.015	0.011	0.127	0.005	0.356	0.008	Poisson(log)	80	1000
0.998	0.001	-0.007	0.008	0.069	0.003	0.263	0.006	squadP	80	995
0.994	0.003	-0.036	0.008	0.052	0.002	0.224	0.006	EM	100	808
0.963	0.006	0.009	0.010	0.088	0.004	0.297	0.007	BFGS	100	917
0.947	0.007	0.002	0.010	0.090	0.004	0.300	0.007	Fisher(glm)	100	966
0.962	0.006	0.009	0.010	0.088	0.004	0.297	0.007	Nelder-Mead	100	928
0.986	0.004	0.000	0.010	0.093	0.005	0.305	0.007	Poisson(log)	100	1000
0.999	0.001	-0.018	0.007	0.056	0.002	0.235	0.005	squadP	100	991
0.971	0.005	-0.005	0.004	0.016	0.001	0.125	0.003	EM	500	965
0.948	0.007	0.005	0.004	0.018	0.001	0.134	0.003	BFGS	500	979
0.950	0.007	0.002	0.004	0.018	0.001	0.134	0.003	Fisher(glm)	500	997
0.948	0.007	0.004	0.004	0.018	0.001	0.134	0.003	Nelder-Mead	500	987
0.973	0.005	0.003	0.004	0.018	0.001	0.135	0.003	Poisson(log)	500	1000
0.970	0.005	-0.001	0.004	0.016	0.001	0.126	0.003	squadP	500	999
0.956	0.007	-0.004	0.003	0.009	0.000	0.094	0.002	EM	1000	985
0.946	0.007	0.000	0.003	0.009	0.000	0.097	0.002	BFGS	1000	987
0.939	0.008	-0.001	0.003	0.010	0.000	0.098	0.002	Fisher(glm)	1000	999
0.946	0.007	-0.001	0.003	0.009	0.000	0.097	0.002	Nelder-Mead	1000	993
0.970	0.005	-0.001	0.003	0.010	0.000	0.098	0.002	Poisson(log)	1000	1000
0.952	0.007	-0.002	0.003	0.009	0.000	0.096	0.002	squadP	1000	1000
0.941	0.007	0.000	0.000	0.000	0.000	0.014	0.000	EM	50000	999
0.940	0.008	0.001	0.000	0.000	0.000	0.014	0.000	BFGS	50000	993
0.941	0.007	0.000	0.000	0.000	0.000	0.014	0.000	Fisher(glm)	50000	1000
0.940	0.008	0.000	0.000	0.000	0.000	0.014	0.000	Nelder-Mead	50000	999
0.970	0.005	0.000	0.000	0.000	0.000	0.014	0.000	Poisson(log)	50000	1000
0.941	0.007	0.000	0.000	0.000	0.000	0.014	0.000	squadP	50000	1000

Table 22: Performance measurements of scenarios 19→24 for coef.2.

Coverage		Bias		MSE		EmpSE				
cover.	mcse	bias	mcse	mse	mcse	empse	mcse	model	s.size	converged
0.996	0.002	0.016	0.003	0.005	0.000	0.271	0.007	EM	60	710
0.966	0.006	0.003	0.003	0.011	0.001	0.397	0.009	BFGS	60	885
0.935	0.008	0.003	0.004	0.011	0.001	0.413	0.010	Fisher(glm)	60	914
0.965	0.006	0.003	0.003	0.011	0.001	0.399	0.009	Nelder-Mead	60	893
0.980	0.004	-0.002	0.003	0.012	0.001	0.419	0.009	Poisson(log)	60	1000
0.998	0.001	0.004	0.002	0.006	0.000	0.295	0.007	squadP	60	989
0.999	0.001	0.015	0.002	0.004	0.000	0.255	0.007	EM	80	754
0.964	0.006	0.003	0.003	0.007	0.000	0.341	0.008	BFGS	80	911
0.932	0.008	0.002	0.003	0.008	0.000	0.350	0.008	Fisher(glm)	80	953
0.963	0.006	0.004	0.003	0.007	0.000	0.341	0.008	Nelder-Mead	80	915
0.973	0.005	0.000	0.003	0.008	0.000	0.356	0.008	Poisson(log)	80	1000
0.998	0.001	0.005	0.002	0.004	0.000	0.263	0.006	squadP	80	995
0.998	0.002	0.013	0.002	0.003	0.000	0.224	0.006	EM	100	808
0.956	0.007	0.003	0.002	0.006	0.000	0.297	0.007	BFGS	100	917
0.937	0.008	0.002	0.002	0.006	0.000	0.300	0.007	Fisher(glm)	100	966
0.958	0.007	0.003	0.002	0.006	0.000	0.297	0.007	Nelder-Mead	100	928
0.980	0.004	0.000	0.002	0.006	0.000	0.305	0.007	Poisson(log)	100	1000
0.998	0.001	0.005	0.002	0.004	0.000	0.235	0.005	squadP	100	991
0.971	0.005	0.002	0.001	0.001	0.000	0.125	0.003	EM	500	965
0.949	0.007	-0.001	0.001	0.001	0.000	0.134	0.003	BFGS	500	979
0.947	0.007	0.000	0.001	0.001	0.000	0.134	0.003	Fisher(glm)	500	997
0.949	0.007	0.000	0.001	0.001	0.000	0.134	0.003	Nelder-Mead	500	987
0.977	0.005	-0.001	0.001	0.001	0.000	0.135	0.003	Poisson(log)	500	1000
0.973	0.005	0.000	0.001	0.001	0.000	0.126	0.003	squadP	500	999
0.953	0.007	0.001	0.001	0.001	0.000	0.094	0.002	EM	1000	985
0.951	0.007	0.000	0.001	0.001	0.000	0.097	0.002	BFGS	1000	987
0.945	0.007	0.000	0.001	0.001	0.000	0.098	0.002	Fisher(glm)	1000	999
0.949	0.007	0.000	0.001	0.001	0.000	0.097	0.002	Nelder-Mead	1000	993
0.977	0.005	0.000	0.001	0.001	0.000	0.098	0.002	Poisson(log)	1000	1000
0.956	0.006	0.000	0.001	0.001	0.000	0.096	0.002	squadP	1000	1000
0.936	0.008	0.000	0.000	0.000	0.000	0.014	0.000	EM	50000	999
0.939	0.008	0.000	0.000	0.000	0.000	0.014	0.000	BFGS	50000	993
0.936	0.008	0.000	0.000	0.000	0.000	0.014	0.000	Fisher(glm)	50000	1000
0.937	0.008	0.000	0.000	0.000	0.000	0.014	0.000	Nelder-Mead	50000	999
0.965	0.006	0.000	0.000	0.000	0.000	0.014	0.000	Poisson(log)	50000	1000
0.936	0.008	0.000	0.000	0.000	0.000	0.014	0.000	squadP	50000	1000

5.3.5 Scenarios with event probability 48%

Here the main findings of scenarios from $25 \rightarrow 30$ (6 scenarios) with event probability 48% and different sample sizes of $60, 80, 100, 500, 1000, 50000$ are discussed.

The absolute biases of the estimated log (RR) from the six models in each of the 6 scenarios are shown in figure 20. in the figure, as expected, the decrease of bias is associated with an increase of sample sizes, however, Nelder-Mead method behaved slightly different than other methods (higher biases) even though with larger sample sizes such as 500, and 1000. Additionally, biased estimates from Nelder-Mead method in these two scenarios are associated with lower coverage probabilities as shown in figure 21.

Estimates of squadP method were relatively very near to 0 even though with small sample sizes such as 60, and 80 as shown in the figure for intercept (β_0), coef.1 (β_1), and coef.2 (β_2).

The performance measures in figure 21 show that the two methods squadP and Poisson(log) have the highest convergence rate (approximately 100%) in case of all six scenarios compared with other approaches, while EM-type method has the lowest, reaching approximately 50% convergence rate in case of scenario with 60 sample size.

The statistical methods as expected have similar coverage probability in case of scenario with the large sample 50000 except Poisson(log) which has slightly higher coverage probability associated with higher MSE and empSE as shown in figure 21. In contrast, squadP method and EM-type methods have the lowest MSE and empSE.

The performance measures of the estimated log (RR) from the six models in each of the 6 scenarios discussed above are shown table 23 for the intercept (β_0), table 24 for coef.1 (β_1), and table 25 for coef.2 (β_2).

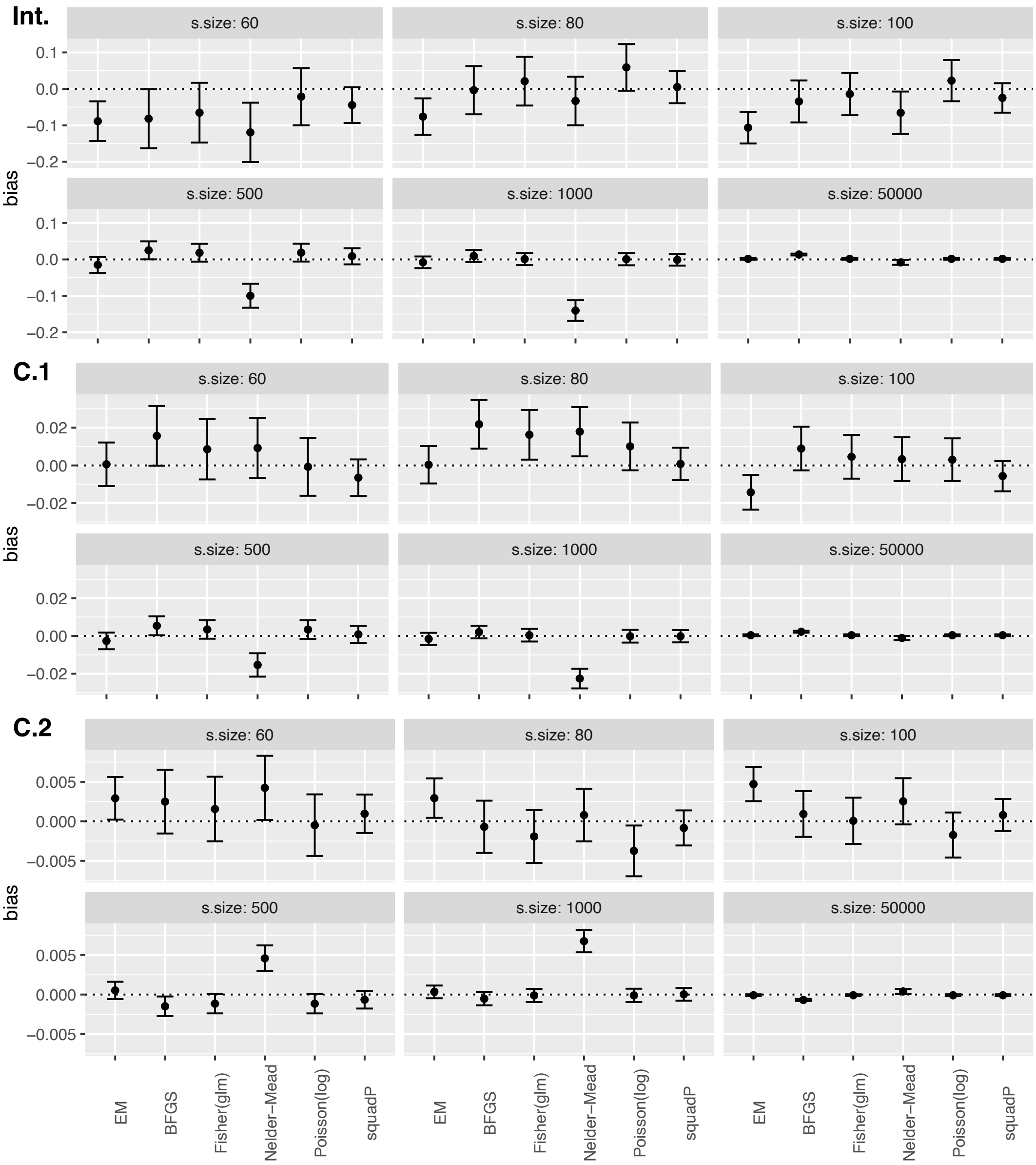

Figure 20: The absolute biases of the estimated log(RR) from the six methods in each of the 6 scenarios with event probability 48% with sample sizes 60, 80, 100, 500, 1000, and 50000. y-axis: bias for intercept (Int), independent variables C.1, and C.2. x-axis: six statistical methods used for each scenario.

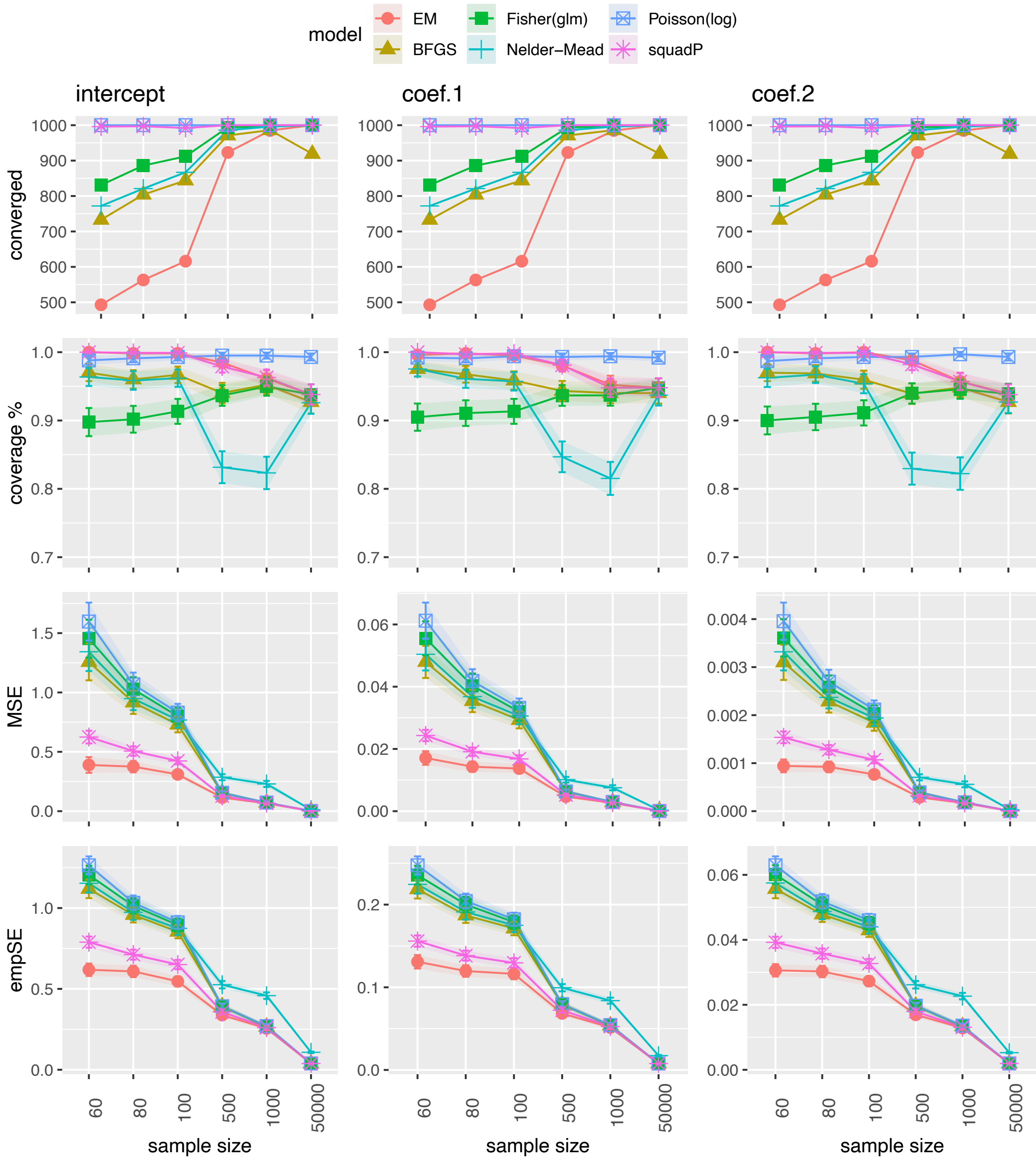

Figure 21: Performance measurements of scenarios with event probability 48%. Convergence rate (at the top), coverage probability, MSE, and empSE (y-axis) for the intercept, independent variables coef.1, and coef.2 using the underlying statistical methods fro each scenario. Each type of shapes with shaded area (confidence interval) is representing a different method. The measurements are done for 6 scenarios that are derived from six different sample sizes 60,80,100,500,1000, and 50000 (x-axis).

Table 23: Performance measurements of scenarios 25→30 for intercept.

Coverage		Bias		MSE		EmpSE				
cover.	mcse	bias	mcse	mse	mcse	empse	mcse	model	s.size	converged
1.000	0.000	-0.089	0.028	0.389	0.034	0.618	0.020	EM	60	493
0.970	0.006	-0.082	0.041	1.257	0.079	1.119	0.029	BFGS	60	733
0.898	0.011	-0.065	0.042	1.453	0.082	1.204	0.030	Fisher(glm)	60	831
0.964	0.007	-0.119	0.042	1.343	0.083	1.153	0.029	Nelder-Mead	60	772
0.988	0.003	-0.021	0.040	1.597	0.082	1.264	0.028	Poisson(log)	60	1000
1.000	0.000	-0.044	0.025	0.624	0.027	0.789	0.018	squadP	60	996
0.998	0.002	-0.076	0.026	0.375	0.025	0.608	0.018	EM	80	563
0.960	0.007	-0.004	0.034	0.916	0.049	0.958	0.024	BFGS	80	804
0.902	0.010	0.021	0.034	1.027	0.052	1.014	0.024	Fisher(glm)	80	886
0.959	0.007	-0.033	0.034	0.948	0.049	0.974	0.024	Nelder-Mead	80	821
0.991	0.003	0.059	0.033	1.069	0.050	1.033	0.023	Poisson(log)	80	1000
0.999	0.001	0.005	0.023	0.506	0.020	0.712	0.016	squadP	80	997
0.998	0.002	-0.107	0.022	0.310	0.015	0.547	0.016	EM	100	616
0.967	0.006	-0.034	0.029	0.729	0.033	0.854	0.021	BFGS	100	844
0.913	0.009	-0.014	0.030	0.801	0.036	0.895	0.021	Fisher(glm)	100	912
0.962	0.006	-0.066	0.030	0.768	0.033	0.874	0.021	Nelder-Mead	100	867
0.993	0.003	0.023	0.029	0.830	0.038	0.911	0.020	Poisson(log)	100	1000
0.999	0.001	-0.025	0.021	0.423	0.015	0.650	0.015	squadP	100	992
0.985	0.004	-0.015	0.011	0.115	0.005	0.338	0.008	EM	500	923
0.940	0.008	0.025	0.013	0.155	0.007	0.393	0.009	BFGS	500	971
0.937	0.008	0.018	0.012	0.154	0.007	0.392	0.009	Fisher(glm)	500	994
0.832	0.012	-0.100	0.017	0.286	0.013	0.526	0.012	Nelder-Mead	500	986
0.995	0.002	0.019	0.012	0.154	0.007	0.393	0.009	Poisson(log)	500	1000
0.979	0.005	0.009	0.011	0.128	0.005	0.358	0.008	squadP	500	1000
0.961	0.006	-0.008	0.008	0.065	0.003	0.256	0.006	EM	1000	985
0.952	0.007	0.009	0.008	0.071	0.003	0.266	0.006	BFGS	1000	986
0.950	0.007	0.001	0.008	0.071	0.003	0.266	0.006	Fisher(glm)	1000	997
0.823	0.012	-0.140	0.015	0.229	0.013	0.458	0.010	Nelder-Mead	1000	996
0.995	0.002	0.001	0.009	0.073	0.003	0.270	0.006	Poisson(log)	1000	1000
0.963	0.006	-0.001	0.008	0.068	0.003	0.260	0.006	squadP	1000	1000
0.938	0.008	0.002	0.001	0.001	0.000	0.039	0.001	EM	50000	1000
0.927	0.009	0.013	0.001	0.002	0.000	0.038	0.001	BFGS	50000	919
0.938	0.008	0.002	0.001	0.001	0.000	0.039	0.001	Fisher(glm)	50000	1000
0.926	0.008	-0.008	0.003	0.012	0.003	0.108	0.002	Nelder-Mead	50000	1000
0.993	0.003	0.002	0.001	0.002	0.000	0.039	0.001	Poisson(log)	50000	1000
0.938	0.008	0.002	0.001	0.001	0.000	0.039	0.001	squadP	50000	1000

Table 24: Performance measurements of scenarios 25→30 for coef.1

Coverage		Bias		MSE		EmpSE				
cover.	mcse	bias	mcse	mse	mcse	empse	mcse	model	s.size	converged
0.996	0.003	0.001	0.006	0.017	0.001	0.131	0.004	EM	60	493
0.975	0.006	0.016	0.008	0.048	0.003	0.219	0.006	BFGS	60	733
0.905	0.010	0.009	0.008	0.056	0.003	0.236	0.006	Fisher(glm)	60	831
0.975	0.006	0.009	0.008	0.050	0.003	0.224	0.006	Nelder-Mead	60	772
0.992	0.003	-0.001	0.008	0.061	0.003	0.248	0.006	Poisson(log)	60	1000
1.000	0.000	-0.006	0.005	0.024	0.001	0.156	0.003	squadP	60	996
0.998	0.002	0.000	0.005	0.014	0.001	0.120	0.004	EM	80	563
0.968	0.006	0.022	0.007	0.035	0.002	0.187	0.005	BFGS	80	804
0.911	0.010	0.016	0.007	0.040	0.002	0.200	0.005	Fisher(glm)	80	886
0.961	0.007	0.018	0.007	0.037	0.002	0.191	0.005	Nelder-Mead	80	821
0.991	0.003	0.010	0.006	0.042	0.002	0.204	0.005	Poisson(log)	80	1000
0.997	0.002	0.001	0.004	0.019	0.001	0.138	0.003	squadP	80	997
0.995	0.003	-0.014	0.005	0.014	0.001	0.116	0.003	EM	100	616
0.959	0.007	0.009	0.006	0.029	0.001	0.171	0.004	BFGS	100	844
0.913	0.009	0.005	0.006	0.032	0.001	0.179	0.004	Fisher(glm)	100	912
0.957	0.007	0.003	0.006	0.031	0.001	0.175	0.004	Nelder-Mead	100	867
0.994	0.002	0.003	0.006	0.033	0.002	0.182	0.004	Poisson(log)	100	1000
0.998	0.001	-0.006	0.004	0.017	0.001	0.130	0.003	squadP	100	992
0.980	0.005	-0.003	0.002	0.005	0.000	0.068	0.002	EM	500	923
0.943	0.007	0.005	0.003	0.006	0.000	0.080	0.002	BFGS	500	971
0.937	0.008	0.003	0.003	0.006	0.000	0.079	0.002	Fisher(glm)	500	994
0.847	0.011	-0.015	0.003	0.010	0.000	0.099	0.002	Nelder-Mead	500	986
0.993	0.003	0.003	0.003	0.006	0.000	0.080	0.002	Poisson(log)	500	1000
0.981	0.004	0.001	0.002	0.005	0.000	0.072	0.002	squadP	500	1000
0.952	0.007	-0.002	0.002	0.003	0.000	0.051	0.001	EM	1000	985
0.940	0.008	0.002	0.002	0.003	0.000	0.054	0.001	BFGS	1000	986
0.937	0.008	0.000	0.002	0.003	0.000	0.054	0.001	Fisher(glm)	1000	997
0.815	0.012	-0.023	0.003	0.008	0.000	0.084	0.002	Nelder-Mead	1000	996
0.994	0.002	0.000	0.002	0.003	0.000	0.054	0.001	Poisson(log)	1000	1000
0.948	0.007	0.000	0.002	0.003	0.000	0.053	0.001	squadP	1000	1000
0.948	0.007	0.000	0.000	0.000	0.000	0.008	0.000	EM	50000	1000
0.940	0.008	0.002	0.000	0.000	0.000	0.008	0.000	BFGS	50000	919
0.948	0.007	0.000	0.000	0.000	0.000	0.008	0.000	Fisher(glm)	50000	1000
0.937	0.008	-0.001	0.001	0.000	0.000	0.017	0.000	Nelder-Mead	50000	1000
0.992	0.003	0.000	0.000	0.000	0.000	0.008	0.000	Poisson(log)	50000	1000
0.948	0.007	0.000	0.000	0.000	0.000	0.008	0.000	squadP	50000	1000

Table 25: Performance measurements of scenarios 25→30 for coef.2

Coverage		Bias		MSE		EmpSE				
cover.	mcse	bias	mcse	mse	mcse	empse	mcse	model	s.size	converged
1.000	0.000	0.003	0.001	0.001	0	0.131	0.004	EM	60	493
0.970	0.006	0.002	0.002	0.003	0	0.219	0.006	BFGS	60	733
0.900	0.010	0.002	0.002	0.004	0	0.236	0.006	Fisher(glm)	60	831
0.962	0.007	0.004	0.002	0.003	0	0.224	0.006	Nelder-Mead	60	772
0.987	0.004	0.000	0.002	0.004	0	0.248	0.006	Poisson(log)	60	1000
1.000	0.000	0.001	0.001	0.002	0	0.156	0.003	squadP	60	996
0.998	0.002	0.003	0.001	0.001	0	0.120	0.004	EM	80	563
0.969	0.006	-0.001	0.002	0.002	0	0.187	0.005	BFGS	80	804
0.905	0.010	-0.002	0.002	0.003	0	0.200	0.005	Fisher(glm)	80	886
0.967	0.006	0.001	0.002	0.002	0	0.191	0.005	Nelder-Mead	80	821
0.991	0.003	-0.004	0.002	0.003	0	0.204	0.005	Poisson(log)	80	1000
0.999	0.001	-0.001	0.001	0.001	0	0.138	0.003	squadP	80	997
1.000	0.000	0.005	0.001	0.001	0	0.116	0.003	EM	100	616
0.960	0.007	0.001	0.001	0.002	0	0.171	0.004	BFGS	100	844
0.911	0.009	0.000	0.001	0.002	0	0.179	0.004	Fisher(glm)	100	912
0.954	0.007	0.003	0.001	0.002	0	0.175	0.004	Nelder-Mead	100	867
0.993	0.003	-0.002	0.001	0.002	0	0.182	0.004	Poisson(log)	100	1000
0.999	0.001	0.001	0.001	0.001	0	0.130	0.003	squadP	100	992
0.988	0.004	0.001	0.001	0.000	0	0.068	0.002	EM	500	923
0.939	0.008	-0.001	0.001	0.000	0	0.080	0.002	BFGS	500	971
0.940	0.008	-0.001	0.001	0.000	0	0.079	0.002	Fisher(glm)	500	994
0.830	0.012	0.005	0.001	0.001	0	0.099	0.002	Nelder-Mead	500	986
0.993	0.003	-0.001	0.001	0.000	0	0.080	0.002	Poisson(log)	500	1000
0.982	0.004	-0.001	0.001	0.000	0	0.072	0.002	squadP	500	1000
0.957	0.006	0.000	0.000	0.000	0	0.051	0.001	EM	1000	985
0.948	0.007	-0.001	0.000	0.000	0	0.054	0.001	BFGS	1000	986
0.946	0.007	0.000	0.000	0.000	0	0.054	0.001	Fisher(glm)	1000	997
0.822	0.012	0.007	0.001	0.001	0	0.084	0.002	Nelder-Mead	1000	996
0.997	0.002	0.000	0.000	0.000	0	0.054	0.001	Poisson(log)	1000	1000
0.957	0.006	0.000	0.000	0.000	0	0.053	0.001	squadP	1000	1000
0.939	0.008	0.000	0.000	0.000	0	0.008	0.000	EM	50000	1000
0.927	0.009	-0.001	0.000	0.000	0	0.008	0.000	BFGS	50000	919
0.939	0.008	0.000	0.000	0.000	0	0.008	0.000	Fisher(glm)	50000	1000
0.927	0.008	0.000	0.000	0.000	0	0.017	0.000	Nelder-Mead	50000	1000
0.993	0.003	0.000	0.000	0.000	0	0.008	0.000	Poisson(log)	50000	1000
0.938	0.008	0.000	0.000	0.000	0	0.008	0.000	squadP	50000	1000

5.4 Monte Carlo simulation results: scenarios with 4 covariates

The current simulation study consists of 30 scenarios with 4 independent variables and variety of event probabilities $6\%, 12\%, 24\%$, and 48%. Simulations consists of 6 different scenarios for each even probability. As explained in *Monte Carlo Simulation* section, the four independent variables are uncorrelated. This Mont Carlo simulation study are summarized and the main findings are discussed in the following points.

5.4.1 Scenarios with event probability 6%

The main findings of scenarios from $31 \rightarrow 36$ (6 scenarios) with event probability 6% and different sample sizes $(250, 300, 400, 500, 1000, 10000)$ are discussed here.

In figure 22, the absolute biases of the estimated log(RR) from the six methods in each of the 6 scenarios are shown. Estimated biases follow relatively similar patterns. The decrease of biases is associated with an increase of sample sizes to reach approximately 0 bias when the sample size is 50000 for all scenarios.

As shown in figure 22, the estimated biases of coefficient C.3 in case of scenarios with sample sizes 250, 300 vary from a method to another. squadP and EM-type methods have lowest bias compared with other methods. On the other hand, EM-type method shows the lowest convergence rate compared with other methods that have approximately 100% convergence including squadP. The six methods have relatively similar coverage probability. However, Poisson(log), Fisher GLM, and Nelder-Mead have significantly higher MSE and empSE in case of coef.3, while squadP and EM-type algorithm have the least as shown in figure 22.

Data is shown in table 26 for coef.1 , table 27 for coef.2, table 28 for coef.3, and table 29 for coef.4

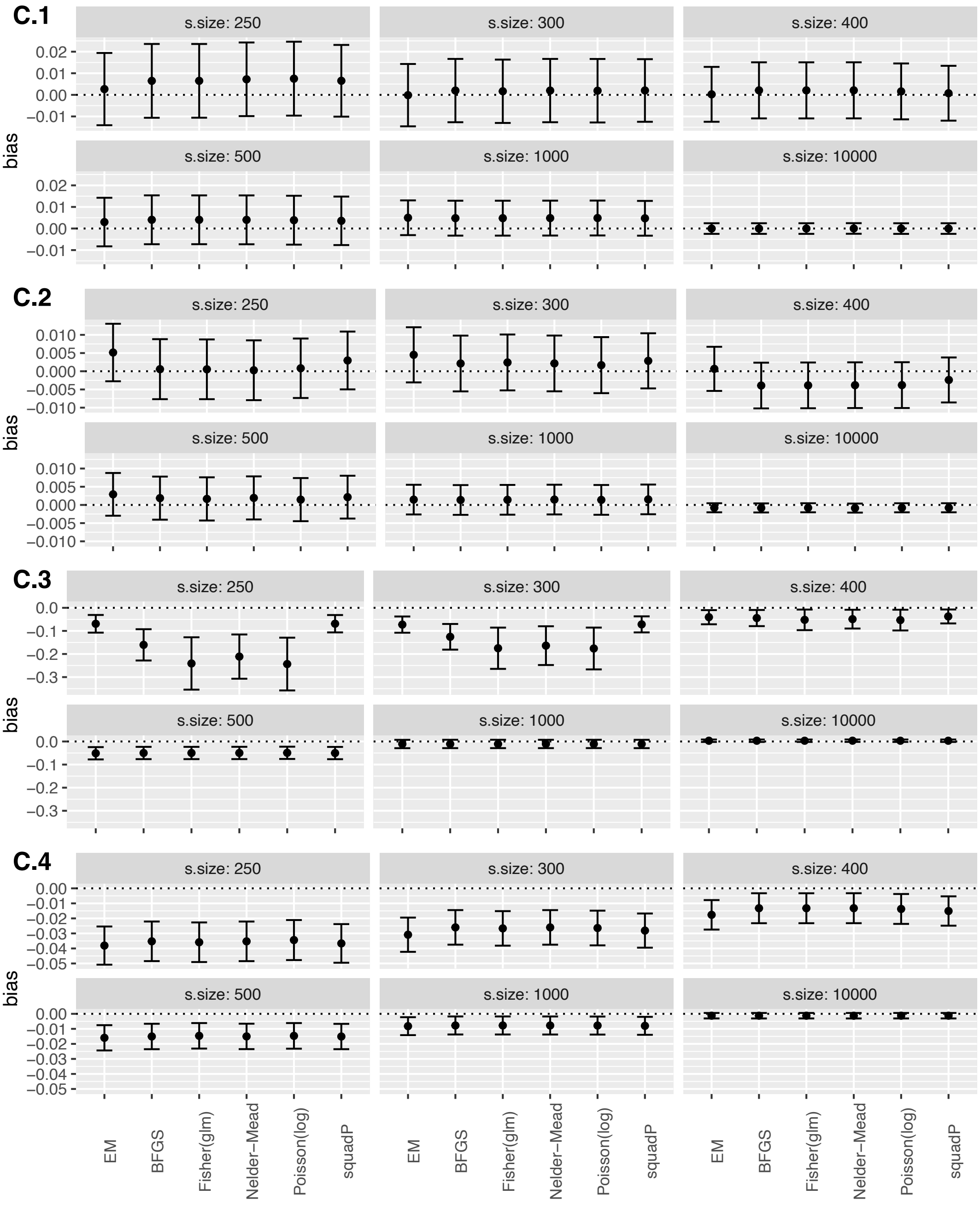

Figure 22: The absolute biases of the estimated log(RR) from the six methods in each of the 6 scenarios with event probability 6% with sample sizes 250, 300, 400, 500, 1000, and 10000. y-axis: bias for coef.1, coef.2, coef.3, and coef.4. x-axis: The six statistical methods used for each scenario.

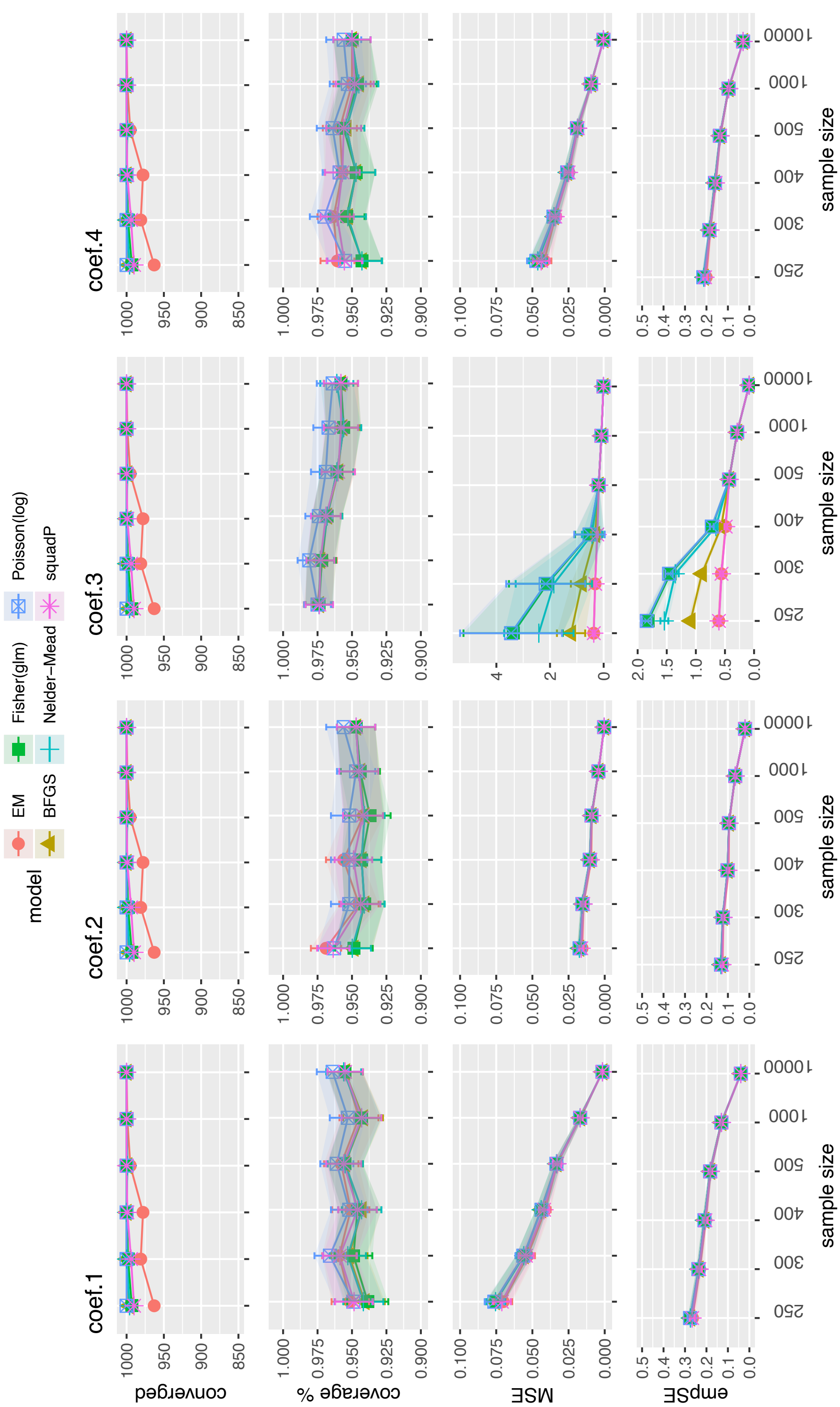

Figure 23: Performance measurements of 6 scenarios with event probability 6%. Convergence rate (at the top), coverage probability, MSE, and EmpSE (y-axis) for the coefficients (coef.1, coef.2, coef.3, and coef.4) using the underlying statistical methods for each scenario. Each type of shapes with shaded area (confidence interval) is representing a different method. The measurements are done for 6 scenarios that are derived from six different sample sizes 250,300,400,500,1000, and 10000 on x-axis.

Table 26: Performance measurements of scenarios 31→36 for coef.1.

Coverage		Bias		MSE		EmpSE				
cover.	mcse	bias	mcse	mse	mcse	empse	mcse	model	s.size	converged
0.951	0.007	0.003	0.009	0.070	0.003	0.265	0.006	EM	250	963
0.941	0.007	0.006	0.009	0.076	0.003	0.276	0.006	BFGS	250	997
0.939	0.008	0.006	0.009	0.075	0.003	0.275	0.006	Fisher(glm)	250	993
0.942	0.007	0.007	0.009	0.075	0.003	0.275	0.006	Nelder-Mead	250	996
0.949	0.007	0.007	0.009	0.076	0.003	0.276	0.006	Poisson(log)	250	1000
0.948	0.007	0.007	0.008	0.071	0.003	0.267	0.006	squadP	250	990
0.959	0.006	0.000	0.007	0.053	0.003	0.231	0.005	EM	300	981
0.952	0.007	0.002	0.007	0.056	0.003	0.237	0.005	BFGS	300	1000
0.949	0.007	0.002	0.007	0.056	0.003	0.237	0.005	Fisher(glm)	300	998
0.953	0.007	0.002	0.007	0.056	0.003	0.237	0.005	Nelder-Mead	300	1000
0.966	0.006	0.002	0.008	0.056	0.003	0.237	0.005	Poisson(log)	300	1000
0.959	0.006	0.002	0.007	0.054	0.003	0.233	0.005	squadP	300	994
0.951	0.007	0.000	0.006	0.041	0.002	0.203	0.005	EM	400	978
0.943	0.007	0.002	0.007	0.044	0.002	0.210	0.005	BFGS	400	1000
0.946	0.007	0.002	0.007	0.044	0.002	0.210	0.005	Fisher(glm)	400	1000
0.943	0.007	0.002	0.007	0.044	0.002	0.210	0.005	Nelder-Mead	400	1000
0.952	0.007	0.002	0.007	0.044	0.002	0.209	0.005	Poisson(log)	400	1000
0.946	0.007	0.001	0.006	0.042	0.002	0.205	0.005	squadP	400	999
0.958	0.006	0.003	0.006	0.033	0.001	0.181	0.004	EM	500	995
0.955	0.007	0.004	0.006	0.033	0.001	0.183	0.004	BFGS	500	999
0.955	0.007	0.004	0.006	0.033	0.001	0.182	0.004	Fisher(glm)	500	1000
0.955	0.007	0.004	0.006	0.033	0.001	0.183	0.004	Nelder-Mead	500	999
0.961	0.006	0.004	0.006	0.033	0.001	0.183	0.004	Poisson(log)	500	1000
0.956	0.006	0.004	0.006	0.033	0.001	0.181	0.004	squadP	500	1000
0.945	0.007	0.005	0.004	0.017	0.001	0.130	0.003	EM	1000	999
0.942	0.007	0.005	0.004	0.017	0.001	0.130	0.003	BFGS	1000	1000
0.943	0.007	0.005	0.004	0.017	0.001	0.130	0.003	Fisher(glm)	1000	1000
0.943	0.007	0.005	0.004	0.017	0.001	0.130	0.003	Nelder-Mead	1000	1000
0.953	0.007	0.005	0.004	0.017	0.001	0.130	0.003	Poisson(log)	1000	1000
0.943	0.007	0.005	0.004	0.017	0.001	0.130	0.003	squadP	1000	1000
0.955	0.007	0.000	0.001	0.002	0.000	0.040	0.001	EM	10000	1000
0.955	0.007	0.000	0.001	0.002	0.000	0.040	0.001	BFGS	10000	1000
0.955	0.007	0.000	0.001	0.002	0.000	0.040	0.001	Fisher(glm)	10000	1000
0.956	0.006	0.000	0.001	0.002	0.000	0.040	0.001	Nelder-Mead	10000	1000
0.964	0.006	0.000	0.001	0.002	0.000	0.040	0.001	Poisson(log)	10000	1000
0.955	0.007	0.000	0.001	0.002	0.000	0.040	0.001	squadP	10000	1000

Table 27: Performance measurements of scenarios 31→36 for coef.2.

Coverage		Bias		MSE		EmpSE				
cover.	mcse	bias	mcse	mse	mcse	empse	mcse	model	s.size	converged
0.969	0.006	0.005	0.004	0.016	0.001	0.125	0.003	EM	250	963
0.949	0.007	0.001	0.004	0.018	0.001	0.133	0.003	BFGS	250	997
0.949	0.007	0.001	0.004	0.017	0.001	0.132	0.003	Fisher(glm)	250	993
0.950	0.007	0.000	0.004	0.017	0.001	0.132	0.003	Nelder-Mead	250	996
0.963	0.006	0.001	0.004	0.017	0.001	0.132	0.003	Poisson(log)	250	1000
0.964	0.006	0.003	0.004	0.016	0.001	0.128	0.003	squadP	250	990
0.945	0.007	0.005	0.004	0.015	0.001	0.121	0.003	EM	300	981
0.941	0.007	0.002	0.004	0.015	0.001	0.124	0.003	BFGS	300	1000
0.941	0.007	0.002	0.004	0.015	0.001	0.124	0.003	Fisher(glm)	300	998
0.941	0.007	0.002	0.004	0.015	0.001	0.124	0.003	Nelder-Mead	300	1000
0.952	0.007	0.002	0.004	0.015	0.001	0.124	0.003	Poisson(log)	300	1000
0.944	0.007	0.003	0.004	0.015	0.001	0.122	0.003	squadP	300	994
0.956	0.007	0.001	0.003	0.009	0.000	0.097	0.002	EM	400	978
0.943	0.007	-0.004	0.003	0.010	0.000	0.101	0.002	BFGS	400	1000
0.943	0.007	-0.004	0.003	0.010	0.000	0.101	0.002	Fisher(glm)	400	1000
0.943	0.007	-0.004	0.003	0.010	0.000	0.101	0.002	Nelder-Mead	400	1000
0.952	0.007	-0.004	0.003	0.010	0.000	0.102	0.002	Poisson(log)	400	1000
0.949	0.007	-0.002	0.003	0.010	0.000	0.100	0.002	squadP	400	999
0.942	0.007	0.003	0.003	0.009	0.000	0.094	0.002	EM	500	995
0.941	0.007	0.002	0.003	0.009	0.000	0.095	0.002	BFGS	500	999
0.937	0.008	0.002	0.003	0.009	0.000	0.096	0.002	Fisher(glm)	500	1000
0.941	0.007	0.002	0.003	0.009	0.000	0.095	0.002	Nelder-Mead	500	999
0.952	0.007	0.001	0.003	0.009	0.000	0.096	0.002	Poisson(log)	500	1000
0.942	0.007	0.002	0.003	0.009	0.000	0.095	0.002	squadP	500	1000
0.945	0.007	0.001	0.002	0.004	0.000	0.066	0.001	EM	1000	999
0.944	0.007	0.001	0.002	0.004	0.000	0.066	0.001	BFGS	1000	1000
0.944	0.007	0.001	0.002	0.004	0.000	0.066	0.001	Fisher(glm)	1000	1000
0.945	0.007	0.001	0.002	0.004	0.000	0.066	0.001	Nelder-Mead	1000	1000
0.947	0.007	0.001	0.002	0.004	0.000	0.066	0.001	Poisson(log)	1000	1000
0.945	0.007	0.002	0.002	0.004	0.000	0.066	0.001	squadP	1000	1000
0.947	0.007	-0.001	0.001	0.000	0.000	0.020	0.000	EM	10000	1000
0.947	0.007	-0.001	0.001	0.000	0.000	0.020	0.000	BFGS	10000	1000
0.947	0.007	-0.001	0.001	0.000	0.000	0.020	0.000	Fisher(glm)	10000	1000
0.947	0.007	-0.001	0.001	0.000	0.000	0.020	0.000	Nelder-Mead	10000	1000
0.956	0.006	-0.001	0.001	0.000	0.000	0.020	0.000	Poisson(log)	10000	1000
0.947	0.007	-0.001	0.001	0.000	0.000	0.020	0.000	squadP	10000	1000

Table 28: Performance measurements of scenarios 31→36 for coef.3.

Coverage		Bias		MSE		EmpSE				
cover.	mcse	bias	mcse	mse	mcse	empse	mcse	model	s.size	converged
0.975	0.005	-0.069	0.019	0.370	0.019	0.605	0.014	EM	250	963
0.974	0.005	-0.160	0.035	1.216	0.269	1.092	0.024	BFGS	250	997
0.974	0.005	-0.241	0.058	3.375	0.945	1.822	0.041	Fisher(glm)	250	993
0.974	0.005	-0.211	0.049	2.417	0.660	1.541	0.035	Nelder-Mead	250	996
0.975	0.005	-0.244	0.058	3.453	0.969	1.843	0.041	Poisson(log)	250	1000
0.974	0.005	-0.069	0.019	0.369	0.019	0.604	0.014	squadP	250	990
0.971	0.005	-0.073	0.018	0.320	0.018	0.562	0.013	EM	300	981
0.973	0.005	-0.126	0.028	0.818	0.207	0.896	0.020	BFGS	300	1000
0.972	0.005	-0.175	0.046	2.105	0.727	1.441	0.032	Fisher(glm)	300	998
0.973	0.005	-0.164	0.043	1.858	0.725	1.354	0.030	Nelder-Mead	300	1000
0.981	0.004	-0.176	0.046	2.162	0.751	1.461	0.033	Poisson(log)	300	1000
0.973	0.005	-0.072	0.018	0.318	0.018	0.559	0.013	squadP	300	994
0.968	0.006	-0.041	0.016	0.241	0.013	0.490	0.011	EM	400	978
0.968	0.006	-0.044	0.018	0.320	0.080	0.564	0.013	BFGS	400	1000
0.968	0.006	-0.052	0.023	0.520	0.280	0.719	0.016	Fisher(glm)	400	1000
0.968	0.006	-0.049	0.021	0.432	0.192	0.656	0.015	Nelder-Mead	400	1000
0.974	0.005	-0.053	0.023	0.529	0.290	0.726	0.016	Poisson(log)	400	1000
0.969	0.005	-0.037	0.015	0.238	0.013	0.487	0.011	squadP	400	999
0.961	0.006	-0.051	0.014	0.187	0.011	0.429	0.010	EM	500	995
0.960	0.006	-0.050	0.014	0.186	0.011	0.429	0.010	BFGS	500	999
0.961	0.006	-0.050	0.014	0.186	0.011	0.429	0.010	Fisher(glm)	500	1000
0.961	0.006	-0.050	0.014	0.186	0.011	0.429	0.010	Nelder-Mead	500	999
0.969	0.005	-0.049	0.014	0.186	0.011	0.429	0.010	Poisson(log)	500	1000
0.960	0.006	-0.050	0.014	0.186	0.011	0.428	0.010	squadP	500	1000
0.956	0.006	-0.011	0.009	0.085	0.004	0.291	0.007	EM	1000	999
0.957	0.006	-0.011	0.009	0.085	0.004	0.291	0.007	BFGS	1000	1000
0.956	0.006	-0.011	0.009	0.085	0.004	0.291	0.007	Fisher(glm)	1000	1000
0.956	0.006	-0.011	0.009	0.084	0.004	0.291	0.007	Nelder-Mead	1000	1000
0.967	0.006	-0.011	0.009	0.084	0.004	0.290	0.006	Poisson(log)	1000	1000
0.958	0.006	-0.011	0.009	0.084	0.004	0.291	0.006	squadP	1000	1000
0.958	0.006	0.003	0.003	0.007	0.000	0.085	0.002	EM	10000	1000
0.958	0.006	0.003	0.003	0.007	0.000	0.085	0.002	BFGS	10000	1000
0.958	0.006	0.003	0.003	0.007	0.000	0.085	0.002	Fisher(glm)	10000	1000
0.961	0.006	0.003	0.003	0.007	0.000	0.085	0.002	Nelder-Mead	10000	1000
0.964	0.006	0.003	0.003	0.007	0.000	0.085	0.002	Poisson(log)	10000	1000
0.958	0.006	0.003	0.003	0.007	0.000	0.085	0.002	squadP	10000	1000

Table 29: Performance measurements of scenarios 31→36 for coef.4.

Coverage		Bias		MSE		EmpSE				
cover.	mcse	bias	mcse	mse	mcse	empse	mcse	model	s.size	converged
0.961	0.006	-0.038	0.006	0.042	0.003	0.201	0.005	EM	250	963
0.943	0.007	-0.035	0.007	0.046	0.003	0.212	0.005	BFGS	250	997
0.943	0.007	-0.036	0.007	0.046	0.003	0.212	0.005	Fisher(glm)	250	993
0.943	0.007	-0.035	0.007	0.046	0.003	0.212	0.005	Nelder-Mead	250	996
0.955	0.007	-0.034	0.007	0.047	0.003	0.215	0.005	Poisson(log)	250	1000
0.956	0.007	-0.037	0.007	0.044	0.003	0.206	0.005	squadP	250	990
0.963	0.006	-0.031	0.006	0.034	0.002	0.182	0.004	EM	300	981
0.953	0.007	-0.026	0.006	0.035	0.002	0.186	0.004	BFGS	300	1000
0.954	0.007	-0.027	0.006	0.035	0.002	0.185	0.004	Fisher(glm)	300	998
0.953	0.007	-0.026	0.006	0.035	0.002	0.186	0.004	Nelder-Mead	300	1000
0.970	0.005	-0.026	0.006	0.035	0.002	0.186	0.004	Poisson(log)	300	1000
0.961	0.006	-0.028	0.006	0.034	0.002	0.183	0.004	squadP	300	994
0.957	0.006	-0.018	0.005	0.025	0.001	0.157	0.004	EM	400	978
0.947	0.007	-0.013	0.005	0.026	0.001	0.161	0.004	BFGS	400	1000
0.947	0.007	-0.013	0.005	0.026	0.001	0.161	0.004	Fisher(glm)	400	1000
0.947	0.007	-0.013	0.005	0.026	0.001	0.161	0.004	Nelder-Mead	400	1000
0.959	0.006	-0.014	0.005	0.026	0.001	0.160	0.004	Poisson(log)	400	1000
0.957	0.006	-0.015	0.005	0.025	0.001	0.157	0.004	squadP	400	999
0.959	0.006	-0.016	0.004	0.019	0.001	0.136	0.003	EM	500	995
0.954	0.007	-0.015	0.004	0.019	0.001	0.136	0.003	BFGS	500	999
0.956	0.006	-0.015	0.004	0.019	0.001	0.137	0.003	Fisher(glm)	500	1000
0.954	0.007	-0.015	0.004	0.019	0.001	0.136	0.003	Nelder-Mead	500	999
0.964	0.006	-0.015	0.004	0.019	0.001	0.138	0.003	Poisson(log)	500	1000
0.956	0.006	-0.015	0.004	0.019	0.001	0.136	0.003	squadP	500	1000
0.950	0.007	-0.008	0.003	0.009	0.000	0.096	0.002	EM	1000	999
0.945	0.007	-0.008	0.003	0.009	0.000	0.097	0.002	BFGS	1000	1000
0.946	0.007	-0.008	0.003	0.009	0.000	0.097	0.002	Fisher(glm)	1000	1000
0.945	0.007	-0.008	0.003	0.009	0.000	0.097	0.002	Nelder-Mead	1000	1000
0.953	0.007	-0.008	0.003	0.009	0.000	0.097	0.002	Poisson(log)	1000	1000
0.948	0.007	-0.008	0.003	0.009	0.000	0.096	0.002	squadP	1000	1000
0.950	0.007	-0.001	0.001	0.001	0.000	0.029	0.001	EM	10000	1000
0.950	0.007	-0.001	0.001	0.001	0.000	0.029	0.001	BFGS	10000	1000
0.950	0.007	-0.001	0.001	0.001	0.000	0.029	0.001	Fisher(glm)	10000	1000
0.950	0.007	-0.001	0.001	0.001	0.000	0.029	0.001	Nelder-Mead	10000	1000
0.956	0.006	-0.001	0.001	0.001	0.000	0.029	0.001	Poisson(log)	10000	1000
0.950	0.007	-0.001	0.001	0.001	0.000	0.029	0.001	squadP	10000	1000

5.4.2 Scenarios with event probability 12%

Scenarios from 37 $\rightarrow$ 42 with event probability 12% and different sample sizes $(250, 300, 400, 500, 1000, 10000)$ are showing relatively similar pattern results when comparing the performance measurements of the estimated log(RR) in log scale from the six models in each of the 6 scenarios.

The performance measurements and comparison between the methods and scenarios are shown in figures 34 and 35, and tables 37 for coef.1, 38 for coef.2, 39 for coef.3, and 40 for coef.4 in Appendix_A section.

5.4.3 Scenarios with event probability 24%

Comparison of scenarios from 43 $\rightarrow$ 48 with event probability 24% and different sample sizes $(250, 300, 400, 500, 1000, 10000)$ using the six methods being studied showed an approximately 0 bias for all methods. However, it is noticeable the low convergence rate of EM-type method compared with the other approaches.

The performance measurements are shown in figures 36 and 37, and tables 41 for coef.1, 42for coef.2, 43for coef.3, and 44 for coef.4 in Appendix_A section.

5.4.4 Scenarios with event probability 48%

Scenarios from 49 $\rightarrow$ 54 (6 scenarios) with event probability 48% and varieties of sample sizes $(250, 300, 400, 500, 1000, 10000)$ are showing similar results in regard to absolute biases of the estimated log(RR) from the six statistical methods. However, it is notable to mention that EM-type method behaved partly different from the other method with slightly higher bias in case of estimates of coef.1, coef.3, and coef.4 (scenarios with sample sizes 250, 300, 400) as shown in figure 24.

In figure 25 a lower convergence rate from EM-type method (approximately 70%) compared with the other methods is observed, while squadP and Poisson(log) methods have 100% convergence. All methods have relatively similar MSE and empSE. Additionally, the estimates from Fisher GLM method in coef.4 $(250, 300, 400, 500, 1000$ sample sizes) have lower coverage probability compared with other methods as shown in figure 25.

The performance measurements (with Monte Carlo standard error) such as bias, coverage probability, MSE, EmpSE and convergence are shown in table 30 for coef.1, tabel 31 for coef.2, table 32 for coef.3, and table 33 for coef.4.

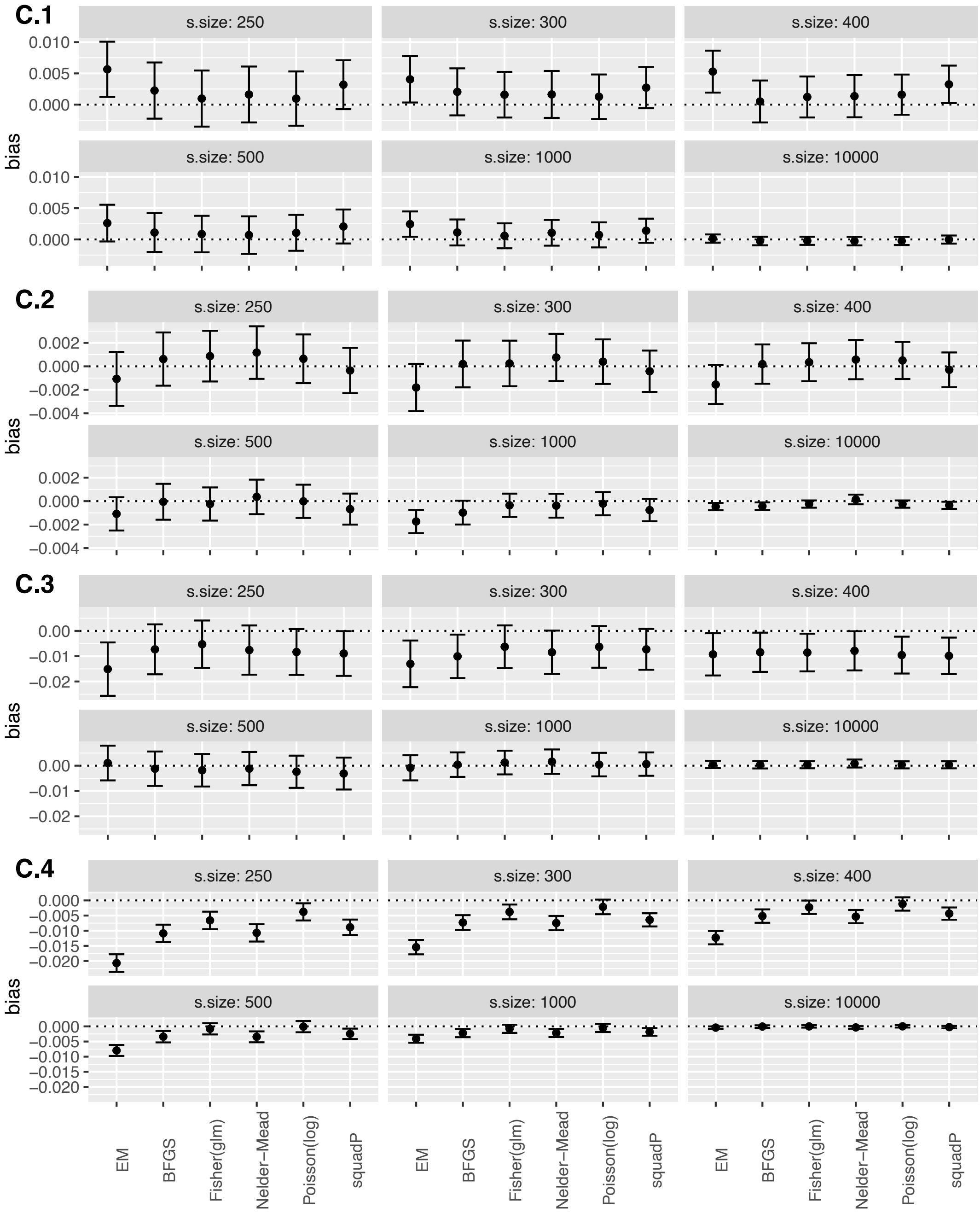

Figure 24: The absolute biases of the estimated log(RR) from the six methods in each of the scenarios (49→54) with event probability 48% with sample sizes 250, 300, 400, 500, 1000, and 10000. y-axis: bias for coef.1, coef.2, coef.3, and coef.4. x-axis: The six statistical methods used for each scenario.

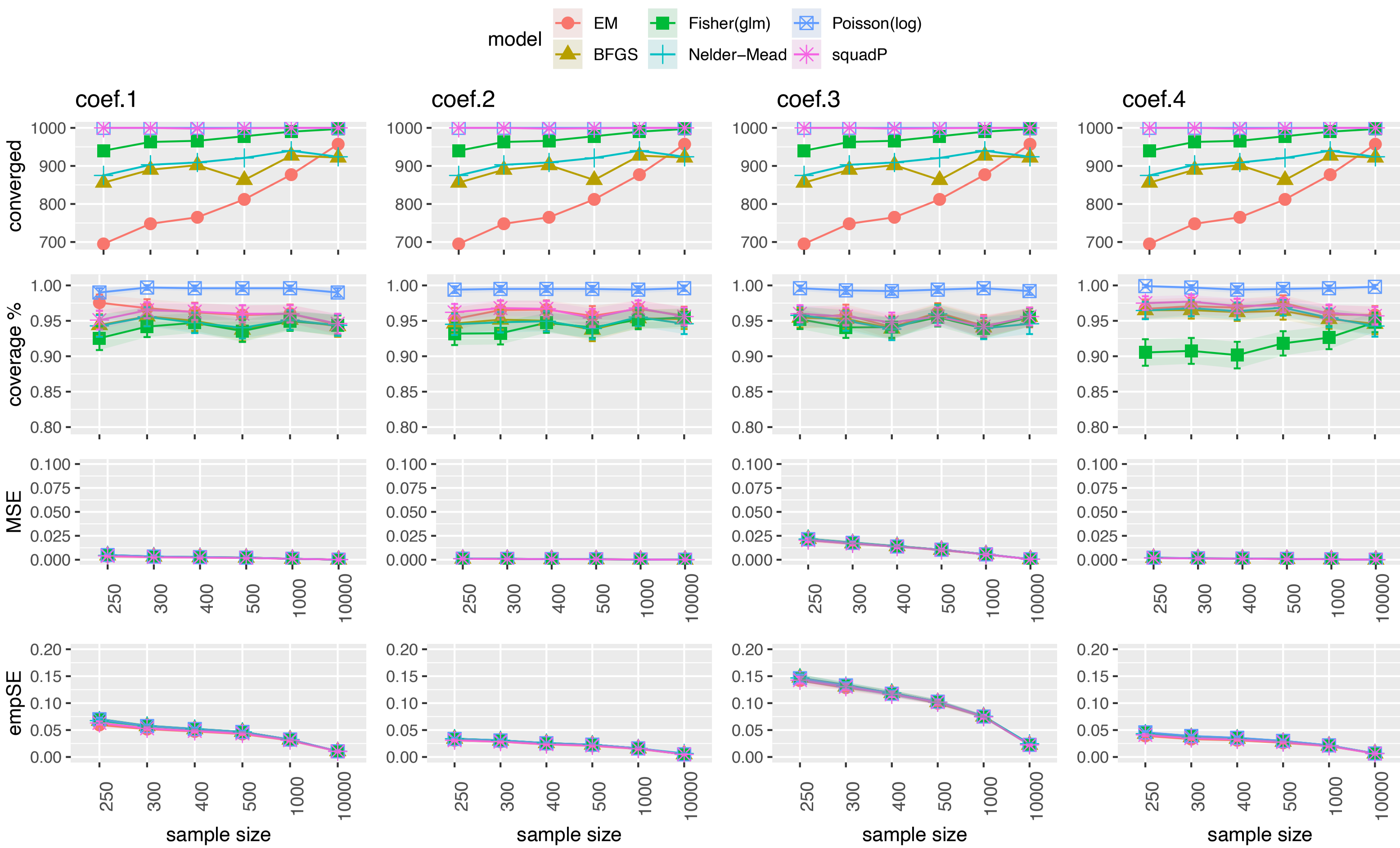

Figure 25: Performance measurements of scenarios (49→54) with event probability 6%. Convergence rate (at the top), coverage probability, MSE, and EmpSE (y-axis) for the coefficients (coef.1, coef.2, coef.3, and coef.4) using the underlying statistical methods for each scenario. Each type of shapes with shaded area (confidence interval) is representing a different method. The measurements are done for 6 scenarios that are derived from six different sample sizes 250,300,400,500,1000, and 10000 on x-axis.

Table 30: Performance measurements of scenarios 49→54 for coef.1

Coverage		Bias		MSE		EmpSE				
cover.	mcse	bias	mcse	mse	mcse	empse	mcse	model	s.size	converged
0.976	0.006	0.006	0.002	0.004	0	0.059	0.002	EM	250	695
0.944	0.008	0.002	0.002	0.005	0	0.067	0.002	BFGS	250	856
0.926	0.009	0.001	0.002	0.005	0	0.070	0.002	Fisher(glm)	250	940
0.943	0.008	0.002	0.002	0.005	0	0.068	0.002	Nelder-Mead	250	875
0.990	0.003	0.001	0.002	0.005	0	0.070	0.002	Poisson(log)	250	1000
0.951	0.007	0.003	0.002	0.004	0	0.063	0.001	squadP	250	1000
0.968	0.006	0.004	0.002	0.003	0	0.052	0.001	EM	300	748
0.956	0.007	0.002	0.002	0.003	0	0.057	0.001	BFGS	300	890
0.942	0.008	0.002	0.002	0.003	0	0.058	0.001	Fisher(glm)	300	963
0.956	0.007	0.002	0.002	0.003	0	0.058	0.001	Nelder-Mead	300	903
0.997	0.002	0.001	0.002	0.003	0	0.057	0.001	Poisson(log)	300	1000
0.965	0.006	0.003	0.002	0.003	0	0.053	0.001	squadP	300	1000
0.962	0.007	0.005	0.002	0.002	0	0.047	0.001	EM	400	765
0.950	0.007	0.001	0.002	0.003	0	0.051	0.001	BFGS	400	902
0.947	0.007	0.001	0.002	0.003	0	0.052	0.001	Fisher(glm)	400	966
0.947	0.007	0.001	0.002	0.003	0	0.052	0.001	Nelder-Mead	400	909
0.996	0.002	0.002	0.002	0.003	0	0.052	0.001	Poisson(log)	400	1000
0.963	0.006	0.003	0.002	0.002	0	0.048	0.001	squadP	400	998
0.958	0.007	0.003	0.001	0.002	0	0.043	0.001	EM	500	812
0.937	0.008	0.001	0.002	0.002	0	0.047	0.001	BFGS	500	863
0.936	0.008	0.001	0.001	0.002	0	0.047	0.001	Fisher(glm)	500	978
0.940	0.008	0.001	0.002	0.002	0	0.046	0.001	Nelder-Mead	500	921
0.996	0.002	0.001	0.001	0.002	0	0.046	0.001	Poisson(log)	500	1000
0.960	0.006	0.002	0.001	0.002	0	0.044	0.001	squadP	500	999
0.960	0.007	0.002	0.001	0.001	0	0.031	0.001	EM	1000	877
0.951	0.007	0.001	0.001	0.001	0	0.032	0.001	BFGS	1000	927
0.949	0.007	0.001	0.001	0.001	0	0.032	0.001	Fisher(glm)	1000	990
0.951	0.007	0.001	0.001	0.001	0	0.032	0.001	Nelder-Mead	1000	940
0.996	0.002	0.001	0.001	0.001	0	0.032	0.001	Poisson(log)	1000	1000
0.960	0.006	0.001	0.001	0.001	0	0.031	0.001	squadP	1000	1000
0.945	0.007	0.000	0.000	0.000	0	0.010	0.000	EM	10000	957
0.943	0.008	0.000	0.000	0.000	0	0.011	0.000	BFGS	10000	922
0.944	0.007	0.000	0.000	0.000	0	0.011	0.000	Fisher(glm)	10000	997
0.944	0.008	0.000	0.000	0.000	0	0.011	0.000	Nelder-Mead	10000	924
0.990	0.003	0.000	0.000	0.000	0	0.011	0.000	Poisson(log)	10000	1000
0.946	0.007	0.000	0.000	0.000	0	0.010	0.000	squadP	10000	1000

Table 31: Performance measurements of scenarios 49→54 for coef.2

Coverage		Bias		MSE		EmpSE				
cover.	mcse	bias	mcse	mse	mcse	empse	mcse	model	s.size	converged
0.953	0.008	-0.001	0.001	0.001	0	0.031	0.001	EM	250	695
0.946	0.008	0.001	0.001	0.001	0	0.034	0.001	BFGS	250	856
0.932	0.008	0.001	0.001	0.001	0	0.034	0.001	Fisher(glm)	250	940
0.945	0.008	0.001	0.001	0.001	0	0.034	0.001	Nelder-Mead	250	875
0.994	0.002	0.001	0.001	0.001	0	0.033	0.001	Poisson(log)	250	1000
0.962	0.006	0.000	0.001	0.001	0	0.031	0.001	squadP	250	1000
0.965	0.007	-0.002	0.001	0.001	0	0.028	0.001	EM	300	748
0.952	0.007	0.000	0.001	0.001	0	0.030	0.001	BFGS	300	890
0.933	0.008	0.000	0.001	0.001	0	0.031	0.001	Fisher(glm)	300	963
0.948	0.007	0.001	0.001	0.001	0	0.031	0.001	Nelder-Mead	300	903
0.995	0.002	0.000	0.001	0.001	0	0.031	0.001	Poisson(log)	300	1000
0.968	0.006	0.000	0.001	0.001	0	0.028	0.001	squadP	300	1000
0.966	0.007	-0.002	0.001	0.001	0	0.023	0.001	EM	400	765
0.950	0.007	0.000	0.001	0.001	0	0.026	0.001	BFGS	400	902
0.947	0.007	0.000	0.001	0.001	0	0.026	0.001	Fisher(glm)	400	966
0.949	0.007	0.001	0.001	0.001	0	0.026	0.001	Nelder-Mead	400	909
0.995	0.002	0.001	0.001	0.001	0	0.026	0.001	Poisson(log)	400	1000
0.967	0.006	0.000	0.001	0.001	0	0.024	0.001	squadP	400	998
0.957	0.007	-0.001	0.001	0.000	0	0.021	0.001	EM	500	812
0.937	0.008	0.000	0.001	0.001	0	0.023	0.001	BFGS	500	863
0.941	0.008	0.000	0.001	0.001	0	0.023	0.001	Fisher(glm)	500	978
0.939	0.008	0.000	0.001	0.001	0	0.023	0.001	Nelder-Mead	500	921
0.995	0.002	0.000	0.001	0.001	0	0.023	0.001	Poisson(log)	500	1000
0.954	0.007	-0.001	0.001	0.000	0	0.021	0.000	squadP	500	999
0.967	0.006	-0.002	0.001	0.000	0	0.015	0.000	EM	1000	877
0.954	0.007	-0.001	0.001	0.000	0	0.016	0.000	BFGS	1000	927
0.952	0.007	0.000	0.001	0.000	0	0.016	0.000	Fisher(glm)	1000	990
0.955	0.007	0.000	0.001	0.000	0	0.016	0.000	Nelder-Mead	1000	940
0.994	0.002	0.000	0.001	0.000	0	0.016	0.000	Poisson(log)	1000	1000
0.968	0.006	-0.001	0.000	0.000	0	0.015	0.000	squadP	1000	1000
0.957	0.007	0.000	0.000	0.000	0	0.005	0.000	EM	10000	957
0.952	0.007	0.000	0.000	0.000	0	0.005	0.000	BFGS	10000	922
0.955	0.007	0.000	0.000	0.000	0	0.005	0.000	Fisher(glm)	10000	997
0.946	0.007	0.000	0.000	0.000	0	0.006	0.000	Nelder-Mead	10000	924
0.996	0.002	0.000	0.000	0.000	0	0.005	0.000	Poisson(log)	10000	1000
0.956	0.006	0.000	0.000	0.000	0	0.005	0.000	squadP	10000	1000

Table 32: Performance measurements of scenarios 49→54 for coef.3

Coverage		Bias		MSE		EmpSE				
cover.	mcse	bias	mcse	mse	mcse	empse	mcse	model	s.size	converged
0.954	0.008	-0.015	0.005	0.020	0.001	0.142	0.004	EM	250	695
0.954	0.007	-0.007	0.005	0.022	0.001	0.147	0.004	BFGS	250	856
0.952	0.007	-0.005	0.005	0.022	0.001	0.147	0.003	Fisher(glm)	250	940
0.958	0.007	-0.008	0.005	0.022	0.001	0.147	0.004	Nelder-Mead	250	875
0.996	0.002	-0.008	0.005	0.021	0.001	0.146	0.003	Poisson(log)	250	1000
0.960	0.006	-0.009	0.005	0.020	0.001	0.143	0.003	squadP	250	1000
0.959	0.007	-0.013	0.005	0.017	0.001	0.129	0.003	EM	300	748
0.953	0.007	-0.010	0.004	0.017	0.001	0.131	0.003	BFGS	300	890
0.941	0.008	-0.006	0.004	0.018	0.001	0.134	0.003	Fisher(glm)	300	963
0.950	0.007	-0.008	0.004	0.017	0.001	0.131	0.003	Nelder-Mead	300	903
0.993	0.003	-0.006	0.004	0.018	0.001	0.133	0.003	Poisson(log)	300	1000
0.956	0.006	-0.007	0.004	0.017	0.001	0.130	0.003	squadP	300	1000
0.941	0.009	-0.009	0.004	0.014	0.001	0.118	0.003	EM	400	765
0.939	0.008	-0.008	0.004	0.014	0.001	0.119	0.003	BFGS	400	902
0.941	0.008	-0.009	0.004	0.014	0.001	0.118	0.003	Fisher(glm)	400	966
0.938	0.008	-0.008	0.004	0.014	0.001	0.119	0.003	Nelder-Mead	400	909
0.992	0.003	-0.010	0.004	0.014	0.001	0.117	0.003	Poisson(log)	400	1000
0.948	0.007	-0.010	0.004	0.014	0.001	0.116	0.003	squadP	400	998
0.962	0.007	0.001	0.003	0.010	0.001	0.100	0.002	EM	500	812
0.961	0.007	-0.001	0.003	0.010	0.001	0.102	0.002	BFGS	500	863
0.955	0.007	-0.002	0.003	0.011	0.001	0.103	0.002	Fisher(glm)	500	978
0.960	0.006	-0.001	0.003	0.010	0.001	0.102	0.002	Nelder-Mead	500	921
0.994	0.002	-0.002	0.003	0.011	0.001	0.103	0.002	Poisson(log)	500	1000
0.956	0.006	-0.003	0.003	0.010	0.001	0.102	0.002	squadP	500	999
0.943	0.008	-0.001	0.003	0.006	0.000	0.075	0.002	EM	1000	877
0.941	0.008	0.000	0.002	0.006	0.000	0.075	0.002	BFGS	1000	927
0.939	0.008	0.001	0.002	0.006	0.000	0.075	0.002	Fisher(glm)	1000	990
0.939	0.008	0.002	0.002	0.006	0.000	0.076	0.002	Nelder-Mead	1000	940
0.996	0.002	0.000	0.002	0.006	0.000	0.075	0.002	Poisson(log)	1000	1000
0.942	0.007	0.001	0.002	0.006	0.000	0.074	0.002	squadP	1000	1000
0.954	0.007	0.000	0.001	0.001	0.000	0.023	0.001	EM	10000	957
0.956	0.007	0.000	0.001	0.001	0.000	0.023	0.001	BFGS	10000	922
0.955	0.007	0.000	0.001	0.001	0.000	0.022	0.001	Fisher(glm)	10000	997
0.946	0.007	0.001	0.001	0.001	0.000	0.024	0.001	Nelder-Mead	10000	924
0.992	0.003	0.000	0.001	0.001	0.000	0.023	0.001	Poisson(log)	10000	1000
0.956	0.006	0.000	0.001	0.001	0.000	0.022	0.001	squadP	10000	1000

Table 33: Performance measurements of scenarios 49→54 for coef.4

Coverage		Bias		MSE		EmpSE				
cover.	mcse	bias	mcse	mse	mcse	empse	mcse	model	s.size	converged
0.967	0.007	-0.021	0.001	0.002	0	0.039	0.001	EM	250	695
0.965	0.006	-0.011	0.001	0.002	0	0.043	0.001	BFGS	250	856
0.905	0.010	-0.007	0.001	0.002	0	0.045	0.001	Fisher(glm)	250	940
0.965	0.006	-0.011	0.001	0.002	0	0.043	0.001	Nelder-Mead	250	875
0.999	0.001	-0.004	0.001	0.002	0	0.046	0.001	Poisson(log)	250	1000
0.975	0.005	-0.009	0.001	0.002	0	0.041	0.001	squadP	250	1000
0.971	0.006	-0.015	0.001	0.001	0	0.033	0.001	EM	300	748
0.965	0.006	-0.007	0.001	0.001	0	0.037	0.001	BFGS	300	890
0.908	0.009	-0.004	0.001	0.002	0	0.039	0.001	Fisher(glm)	300	963
0.968	0.006	-0.007	0.001	0.001	0	0.036	0.001	Nelder-Mead	300	903
0.997	0.002	-0.002	0.001	0.002	0	0.039	0.001	Poisson(log)	300	1000
0.977	0.005	-0.006	0.001	0.001	0	0.035	0.001	squadP	300	1000
0.969	0.006	-0.012	0.001	0.001	0	0.031	0.001	EM	400	765
0.962	0.006	-0.005	0.001	0.001	0	0.034	0.001	BFGS	400	902
0.902	0.010	-0.002	0.001	0.001	0	0.035	0.001	Fisher(glm)	400	966
0.964	0.006	-0.005	0.001	0.001	0	0.034	0.001	Nelder-Mead	400	909
0.994	0.002	-0.001	0.001	0.001	0	0.036	0.001	Poisson(log)	400	1000
0.971	0.005	-0.004	0.001	0.001	0	0.033	0.001	squadP	400	998
0.975	0.005	-0.008	0.001	0.001	0	0.026	0.001	EM	500	812
0.964	0.006	-0.003	0.001	0.001	0	0.028	0.001	BFGS	500	863
0.918	0.009	-0.001	0.001	0.001	0	0.030	0.001	Fisher(glm)	500	978
0.969	0.006	-0.003	0.001	0.001	0	0.028	0.001	Nelder-Mead	500	921
0.995	0.002	0.000	0.001	0.001	0	0.030	0.001	Poisson(log)	500	1000
0.972	0.005	-0.002	0.001	0.001	0	0.028	0.001	squadP	500	999
0.959	0.007	-0.004	0.001	0.000	0	0.020	0.000	EM	1000	877
0.953	0.007	-0.002	0.001	0.000	0	0.021	0.000	BFGS	1000	927
0.926	0.008	-0.001	0.001	0.000	0	0.022	0.000	Fisher(glm)	1000	990
0.954	0.007	-0.002	0.001	0.000	0	0.021	0.000	Nelder-Mead	1000	940
0.996	0.002	-0.001	0.001	0.000	0	0.022	0.000	Poisson(log)	1000	1000
0.961	0.006	-0.002	0.001	0.000	0	0.020	0.000	squadP	1000	1000
0.958	0.006	0.000	0.000	0.000	0	0.006	0.000	EM	10000	957
0.946	0.007	0.000	0.000	0.000	0	0.006	0.000	BFGS	10000	922
0.948	0.007	0.000	0.000	0.000	0	0.007	0.000	Fisher(glm)	10000	997
0.943	0.008	0.000	0.000	0.000	0	0.007	0.000	Nelder-Mead	10000	924
0.998	0.001	0.000	0.000	0.000	0	0.007	0.000	Poisson(log)	10000	1000
0.957	0.006	0.000	0.000	0.000	0	0.006	0.000	squadP	10000	1000

5.5 Monte Carlo simulation results: scenarios with 8 covariates

The results of the simulation study which consists of 9 scenarios with 8 variables and event probabilities $12\%, 24\%$, and 48% are presented here. Correlations between the different variables are previously explained in *Monte Carlo Simulation* section, while here a summary of the main findings is represented.

EM-type method was excluded from the current simulation study (with 8 covariates) for computational reasons. The estimated time that EM-type algorithm required to analyse the scenarios was approximately $\approx$ 34d 01h 48m 06s for each scenario using our local server with 12 cores. As estimated, EM-type method can take $\approx$ 306 days running on our local server continuously to analyse the scenarios with 8 covariates.

The data of the current simulation (9 scenarios) excluding EM-type method is shown in eight different tables in Appendix_A. Tables 45, 46, 47, 48, 49, 50, 51, and 52 show performance measurements such as coverage probability, absolute bias, MSE and convergence rate for coef.1, coef.2, coef.3, coef.4, coef.5, coef.6, coef.7, and coef.8.

5.5.1 Scenarios with event probability 12%

The main findings of scenarios from $55 \rightarrow 57$ (3 scenarios) with event probability 12% and different sample sizes $(500, 1000, 10000)$ are represented here.

In figure 26, the absolute biases, mean squared error, and empirical standard error of the estimated log(RR) from the five methods (EM-type method was excluded as explained above) in each of the 3 scenarios are shown. Estimated biases follow relatively similar patterns except that Nelder-Mead method is showing significantly higher biases in coefficients c.2, c.4, and c.6. While Fisher GLM and Poisson(log) had significantly larger MES and empSE in coefficients c.6.

As shown in figure 27, squadP and Poisson(log) methods have the highest convergence rate compared with other methods, while, Nelder-Mead and BFGS methods showed the lowest convergence rate.

The estimated coverage probability of all coefficients in case of scenarios with sample sizes 500, 1000 showed a high probability of coverage. However, Nelder-Mead method in case of the scenario with large sample sizes 1000 showed significantly lower coverage probability in coefficients c.1, c.2, c.4, c.6, c.7, and c.8 as shown in figure 27

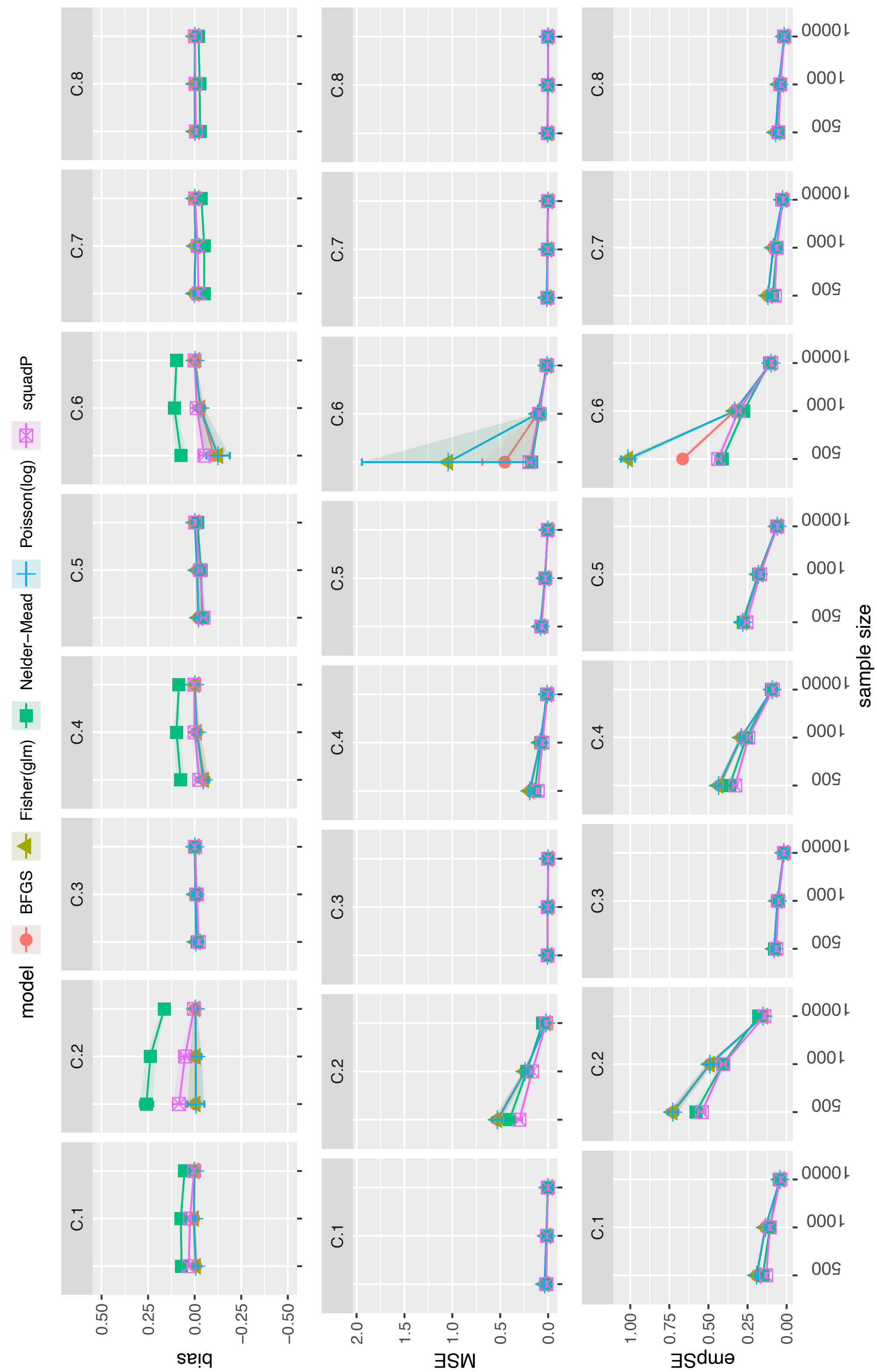

Figure 26: Performance measurements comparison from the five statistical methods. Absolute bias, MSE, and empSE (y-axis) of scenarios (55→57) with 12% event probability and 8 covariates. On x-axis are the sample sizes.

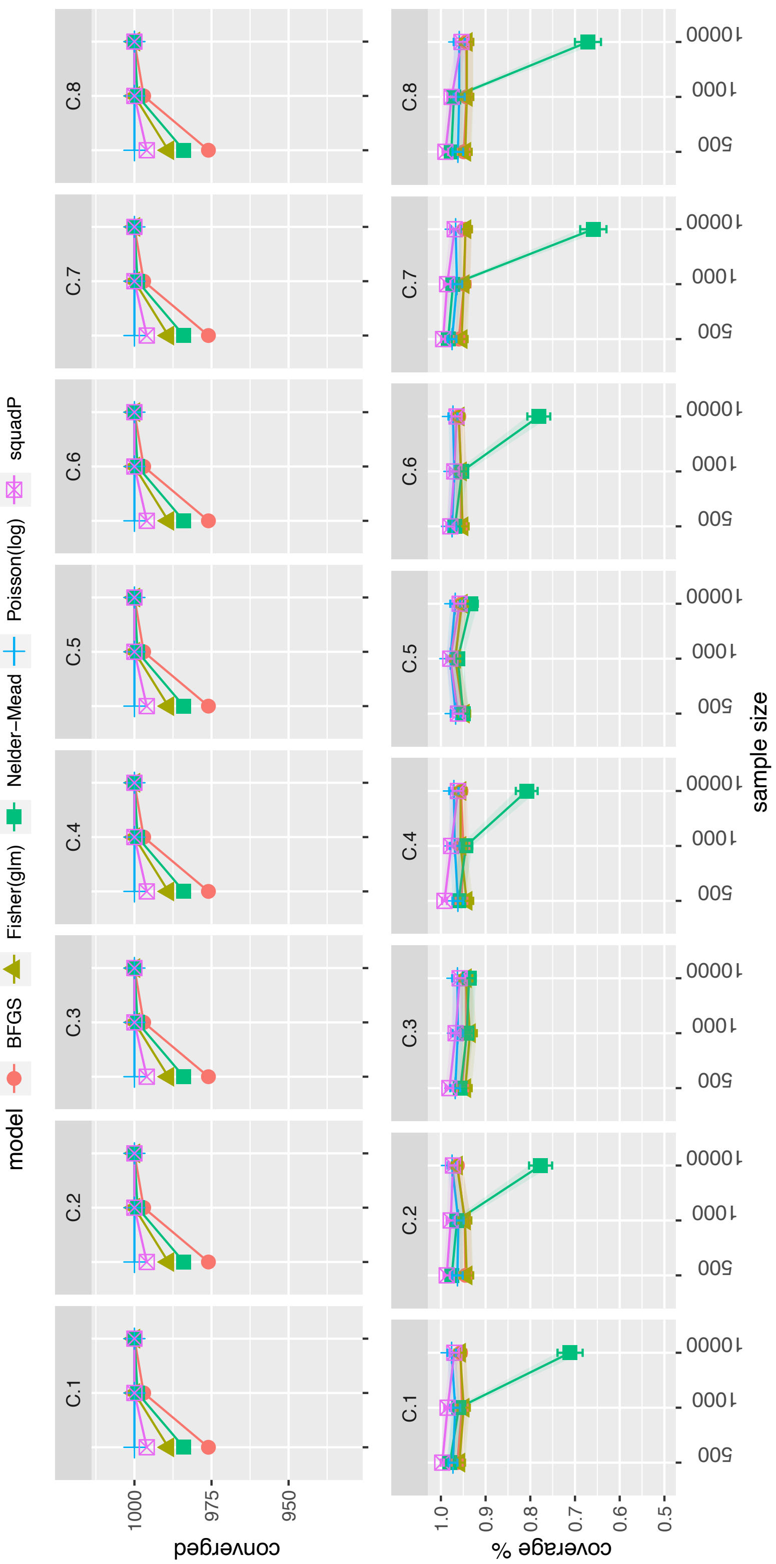

Figure 27: Performance measurements comparison from the five statistical methods. Convergence rate and coverage probability (y-axis) of scenarios (55→57) with 12% event probability and 8 covariates. On x-axis are the sample sizes.

5.5.2 Scenarios with event probability 24%

The performance measurements of the three scenarios from $58 \rightarrow 60$ with event probability 24% and different sample sizes $(500, 1000, 10000)$ are shown in the Appendix_A. In figure 38 the absolute bias, MSE, and empSE are shown, and in figure 39 the convergence rate and coverage probabilities are shown.

5.5.3 Scenarios with event probability 48%

The main findings of scenarios from $61 \rightarrow 63$ (3 scenarios) with event probability 48% and sample sizes of $500, 1000$, and 10000 are shown here.

In figure 28, the absolute biases, MSE, and empSE of the estimated log(RR) from the five methods being studied in each of the 3 scenarios are shown. In the figure, all methods showed relatively low bias estimates with lower MSE and empSE from squadP method compared with other methods in coefficients c.1, c.2, c.4, and c.6.

squadP and Poisson(log) methods showed the highest convergence rate ($\approx$ 100%) and coverage probability as shown in figure 29, while BFGS method had significantly lower convergence rate. Nelder-Mead showed significantly lower coverage probability compared with other methods, particularly in case of scenario with sample size 10000, to reach the lowest probability of $\approx$ 35% in coefficient c.8.

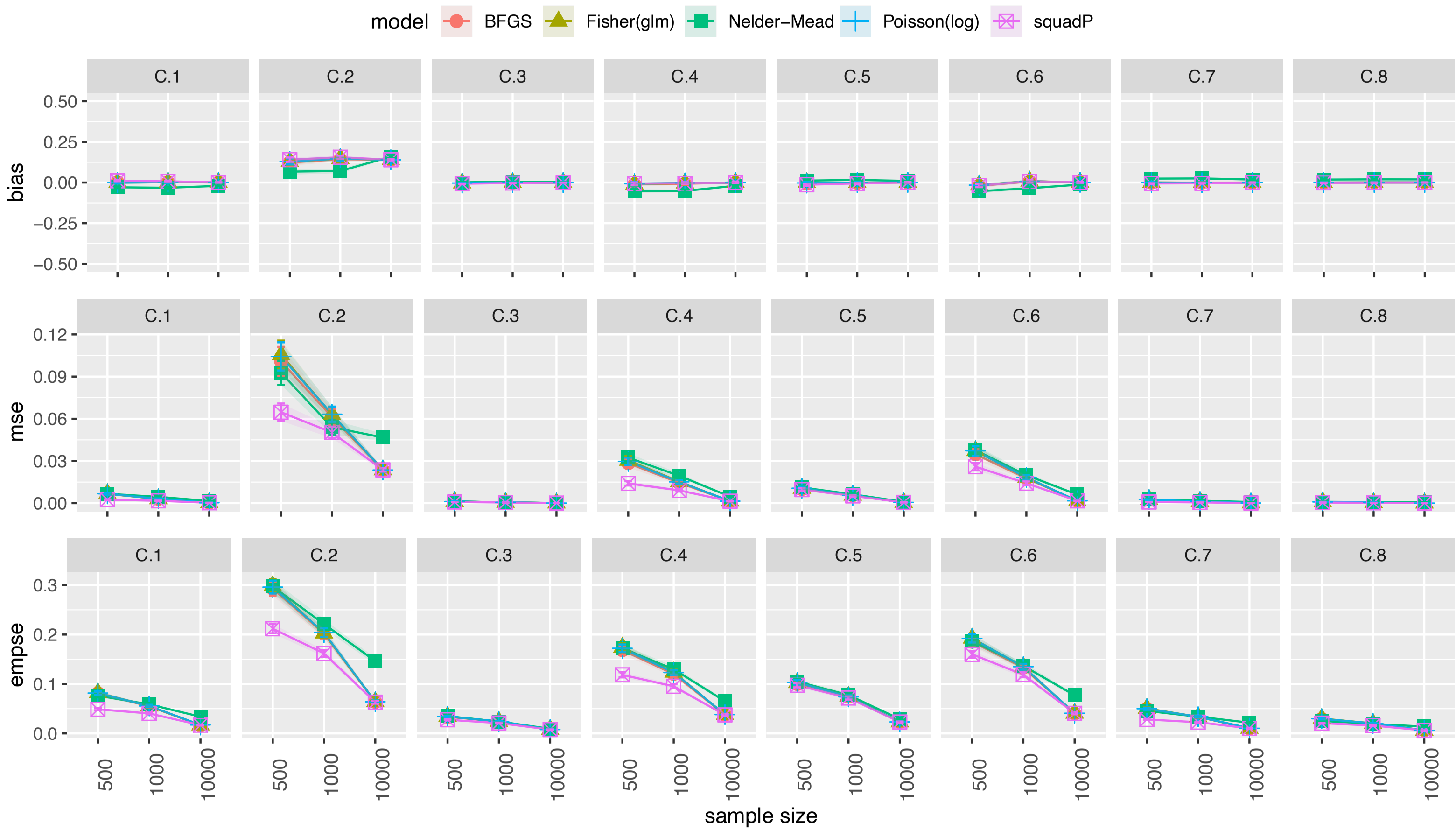

Figure 28: Performance measurements comparison from the five statistical methods. Absolute bias, MSE, and empSE (y-axis) of scenarios (61→63) with 48% event probability and 8 covariates. On x-axis are the sample sizes.

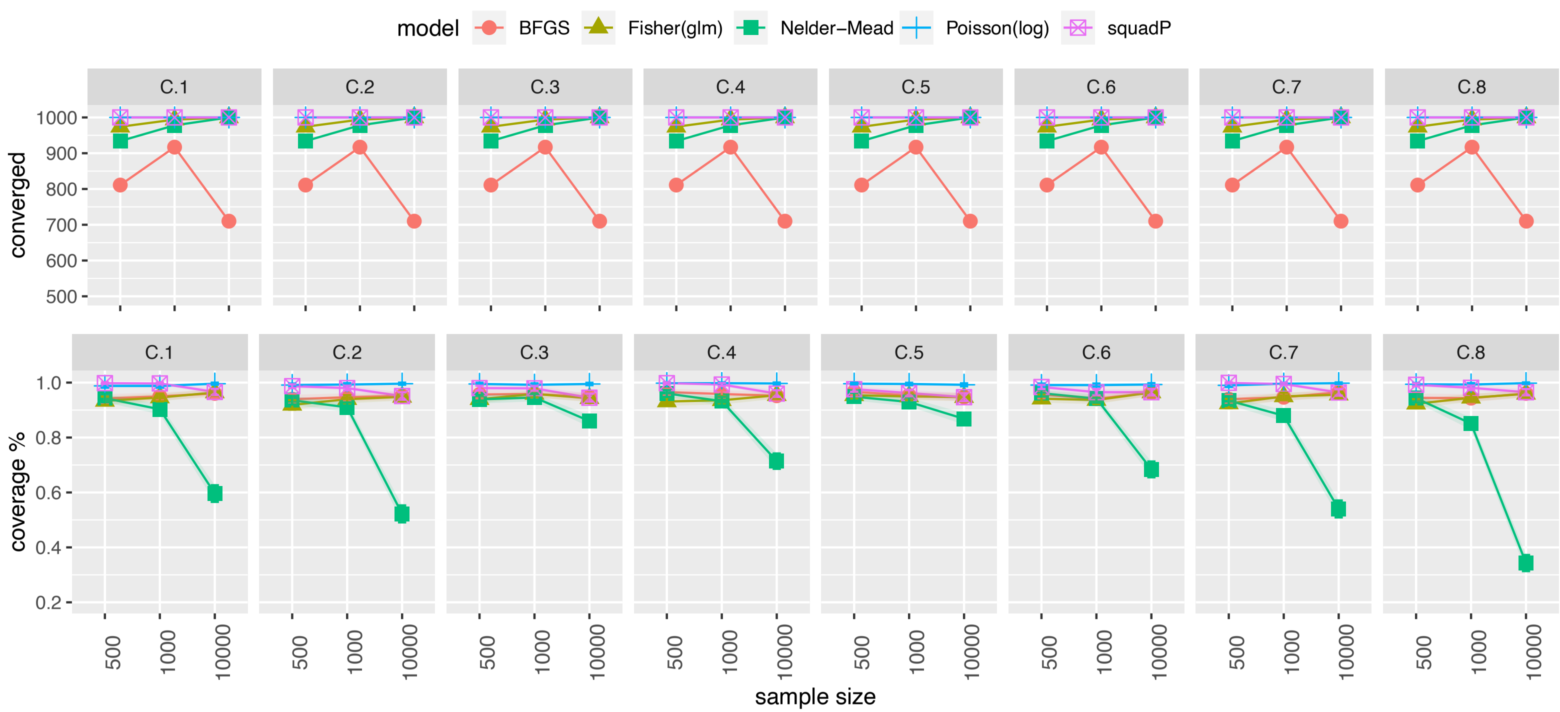

Figure 29: Performance measurements comparison from the five statistical methods. Convergence rate and coverage probability of scenarios (61→63) with 48% event probability and 8 covariates. On x-axis are the sample sizes.

5.6 Monte Carlo simulation results: scenarios under model misspecifications

A comparison of all scenarios presented in the previous chapters (1 $\rightarrow$ 63) was carried out from the methods under correct model specifications. In this Monte Carlo simulation, comparing performance between the six underlying regression models being studied for estimating RR in log scale was executed under model misspecification as explained in *Monte Carlo Simulation* section.

The results of simulation study under model misspecification consist of 39 scenarios (67 $\rightarrow$ 105). 30 scenarios are with 2 covariates and event probabilities of $3\%, 6\%, 12\%, 24\%, 48\%$, and 9 scenarios with 8 covariates and event probabilities of $12\%, 24\%, 48\%$.

This simulation study results (performance measurements) are summarized and represented in figures and data are shown in tables in Appendix_A (Scenarios under model misspecification). A general investigation of the results from scenarios where the models were misspecified will be discussed in the discussion and conclusion section.

5.7 Monte Carlo simulation results: large sample size (1 million) comparison

This simulation study consists of 6 scenarios. One group has 3 scenarios (64 $\rightarrow$ 66) under correct model specifications, and the other group has 3 scenarios (106 $\rightarrow$ 108) under model misspecifications. The 6 scenarios are derived from 12%, 24%, 48% event probabilities with one million sample size for each.

Here in figure 30 the convergence rate and absolute biases was compared for both groups of scenarios with and without model misspecification. The five statistical methods showed high convergence rate in case of scenario with 12% event probability with and without model misspecification. However, BFGS method had a significantly lower convergence rate in case of scenario with 24% event probability under model misspecification.

Methods are showing relatively similar results in regard to absolute biases of the estimated log(RR) with and without model misspecification, however, it is noticeable that Nelder-Mead method behaved partly different from the other method with higher bias in case of estimates of c.1, c.2, c.4, c.6, and c.7 with and without model misspecification as shown in figure 30

Performance measurements for scenarios with one million sample size are shown in tables 76, 77, 78, and 79 under correct model specifications, and tables 80, 81, 82, 83 under model misspecifications for the coefficients c.1, c.2, c.4, c.5, c.6, c.7 and c.8 in Appendix_A.

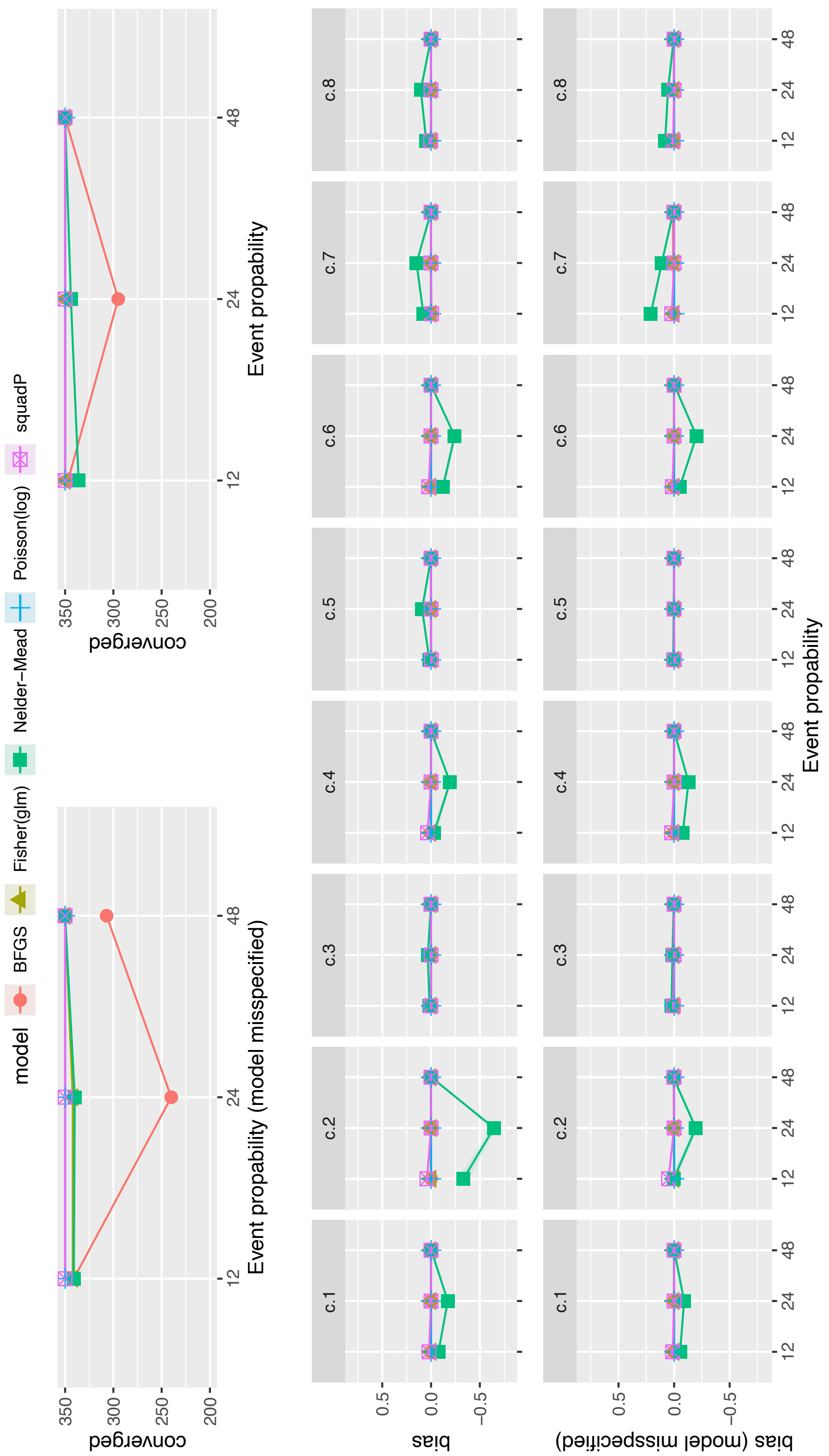

Figure 30: Absolute bias and MSE comparison between results from the five statistical methods with and without model misspecification for scenarios with 1 million sample size and 12%, 24%, 48% event probability

5.8 Comparison of the execution time

The execution time, in other words, computational cost, of a computer program or a method is an important and challenging issue. It gives us a general idea about the time required for a process to be executed, in addition to a better understanding of behavior of the method being used.

In this chapter, the running time of the methods being studied was calculated, which is the time required by each method to run the analysis till convergence. The running time of the six methods (squadP, Fisher GLM(log), Poisson(log), BFGS, and Nelder-Mead) was carried out using microbenchmark method (developed by Olaf Mersmann) that is implemented in R statistical programming language.

Three different scenarios were designed for this speed test; first scenario was with two covariates and event probability 12%, the second scenario was with for four covariates and event probability 24%, and third scenario was with eight covariates and event probability 48%. All scenarios were executed with sample size of 1000.

The speed test was executed with 800 run per each scenarios and measurements of the execution (running) time of the six methods were calculated and presented as follows.

5.8.1 Scenario with 2 covariates

It is obvious as shown in figure 31 that the EM-type method is extremely slower than other methods for executed the same process, while Poisson(log), and Fisher GLM were relatively faster (with maximum execution time of 387.53 millisecond for Poisson(log)). measurements of all methods are shown in table 34.

model	min	lq	mean	median	uq	max	neval
squadP	120.467	126.016	128.847	127.830	130.059	142.866	800
BFGS	34.946	36.244	36.886	36.846	37.330	46.432	800
Nelder-Mead	33.573	35.021	35.666	35.613	36.091	46.042	800
GLM(log)	7.785	8.322	8.651	8.505	8.750	18.340	800
Poisson(log)	10.246	10.809	11.778	11.026	11.267	387.538	800
EM-type	2835.72	2925.95	2968.73	2950.47	2974.35	3410.25	800

Table 34: Execution time of each method for the scenario with 2 covariates. Models are the methods being tested. The minimum, mean, and maximum execution time per millisecond was taken after the evaluation of 800 run.

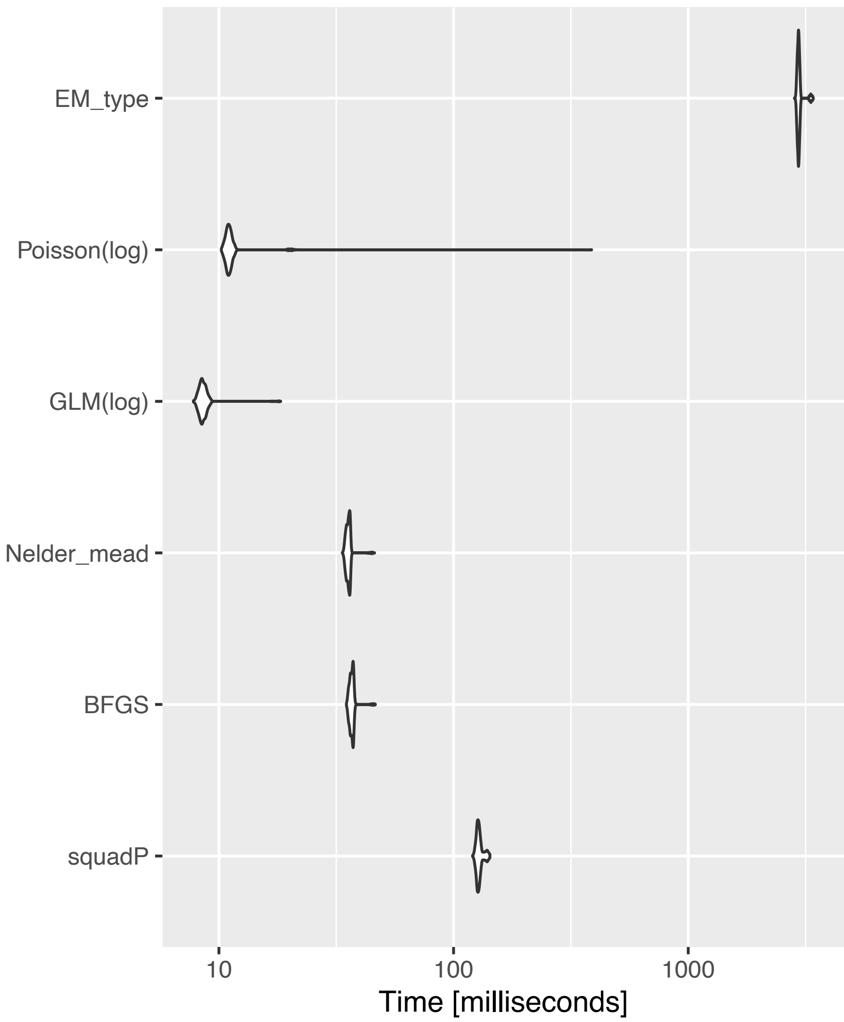

Figure 31: Computational cost (running time) of the six statistical methods on y-axis for the scenario with 2 covariates and event probability 12%. On x-axis is the required time from a method per millisecond till convergence

5.8.2 Scenario with 4 covariates

EM-type method, as shown in figure 32, still the slowest in comparison with the other methods. Nelder-Mead and squadP have relatively similar execution time for analyzing the same dataset. Poisson(log), and Fisher GLM have the least execution time as shown in the measurements table 35.

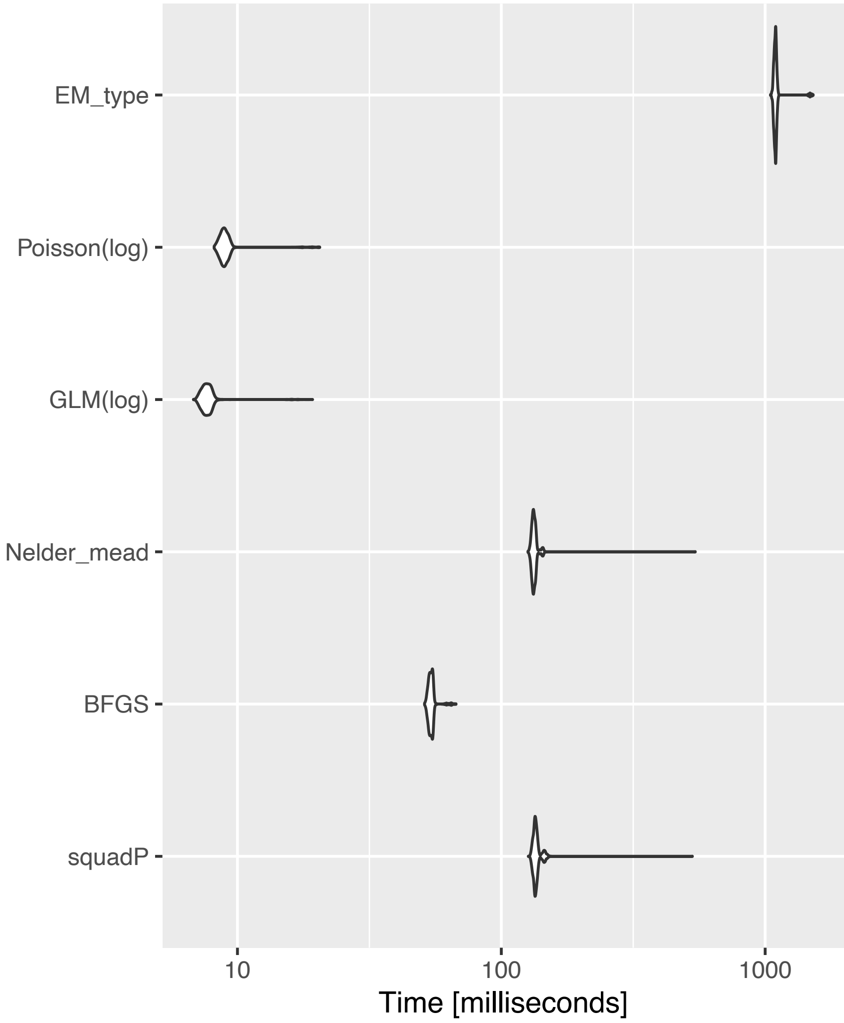

Figure 32: Running time of the six statistical methods on y-axis for the scenario with 4 covariates and event probability 24%. On x-axis is the required time from a method per millisecond till convergence

model	min	lq	mean	median	uq	max	neval
squadP	126.534	133.226	136.337	134.853	136.847	529.898	800
BFGS	51.159	53.235	54.321	54.075	54.869	67.391	800
Nelder-Mead	126.145	131.283	133.771	132.744	134.567	543.562	800
GLM(log)	6.808	7.413	7.789	7.643	7.881	19.269	800
Poisson(log)	8.158	8.691	9.203	8.908	9.158	20.511	800
EM-type	1047.03	1083.6	1102.4	1093.1	1100.5	1520.8	800

Table 35: Execution time of the six methods for the scenario with 4 covariates. The minimum, mean, and maximum execution time per millisecond was taken after the evaluation of 800 run.

5.8.3 Scenario with 8 covariates

The measurements in figure 33 and table 36 show that Poisson(log), and Fisher GLM have the least execution time mean, however with relatively high maximum execution time of 443.74, and 405.62 millisecond, while squadP method has a mean of execution time as 182.67 and maximum 207.47 millisecond. In this scenario with 8 covariates, squadP method showed significantly less execution time than Nelder-Mead and EM-type methods, and relatively similar speed as BFGS.

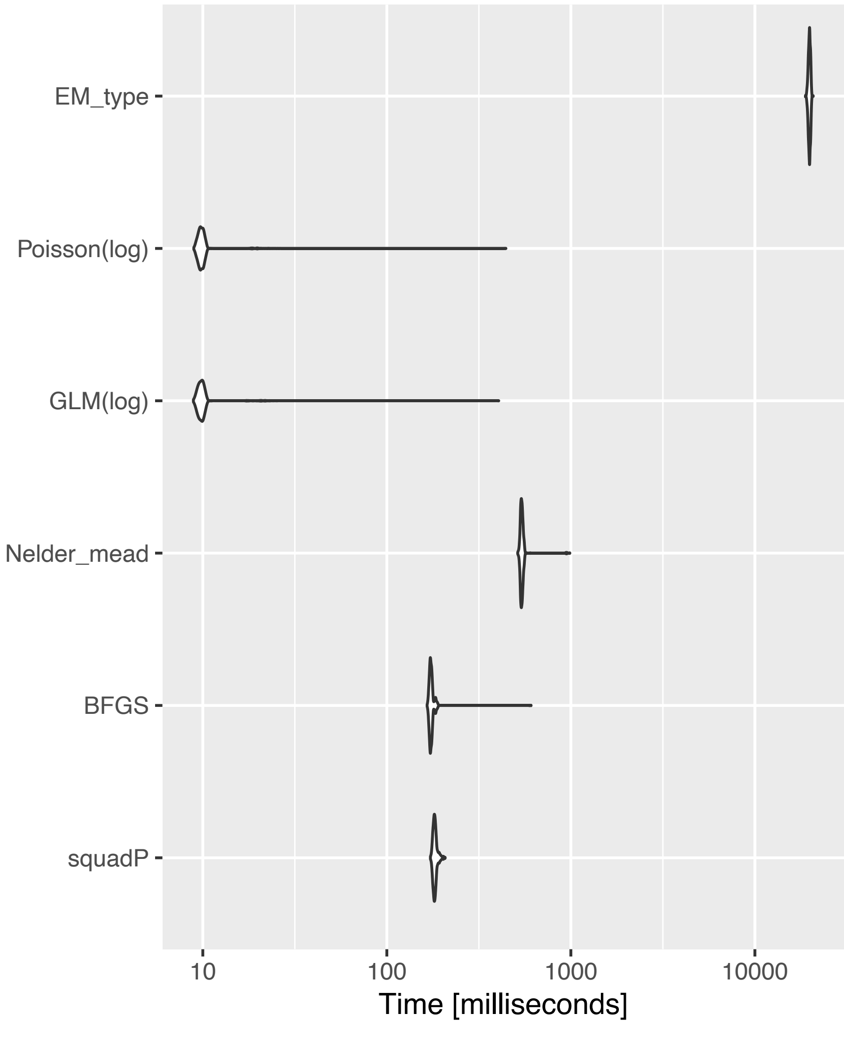

Figure 33: Running time required from the six statistical methods (y-axis) till convergence for the scenario with 8 covariates and event probability 48%. On x-axis is the required time per millisecond.

model	min	lq	mean	median	uq	max	neval
squadP	172.422	179.308	182.673	181.825	184.600	207.471	800
BFGS	165.458	171.403	175.396	173.340	176.002	607.953	800
Nelder-Mead	514.166	534.183	543.639	539.979	546.201	986.503	800
GLM(log)	8.887	9.502	10.578	9.801	10.074	405.628	800
Poisson(log)	8.932	9.539	10.623	9.789	10.073	443.742	800
EM-type	18815.1	19595.5	19775.4	19793.2	19975.5	20739.7	800

Table 36: Execution time of the six methods for the scenario with 8 covariates. The minimum, mean, and maximum execution time per millisecond was taken after the evaluation of 800 run.

6 Discussion and conclusion

For binary outcome data, the relative risk becomes an essential measure of association. Moreover most of the researchers calculate odds ratio which can be estimated directly for prospective studies using the logistic regression model. However, in case of an outcome of common incidence, calculating the odds ratio using the logistic model can overestimate and magnify the risk heavily [23] [58]. Moreover, odds ratio is arguably difficult to interpret. Thus, in such cases odds ratio should be avoided and risk ratio can be directly computed using the log-binomial regression model [50] [2] [54]. However, the standard log-binomial model used to estimate the maximum likelihood can encounter difficulties to converge and find the solution [57]. Williamson, et al. (2013) suggested an approach to deal with observed failed convergence. That is brute force maximization [48] [52] in case of log-binomial regression failed for estimating the maximum of a log-likelihood function. Brute force maximization offers MLE estimation in a similar way to what is done to generate a contour plot. However, this approach does not included standard errors as part of the estimating process. Brute force maximization generates only the point estimates which is insufficient. Furthermore, it requires more computing power. For the mentioned reasons brute force maximization was not included in this study.

The causes of convergence failure need to be studied and the problem to be reparameterized to present a reliable and specialized method for fitting the log-binomial model. Here, in this PhD thesis, a modified Newton-type algorithm (called squadP) is presented to perform not only maximum likelihood estimation for the log-binomial regression model, but also provides a solution for the starting value problem that is auto-generating optimized starting values.

Theoretically, the regression models may converge faster when using the optimized starting value because it is closer to the optimal solution as observed in many statistical analysis performed (data are not shown).

The standard log-binomial regression model, often the estimating algorithms as well, must be supplied with reasonable starting values. If the starting values are not supplied, or not inside the valid parameter space, the estimators then cannot proceed and the estimating procedure will be interrupted and produces an error. The user has to provide the algorithm with a combination of valid starting values inside the parameter space. The presented method in this study (squadP) guarantees a combination of self-generated starting values inside the feasible region. Moreover, the optimized self-generated start values showed a decrease of the number of iterations required by the estimators (Fisher(GLM) and squadP) to converge as expected, because it is not far from the maximum solution in the parameter space.

Providing any estimator with valid starting values inside the parameter space does not guarantee convergence and is insufficient. Wherefore, a further testing and examination of the method used for estimation is required.

In this study, testing of presented “squadP” was carried out. Moreover, a comparison with other popular statistical methods to estimate the maximum likelihood function was performed using a published data set [27]. All methods being studied including squadP converged to similar estimates as presented in figure 11. However, Poisson(log) method showed larger standard error therefore a wider range of confidence interval. Generally, Poisson method has larger standard errors compared with the other statistical methods because of its nature of distributional assumption that is the variance equals the mean. There are some methods to control the mild violation of the distribution assumption such as using robust standard errors as recommended by Cameron and Trivedi (2009) [8]. However it is not used in this study because is not implemented in the standard GLM models regression in R.

A further testing and examination of the approach presented in this study (squadP) was performed to detect measurement errors such as the absolute bias using a simulated example. As shown in figure 10, squadP and EM-type have estimates with the smallest biases (nearest to the true solution) and they showed identical estimates compared with other methods.

Failed convergence can occur often, and if a model fails, a further testing and examination of the possible causes should be carried out. Causes of failure can be reparametrization of the model, improper starting values, or outer parameter space. For the above mentioned reasons, a large simulation study was conducted which allowed many possible scenarios to evaluate the statistical models being studied compared with the proposed method “squadP.” As expected, convergence rate was smaller with small sample sizes in most scenarios which indicates higher probability for a model to fail with small sample size moreover higher bias.

Using real data, most of the studies have small sample sizes, therefore a model with higher certainty is required for estimation.

As observed in figure 12, squadP method and EM-type algorithm have the smallest bias compared with the other methods particularly in case of scenarios with small samples sizes such as 60 and small event probability 3%. We find -0.18, and -0.14 bias for squadP and EM-type methods compared with -0.683, -1.114, -0.884, and -1.450 bias for BFGS, Fisher, Nelder-Mead, and Poisson methods. Scenarios with 80 sample size have similar pattern as shown in table 11. Furthermore, squadP and EM-type methods showed the narrowest confidence interval compared with other methods for the intercept, and coefficients 1 and 2 as shown in tables 11, 12, and 13

As expected, with large samples such as 1000, and 50000 all methods in most scenarios have similar bias near to 0 because they give the model more information about the nature of data which helps for accurate estimation. Certainly, specifying models correctly and fitting them with the proper data types and format ensure generally meaningful results.

As shown in figure 13, squadP and Poisson methods showed the highest and similar convergence rate compared with the other methods particularly in case of scenarios with small sample sizes such as 60, 80 and 100. Moreover, squadP and EM-type showed the highest coverage probability with the smallest empirical standard error and mean squared error. EM-type method has the lowest convergence rate and reached 100% only when the sample size was 50000. This seems to be a pattern. The EM-algorithm has more convergence problems than the other algorithms.

Theoretically, log-binomial regression models are asymptotically unbiased, but that does not guarantee good performance in small sample sizes. Therefore, it is difficult for a model to yield an accurate estimation in small samples and particularly small event probability. Therefore, Researchers and data analysts who are frequently faced with the problem in which the sample size is relatively small, should search for the statistical model of interest that provide less biased estimates because the standard methods may not provide accurate parameter estimates.

squadP in most problematic scenarios had very high convergence rates and coverage probabilities with one of the smallest errors measurements furthermore less bias compared with other methods in the study. Generally, EM-type algorithm has a very high coverage probability as well. However, by looking at all scenarios EM-type algorithm had always the lowest convergence rate which fails the most to converge and find a solution. In conclusion, EM-type didn't perform well particularly when with a small proportion of events and small sample sizes.

EM-type method (implemented combinatorial em algorithm) are the main novel method included in the logbin package in R and very stable. However it has a problem of defining the factor variable as an input to the model which gives results different than when defining the input variable as numeric and hasn't been fixed so far by the maintainer of the logbin package. Furthermore, EM-type method doesn't allow interaction terms between variables which indicate that it will be avoided to use in certain scenarios or particular analysis. It is noted that squadP and EM-type have similar estimates with significant excel of squadP convergence rate even though both methods solve constrained problems.

In contrast, Fisher scoring algorithm and Poisson (log) model, as methods for fitting generalized linear models (GLM) and part of GLM package in R, are solving unconstrained problems by quadratic convergence. In this simulation study, Fisher scoring generally performed better with smaller bias compared with Poisson (log), particularly in scenarios with small sample sizes (such as 60, 80 and 100) and small event probability such as 3% as observed in figure 12. As expected, in case of scenarios with large sample sizes (1000, and 50000) both methods performed similarly. It is noted that Poisson(log) method in most scenarios had significantly a larger convergence rate and coverage probabilities compared with Fisher scoring however with larger error measurements as shown in figure 13. A similar patterns were observed in most scenarios across this simulation study.

In most scenarios across the simulation study, BFGS and Nelder-Mead methods as part of optim package in R showed similar patterns with insignificant bias, coverage probabilities, and errors differences. However, Nelder-mead method in few scenarios showed a bizarre and not understood behavior compared with other methods in the study. For instance, a sudden and significant drop of the coverage probabilities in case of scenarios with event probability 48% and sample sizes 500, and 1000 as showed in figure 21.

Regarding the computational cost, generally the algorithms underlying the adaptive barrier approach have fast convergence particularly in the unconstrained cases. For example, quasi-Newton methods such as BFGS have super-linear convergence (Broyden, et all 1973). In contrast, the EM-type algorithm typically takes a large number of iterations to converge which makes the method significantly slower to converge compared with other methods. As expected, EM-type as a linear convergent algorithm showed the slowest convergence in all tested scenarios compared with other methods. Moreover, EM-type method was excluded in scenarios with 8 covariates for computational reasons. The time estimated time required from EM-type algorithm approximately $\approx$ 34d 01h 48m 06s for each scenario therefore $\approx$ 306 days running continuously on our local server with 12 cores for all scenarios with 8 covariates in the simulation study.

Fisher scoring and Poisson(log) showed fast convergence in all tested scenarios compared with other methods as presented in figures 31, 32, and 33.

In case of scenario with 4 covariates, squadP had similar performance as Nelder-Mead compared with a slightly faster performance of BFGS. In case of scenario with 8 covariates, squadP showed better and faster performance than Nelder-Mead and BFGS. Furthermore, squadP had the smallest range between the minimum and the maximum time required for a single estimate as shown in figure 33.

In this large simulation study, squadP approach presented a better performance in most of the scenarios compared with methods being studied. Furthermore, Execution time of squadP was not affected significantly when samples have larger sizes. Moreover, a further simulated data with more variety of scenarios could be done in the future with the optimized starting value in order to generalize the optimization approach on all of the statistical methods that require starting values.

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8 Appendix A: Study results

8.1 Monte Carlo simulation results: scenarios with 4 covariates

8.1.1 Scenarios with event probability 12%

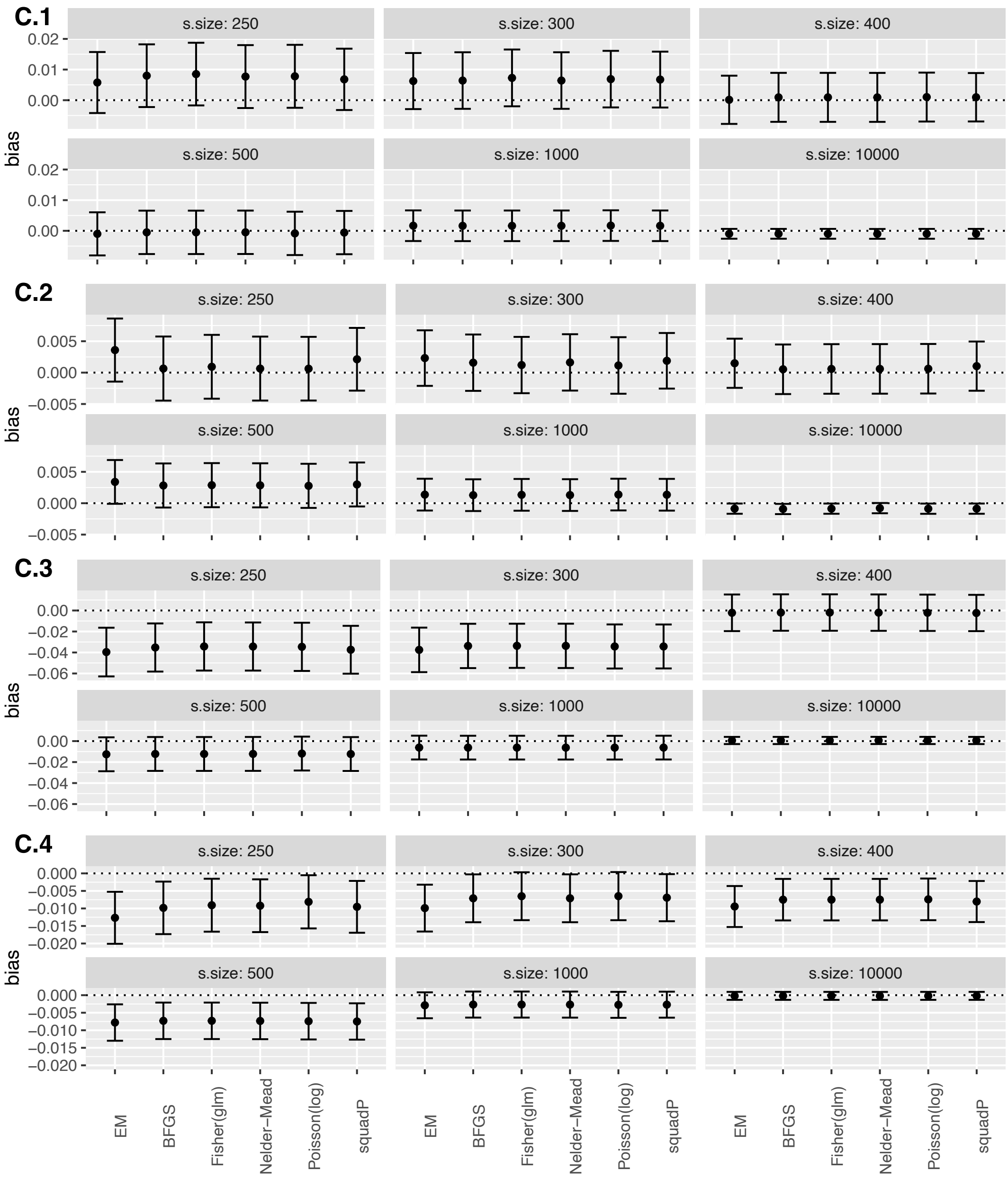

Figure 34: The absolute biases from the six methods in each of the 6 scenarios with event probability 12% with sample sizes 250, 300, 400, 500, 1000, and 10000. y-axis: bias for coef.1, coef.2, coef.3, and coef.4. x-axis: The six statistical methods compared for each scenario.

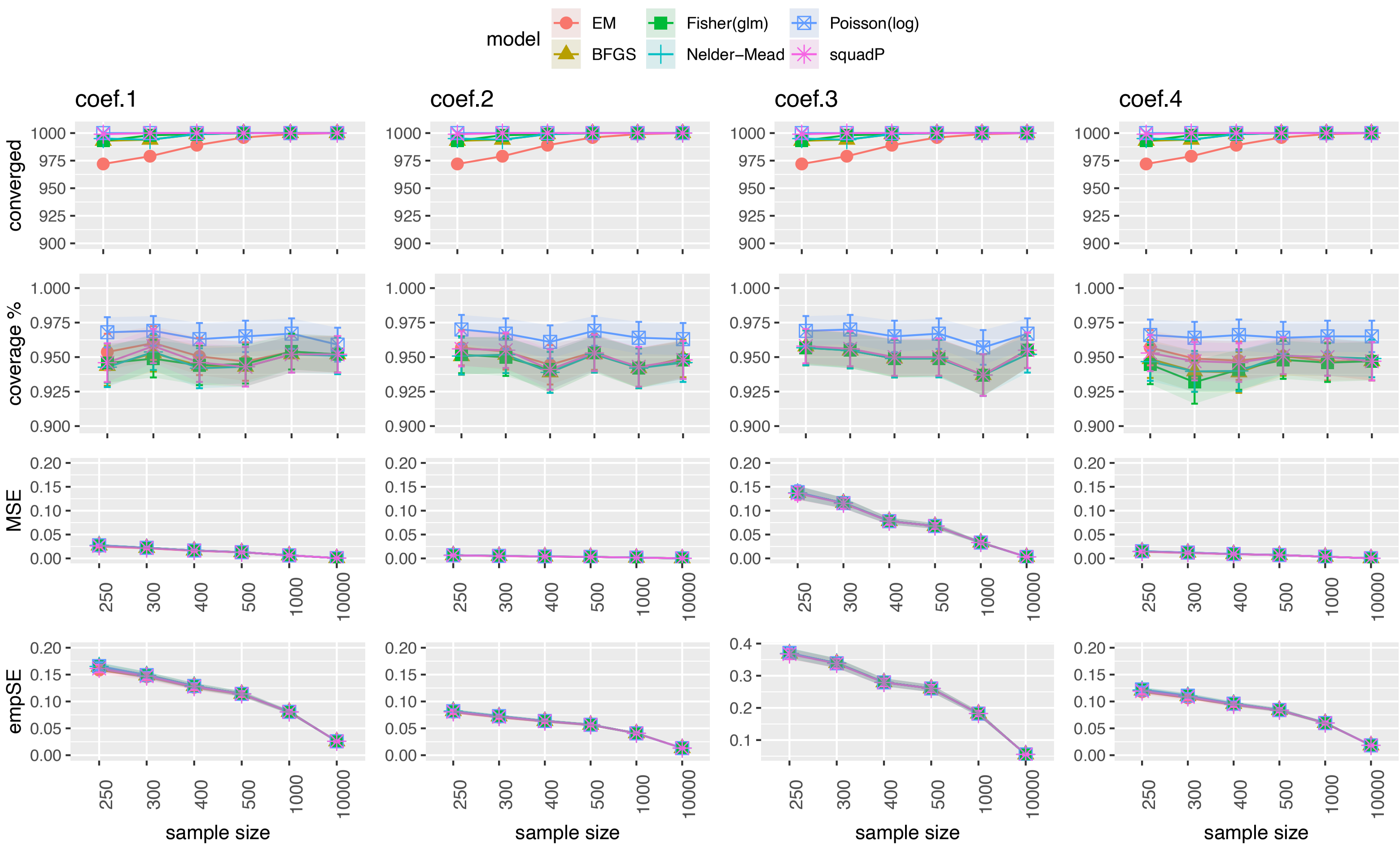

Figure 35: Performance measurments of 6 scenarios with event probability 12%. Convergence rate (at the top), coverage prabability, MSE, and EmpSE (y-axis) for the coeffecints (coef.1, coef.2, coef.3, and coef.4) using the six methods for each scenario. Each type of shapes with shaded area (confidence interval) is represnting a different method. The measurments are done for 6 scenarios that are derived from six different sample sizes 250,300,400,500,1000, and 10000 on x-axis.

Table 37: Performance measurments of scenarios 37→42 for coef.1

Covarage		Bias		MSE		EmpSE				
cover.	mcse	bias	mcse	mse	mcse	empse	mcse	model	s.size	converged
0.954	0.007	0.006	0.005	0.025	0.001	0.158	0.004	EM	250	972
0.944	0.007	0.008	0.005	0.027	0.001	0.165	0.004	BFGS	250	993
0.946	0.007	0.009	0.005	0.027	0.001	0.164	0.004	Fisher(glm)	250	993
0.943	0.007	0.008	0.005	0.027	0.001	0.165	0.004	Nelder-Mead	250	995
0.968	0.006	0.008	0.005	0.028	0.001	0.166	0.004	Poisson(log)	250	1000
0.946	0.007	0.007	0.005	0.026	0.001	0.161	0.004	squadP	250	999
0.960	0.006	0.006	0.005	0.021	0.001	0.146	0.003	EM	300	979
0.953	0.007	0.006	0.005	0.022	0.001	0.148	0.003	BFGS	300	994
0.949	0.007	0.007	0.005	0.022	0.001	0.149	0.003	Fisher(glm)	300	998
0.954	0.007	0.006	0.005	0.022	0.001	0.148	0.003	Nelder-Mead	300	994
0.969	0.005	0.007	0.005	0.022	0.001	0.149	0.003	Poisson(log)	300	1000
0.958	0.006	0.007	0.005	0.022	0.001	0.147	0.003	squadP	300	1000
0.950	0.007	0.000	0.004	0.016	0.001	0.126	0.003	EM	400	989
0.944	0.007	0.001	0.004	0.017	0.001	0.129	0.003	BFGS	400	999
0.944	0.007	0.001	0.004	0.017	0.001	0.129	0.003	Fisher(glm)	400	999
0.942	0.007	0.001	0.004	0.017	0.001	0.129	0.003	Nelder-Mead	400	999
0.963	0.006	0.001	0.004	0.017	0.001	0.129	0.003	Poisson(log)	400	1000
0.946	0.007	0.001	0.004	0.016	0.001	0.127	0.003	squadP	400	1000
0.947	0.007	-0.001	0.004	0.013	0.001	0.113	0.003	EM	500	996
0.943	0.007	-0.001	0.004	0.013	0.001	0.114	0.003	BFGS	500	1000
0.945	0.007	-0.001	0.004	0.013	0.001	0.114	0.003	Fisher(glm)	500	1000
0.943	0.007	0.000	0.004	0.013	0.001	0.114	0.003	Nelder-Mead	500	1000
0.965	0.006	-0.001	0.004	0.013	0.001	0.114	0.003	Poisson(log)	500	1000
0.943	0.007	-0.001	0.004	0.013	0.001	0.114	0.003	squadP	500	1000
0.954	0.007	0.002	0.003	0.006	0.000	0.081	0.002	EM	1000	999
0.952	0.007	0.002	0.003	0.006	0.000	0.081	0.002	BFGS	1000	1000
0.954	0.007	0.002	0.003	0.006	0.000	0.081	0.002	Fisher(glm)	1000	1000
0.952	0.007	0.002	0.003	0.006	0.000	0.081	0.002	Nelder-Mead	1000	1000
0.967	0.006	0.002	0.003	0.006	0.000	0.081	0.002	Poisson(log)	1000	1000
0.952	0.007	0.002	0.003	0.006	0.000	0.081	0.002	squadP	1000	1000
0.952	0.007	-0.001	0.001	0.001	0.000	0.026	0.001	EM	10000	1000
0.952	0.007	-0.001	0.001	0.001	0.000	0.026	0.001	BFGS	10000	1000
0.952	0.007	-0.001	0.001	0.001	0.000	0.026	0.001	Fisher(glm)	10000	1000
0.951	0.007	-0.001	0.001	0.001	0.000	0.026	0.001	Nelder-Mead	10000	1000
0.959	0.006	-0.001	0.001	0.001	0.000	0.026	0.001	Poisson(log)	10000	1000
0.952	0.007	-0.001	0.001	0.001	0.000	0.026	0.001	squadP	10000	1000

Table 38: Performance measurments of scenarios 37→42 for coef.2

Covarage		Bias		MSE		EmpSE				
cover.	mcse	bias	mcse	mse	mcse	empse	mcse	model	s.size	converged
0.957	0.007	0.004	0.003	0.006	0	0.080	0.002	EM	250	972
0.951	0.007	0.001	0.003	0.007	0	0.082	0.002	BFGS	250	993
0.952	0.007	0.001	0.003	0.007	0	0.082	0.002	Fisher(glm)	250	993
0.951	0.007	0.001	0.003	0.007	0	0.082	0.002	Nelder-Mead	250	995
0.970	0.005	0.001	0.003	0.007	0	0.082	0.002	Poisson(log)	250	1000
0.956	0.006	0.002	0.003	0.006	0	0.081	0.002	squadP	250	999
0.954	0.007	0.002	0.002	0.005	0	0.071	0.002	EM	300	979
0.952	0.007	0.002	0.002	0.005	0	0.072	0.002	BFGS	300	994
0.950	0.007	0.001	0.002	0.005	0	0.072	0.002	Fisher(glm)	300	998
0.952	0.007	0.002	0.002	0.005	0	0.072	0.002	Nelder-Mead	300	994
0.967	0.006	0.001	0.002	0.005	0	0.073	0.002	Poisson(log)	300	1000
0.955	0.007	0.002	0.002	0.005	0	0.071	0.002	squadP	300	1000
0.944	0.007	0.001	0.002	0.004	0	0.063	0.001	EM	400	989
0.939	0.008	0.001	0.002	0.004	0	0.064	0.001	BFGS	400	999
0.941	0.007	0.001	0.002	0.004	0	0.064	0.001	Fisher(glm)	400	999
0.939	0.008	0.001	0.002	0.004	0	0.064	0.001	Nelder-Mead	400	999
0.961	0.006	0.001	0.002	0.004	0	0.064	0.001	Poisson(log)	400	1000
0.941	0.007	0.001	0.002	0.004	0	0.063	0.001	squadP	400	1000
0.954	0.007	0.003	0.002	0.003	0	0.056	0.001	EM	500	996
0.952	0.007	0.003	0.002	0.003	0	0.057	0.001	BFGS	500	1000
0.953	0.007	0.003	0.002	0.003	0	0.057	0.001	Fisher(glm)	500	1000
0.952	0.007	0.003	0.002	0.003	0	0.057	0.001	Nelder-Mead	500	1000
0.969	0.005	0.003	0.002	0.003	0	0.057	0.001	Poisson(log)	500	1000
0.953	0.007	0.003	0.002	0.003	0	0.057	0.001	squadP	500	1000
0.942	0.007	0.001	0.001	0.002	0	0.041	0.001	EM	1000	999
0.942	0.007	0.001	0.001	0.002	0	0.041	0.001	BFGS	1000	1000
0.942	0.007	0.001	0.001	0.002	0	0.041	0.001	Fisher(glm)	1000	1000
0.942	0.007	0.001	0.001	0.002	0	0.041	0.001	Nelder-Mead	1000	1000
0.964	0.006	0.001	0.001	0.002	0	0.041	0.001	Poisson(log)	1000	1000
0.943	0.007	0.001	0.001	0.002	0	0.041	0.001	squadP	1000	1000
0.948	0.007	-0.001	0.000	0.000	0	0.013	0.000	EM	10000	1000
0.949	0.007	-0.001	0.000	0.000	0	0.013	0.000	BFGS	10000	1000
0.948	0.007	-0.001	0.000	0.000	0	0.013	0.000	Fisher(glm)	10000	1000
0.946	0.007	-0.001	0.000	0.000	0	0.013	0.000	Nelder-Mead	10000	1000
0.963	0.006	-0.001	0.000	0.000	0	0.013	0.000	Poisson(log)	10000	1000
0.948	0.007	-0.001	0.000	0.000	0	0.013	0.000	squadP	10000	1000

Table 39: Performance measurments of scenarios 37→42 for coef.3

Covarage		Bias		MSE		EmpSE				
cover.	mcse	bias	mcse	mse	mcse	empse	mcse	model	s.size	converged
0.957	0.007	-0.040	0.012	0.138	0.007	0.369	0.008	EM	250	972
0.958	0.006	-0.035	0.012	0.137	0.007	0.369	0.008	BFGS	250	993
0.957	0.006	-0.034	0.012	0.138	0.007	0.370	0.008	Fisher(glm)	250	993
0.957	0.006	-0.034	0.012	0.137	0.007	0.369	0.008	Nelder-Mead	250	995
0.969	0.005	-0.035	0.012	0.139	0.007	0.371	0.008	Poisson(log)	250	1000
0.958	0.006	-0.037	0.012	0.136	0.007	0.368	0.008	squadP	250	999
0.956	0.007	-0.038	0.011	0.116	0.006	0.339	0.008	EM	300	979
0.955	0.007	-0.034	0.011	0.116	0.006	0.340	0.008	BFGS	300	994
0.955	0.007	-0.034	0.011	0.116	0.006	0.339	0.008	Fisher(glm)	300	998
0.955	0.007	-0.034	0.011	0.116	0.006	0.340	0.008	Nelder-Mead	300	994
0.970	0.005	-0.034	0.011	0.116	0.006	0.339	0.008	Poisson(log)	300	1000
0.956	0.006	-0.034	0.011	0.115	0.006	0.338	0.008	squadP	300	1000
0.949	0.007	-0.002	0.009	0.078	0.004	0.280	0.006	EM	400	989
0.949	0.007	-0.002	0.009	0.078	0.004	0.280	0.006	BFGS	400	999
0.949	0.007	-0.002	0.009	0.078	0.004	0.280	0.006	Fisher(glm)	400	999
0.949	0.007	-0.002	0.009	0.078	0.004	0.280	0.006	Nelder-Mead	400	999
0.965	0.006	-0.002	0.009	0.078	0.004	0.279	0.006	Poisson(log)	400	1000
0.950	0.007	-0.002	0.009	0.078	0.004	0.279	0.006	squadP	400	1000
0.950	0.007	-0.013	0.008	0.068	0.003	0.260	0.006	EM	500	996
0.950	0.007	-0.012	0.008	0.068	0.003	0.261	0.006	BFGS	500	1000
0.949	0.007	-0.012	0.008	0.068	0.003	0.261	0.006	Fisher(glm)	500	1000
0.949	0.007	-0.012	0.008	0.068	0.003	0.261	0.006	Nelder-Mead	500	1000
0.967	0.006	-0.012	0.008	0.068	0.003	0.260	0.006	Poisson(log)	500	1000
0.950	0.007	-0.012	0.008	0.068	0.003	0.260	0.006	squadP	500	1000
0.937	0.008	-0.006	0.006	0.033	0.002	0.183	0.004	EM	1000	999
0.937	0.008	-0.006	0.006	0.033	0.002	0.183	0.004	BFGS	1000	1000
0.937	0.008	-0.006	0.006	0.033	0.002	0.183	0.004	Fisher(glm)	1000	1000
0.937	0.008	-0.006	0.006	0.033	0.002	0.183	0.004	Nelder-Mead	1000	1000
0.957	0.006	-0.006	0.006	0.033	0.002	0.182	0.004	Poisson(log)	1000	1000
0.937	0.008	-0.006	0.006	0.033	0.002	0.183	0.004	squadP	1000	1000
0.955	0.007	0.001	0.002	0.003	0.000	0.055	0.001	EM	10000	1000
0.955	0.007	0.001	0.002	0.003	0.000	0.055	0.001	BFGS	10000	1000
0.955	0.007	0.001	0.002	0.003	0.000	0.055	0.001	Fisher(glm)	10000	1000
0.952	0.007	0.001	0.002	0.003	0.000	0.055	0.001	Nelder-Mead	10000	1000
0.967	0.006	0.001	0.002	0.003	0.000	0.055	0.001	Poisson(log)	10000	1000
0.955	0.007	0.001	0.002	0.003	0.000	0.055	0.001	squadP	10000	1000

Table 40: Performance measurments of scenarios 37→42 for coef.4

Covarage		Bias		MSE		EmpSE				
cover.	mcse	bias	mcse	mse	mcse	empse	mcse	model	s.size	converged
0.957	0.007	-0.013	0.004	0.014	0.001	0.118	0.003	EM	250	972
0.949	0.007	-0.010	0.004	0.015	0.001	0.120	0.003	BFGS	250	993
0.945	0.007	-0.009	0.004	0.015	0.001	0.121	0.003	Fisher(glm)	250	993
0.947	0.007	-0.009	0.004	0.015	0.001	0.121	0.003	Nelder-Mead	250	995
0.966	0.006	-0.008	0.004	0.015	0.001	0.122	0.003	Poisson(log)	250	1000
0.953	0.007	-0.010	0.004	0.014	0.001	0.119	0.003	squadP	250	999
0.949	0.007	-0.010	0.003	0.011	0.001	0.107	0.002	EM	300	979
0.940	0.008	-0.007	0.003	0.012	0.001	0.110	0.002	BFGS	300	994
0.932	0.008	-0.007	0.003	0.012	0.001	0.110	0.002	Fisher(glm)	300	998
0.940	0.008	-0.007	0.003	0.012	0.001	0.110	0.002	Nelder-Mead	300	994
0.964	0.006	-0.006	0.004	0.012	0.001	0.111	0.002	Poisson(log)	300	1000
0.947	0.007	-0.007	0.003	0.012	0.001	0.108	0.002	squadP	300	1000
0.947	0.007	-0.009	0.003	0.009	0.000	0.094	0.002	EM	400	989
0.939	0.008	-0.007	0.003	0.009	0.000	0.095	0.002	BFGS	400	999
0.941	0.007	-0.007	0.003	0.009	0.000	0.095	0.002	Fisher(glm)	400	999
0.940	0.008	-0.007	0.003	0.009	0.000	0.095	0.002	Nelder-Mead	400	999
0.966	0.006	-0.007	0.003	0.009	0.000	0.096	0.002	Poisson(log)	400	1000
0.946	0.007	-0.008	0.003	0.009	0.000	0.094	0.002	squadP	400	1000
0.951	0.007	-0.008	0.003	0.007	0.000	0.084	0.002	EM	500	996
0.950	0.007	-0.007	0.003	0.007	0.000	0.084	0.002	BFGS	500	1000
0.948	0.007	-0.007	0.003	0.007	0.000	0.084	0.002	Fisher(glm)	500	1000
0.951	0.007	-0.007	0.003	0.007	0.000	0.084	0.002	Nelder-Mead	500	1000
0.964	0.006	-0.007	0.003	0.007	0.000	0.084	0.002	Poisson(log)	500	1000
0.951	0.007	-0.007	0.003	0.007	0.000	0.084	0.002	squadP	500	1000
0.947	0.007	-0.003	0.002	0.004	0.000	0.060	0.001	EM	1000	999
0.950	0.007	-0.003	0.002	0.004	0.000	0.060	0.001	BFGS	1000	1000
0.946	0.007	-0.003	0.002	0.004	0.000	0.060	0.001	Fisher(glm)	1000	1000
0.950	0.007	-0.003	0.002	0.004	0.000	0.060	0.001	Nelder-Mead	1000	1000
0.965	0.006	-0.003	0.002	0.004	0.000	0.060	0.001	Poisson(log)	1000	1000
0.950	0.007	-0.003	0.002	0.004	0.000	0.060	0.001	squadP	1000	1000
0.947	0.007	0.000	0.001	0.000	0.000	0.018	0.000	EM	10000	1000
0.947	0.007	0.000	0.001	0.000	0.000	0.018	0.000	BFGS	10000	1000
0.947	0.007	0.000	0.001	0.000	0.000	0.018	0.000	Fisher(glm)	10000	1000
0.949	0.007	0.000	0.001	0.000	0.000	0.018	0.000	Nelder-Mead	10000	1000
0.965	0.006	0.000	0.001	0.000	0.000	0.018	0.000	Poisson(log)	10000	1000
0.947	0.007	0.000	0.001	0.000	0.000	0.018	0.000	squadP	10000	1000

8.1.2 Scenarios with event probability 24%

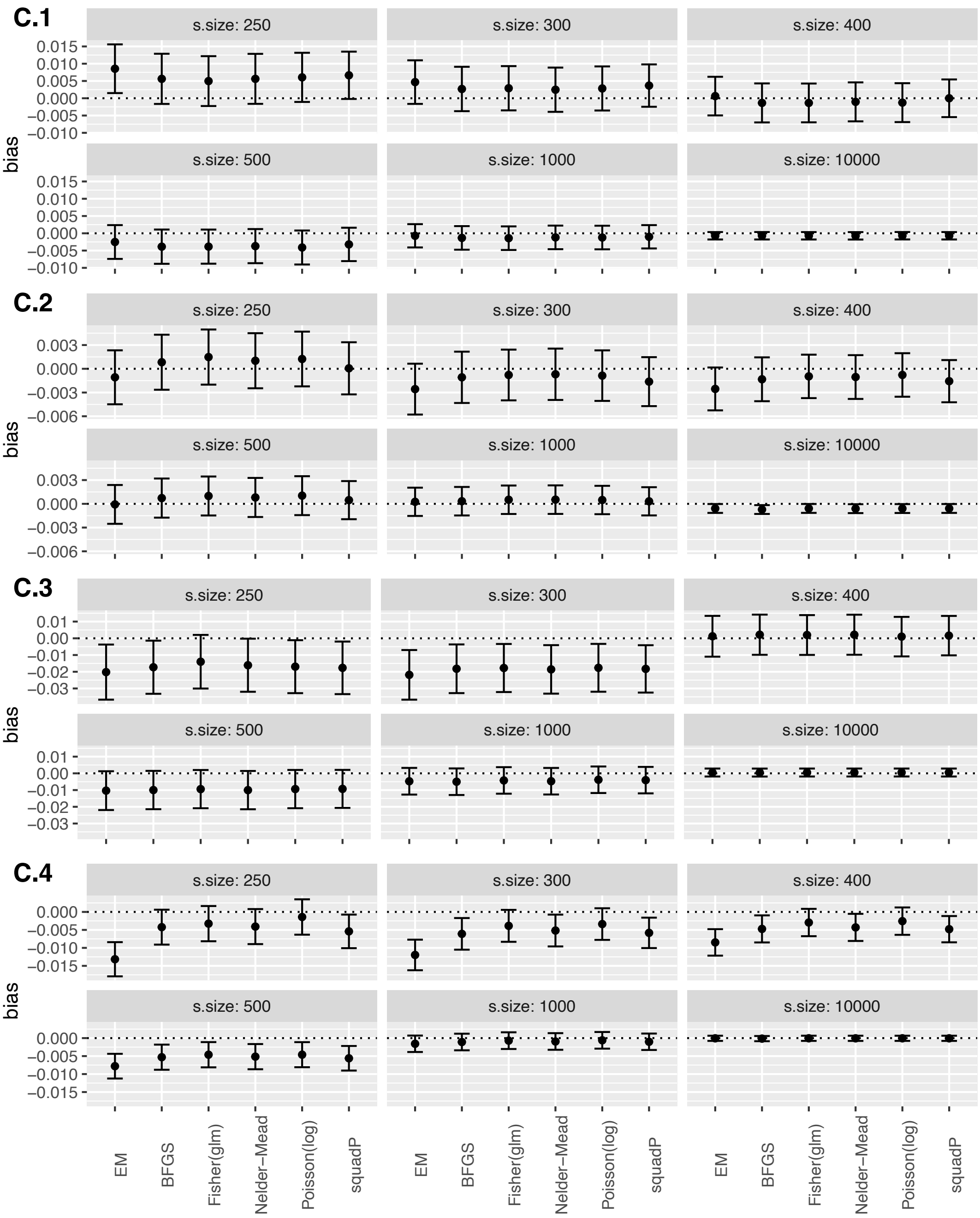

Figure 36: The absolute biases of the estimated log(RR) from the six methods in each of the 6 scenarios with event probability 24% with sample sizes 250, 300, 400, 500, 1000, and 10000. y-axis: bias for coef.1, coef.2, coef.3, and coef.4. x-axis: The six statistical methods used for each scenario.

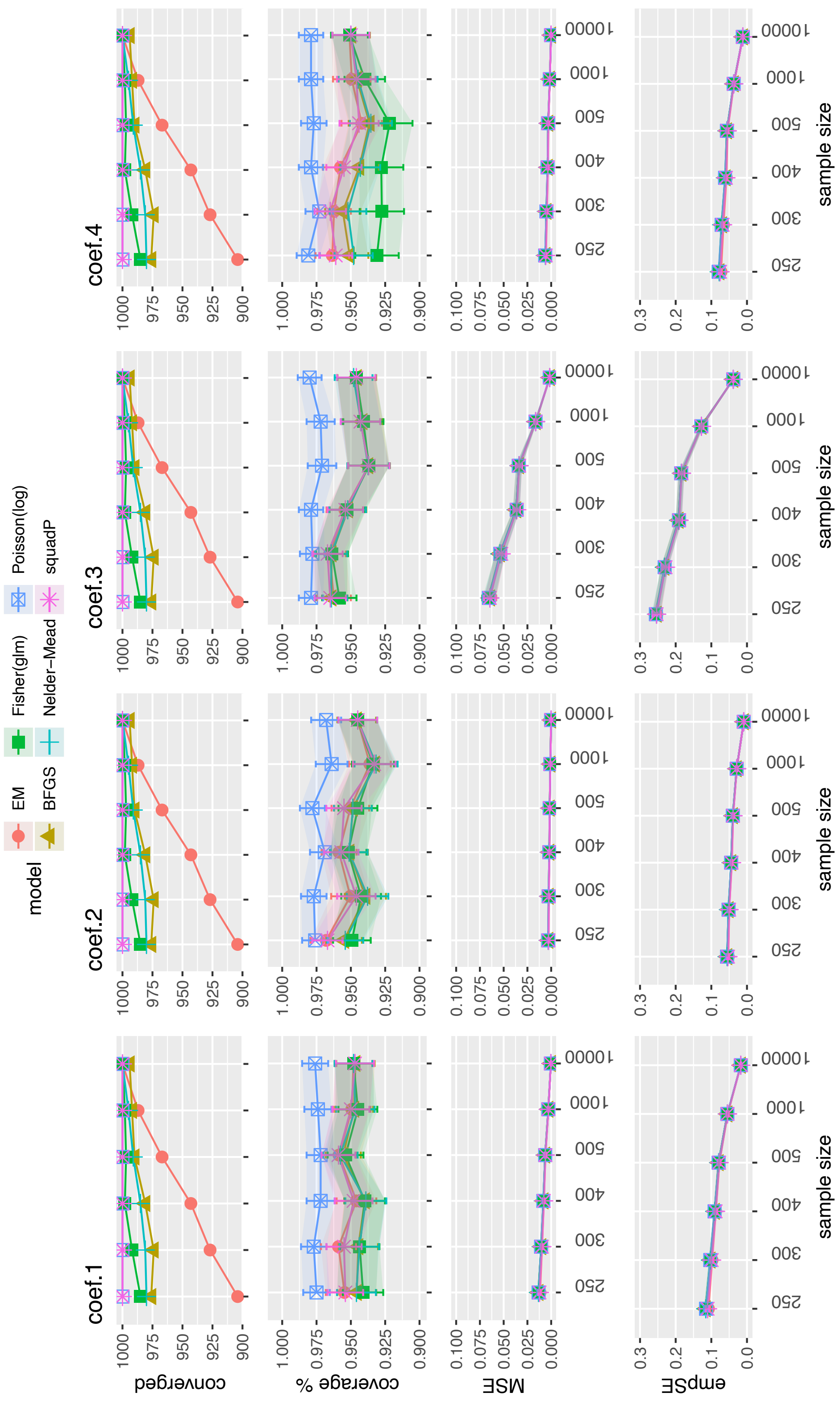

Figure 37: Performance measurments of 6 scenarios with event probability 24%. Convergence rate, coverage prabability, MSE, and EmpSE (y-axis) for coeffecints (coef.1, coef.2, coef.3, and coef.4) using the six methods for each scenario. Each type of shapes with shaded area (confidence interval) is representing a different method. The measurments are done for 6 scenarios that are derived from six different sample sizes 250,300,400,500,1000, and 10000 on x-axis.

Table 41: Performance measurments of scenarios 43→48 for coef.1

Covarage		Bias		MSE		EmpSE				
cover.	mcse	bias	mcse	mse	mcse	empse	mcse	model	s.size	converged
0.955	0.007	0.009	0.004	0.012	0.001	0.108	0.003	EM	250	904
0.946	0.007	0.006	0.004	0.013	0.001	0.116	0.003	BFGS	250	976
0.941	0.008	0.005	0.004	0.013	0.001	0.116	0.003	Fisher(glm)	250	985
0.946	0.007	0.006	0.004	0.013	0.001	0.116	0.003	Nelder-Mead	250	980
0.975	0.005	0.006	0.004	0.013	0.001	0.115	0.003	Poisson(log)	250	1000
0.954	0.007	0.007	0.003	0.012	0.001	0.110	0.002	squadP	250	1000
0.959	0.007	0.005	0.003	0.010	0.000	0.098	0.002	EM	300	927
0.945	0.007	0.003	0.003	0.010	0.000	0.102	0.002	BFGS	300	974
0.944	0.007	0.003	0.003	0.011	0.000	0.103	0.002	Fisher(glm)	300	992
0.944	0.007	0.002	0.003	0.010	0.000	0.102	0.002	Nelder-Mead	300	981
0.977	0.005	0.003	0.003	0.011	0.000	0.103	0.002	Poisson(log)	300	1000
0.955	0.007	0.004	0.003	0.010	0.000	0.099	0.002	squadP	300	1000
0.946	0.007	0.001	0.003	0.008	0.000	0.088	0.002	EM	400	943
0.940	0.008	-0.001	0.003	0.008	0.000	0.090	0.002	BFGS	400	981
0.940	0.008	-0.001	0.003	0.008	0.000	0.091	0.002	Fisher(glm)	400	998
0.939	0.008	-0.001	0.003	0.008	0.000	0.090	0.002	Nelder-Mead	400	985
0.972	0.005	-0.001	0.003	0.008	0.000	0.091	0.002	Poisson(log)	400	1000
0.948	0.007	0.000	0.003	0.008	0.000	0.088	0.002	squadP	400	1000
0.959	0.006	-0.003	0.002	0.006	0.000	0.078	0.002	EM	500	967
0.956	0.007	-0.004	0.003	0.006	0.000	0.080	0.002	BFGS	500	990
0.954	0.007	-0.004	0.003	0.006	0.000	0.080	0.002	Fisher(glm)	500	997
0.958	0.006	-0.004	0.003	0.006	0.000	0.079	0.002	Nelder-Mead	500	991
0.972	0.005	-0.004	0.003	0.006	0.000	0.079	0.002	Poisson(log)	500	1000
0.959	0.006	-0.003	0.002	0.006	0.000	0.078	0.002	squadP	500	1000
0.949	0.007	-0.001	0.002	0.003	0.000	0.054	0.001	EM	1000	987
0.948	0.007	-0.001	0.002	0.003	0.000	0.055	0.001	BFGS	1000	992
0.945	0.007	-0.001	0.002	0.003	0.000	0.055	0.001	Fisher(glm)	1000	998
0.947	0.007	-0.001	0.002	0.003	0.000	0.055	0.001	Nelder-Mead	1000	995
0.974	0.005	-0.001	0.002	0.003	0.000	0.055	0.001	Poisson(log)	1000	1000
0.950	0.007	-0.001	0.002	0.003	0.000	0.054	0.001	squadP	1000	1000
0.948	0.007	-0.001	0.001	0.000	0.000	0.017	0.000	EM	10000	1000
0.947	0.007	-0.001	0.001	0.000	0.000	0.017	0.000	BFGS	10000	994
0.948	0.007	-0.001	0.001	0.000	0.000	0.017	0.000	Fisher(glm)	10000	1000
0.948	0.007	-0.001	0.001	0.000	0.000	0.017	0.000	Nelder-Mead	10000	1000
0.976	0.005	-0.001	0.001	0.000	0.000	0.017	0.000	Poisson(log)	10000	1000
0.947	0.007	-0.001	0.001	0.000	0.000	0.017	0.000	squadP	10000	1000

Table 42: Performance measurments of scenarios 43→48 for coef.2

Covarage		Bias		MSE		EmpSE				
cover.	mcse	bias	mcse	mse	mcse	empse	mcse	model	s.size	converged
0.968	0.006	-0.001	0.002	0.003	0	0.052	0.001	EM	250	904
0.955	0.007	0.001	0.002	0.003	0	0.055	0.001	BFGS	250	976
0.949	0.007	0.001	0.002	0.003	0	0.056	0.001	Fisher(glm)	250	985
0.954	0.007	0.001	0.002	0.003	0	0.056	0.001	Nelder-Mead	250	980
0.976	0.005	0.001	0.002	0.003	0	0.056	0.001	Poisson(log)	250	1000
0.967	0.006	0.000	0.002	0.003	0	0.053	0.001	squadP	250	1000
0.950	0.007	-0.003	0.002	0.002	0	0.050	0.001	EM	300	927
0.939	0.008	-0.001	0.002	0.003	0	0.052	0.001	BFGS	300	974
0.943	0.007	-0.001	0.002	0.003	0	0.052	0.001	Fisher(glm)	300	992
0.938	0.008	-0.001	0.002	0.003	0	0.052	0.001	Nelder-Mead	300	981
0.977	0.005	-0.001	0.002	0.003	0	0.052	0.001	Poisson(log)	300	1000
0.946	0.007	-0.002	0.002	0.002	0	0.050	0.001	squadP	300	1000
0.959	0.006	-0.003	0.001	0.002	0	0.042	0.001	EM	400	943
0.951	0.007	-0.001	0.001	0.002	0	0.044	0.001	BFGS	400	981
0.952	0.007	-0.001	0.001	0.002	0	0.044	0.001	Fisher(glm)	400	998
0.951	0.007	-0.001	0.001	0.002	0	0.044	0.001	Nelder-Mead	400	985
0.969	0.005	-0.001	0.001	0.002	0	0.044	0.001	Poisson(log)	400	1000
0.957	0.006	-0.002	0.001	0.002	0	0.043	0.001	squadP	400	1000
0.950	0.007	0.000	0.001	0.002	0	0.039	0.001	EM	500	967
0.948	0.007	0.001	0.001	0.002	0	0.040	0.001	BFGS	500	990
0.945	0.007	0.001	0.001	0.002	0	0.040	0.001	Fisher(glm)	500	997
0.949	0.007	0.001	0.001	0.002	0	0.040	0.001	Nelder-Mead	500	991
0.978	0.005	0.001	0.001	0.002	0	0.040	0.001	Poisson(log)	500	1000
0.955	0.007	0.000	0.001	0.002	0	0.039	0.001	squadP	500	1000
0.936	0.008	0.000	0.001	0.001	0	0.029	0.001	EM	1000	987
0.932	0.008	0.000	0.001	0.001	0	0.029	0.001	BFGS	1000	992
0.934	0.008	0.001	0.001	0.001	0	0.029	0.001	Fisher(glm)	1000	998
0.932	0.008	0.001	0.001	0.001	0	0.029	0.001	Nelder-Mead	1000	995
0.964	0.006	0.000	0.001	0.001	0	0.029	0.001	Poisson(log)	1000	1000
0.933	0.008	0.000	0.001	0.001	0	0.029	0.001	squadP	1000	1000
0.945	0.007	-0.001	0.000	0.000	0	0.009	0.000	EM	10000	1000
0.946	0.007	-0.001	0.000	0.000	0	0.009	0.000	BFGS	10000	994
0.945	0.007	-0.001	0.000	0.000	0	0.009	0.000	Fisher(glm)	10000	1000
0.945	0.007	-0.001	0.000	0.000	0	0.009	0.000	Nelder-Mead	10000	1000
0.968	0.006	-0.001	0.000	0.000	0	0.009	0.000	Poisson(log)	10000	1000
0.945	0.007	-0.001	0.000	0.000	0	0.009	0.000	squadP	10000	1000

Table 43: Performance measurments of scenarios 43→48 for coef.3

Covarage		Bias		MSE		EmpSE				
cover.	mcse	bias	mcse	mse	mcse	empse	mcse	model	s.size	converged
0.962	0.006	-0.020	0.008	0.064	0.003	0.253	0.006	EM	250	904
0.964	0.006	-0.017	0.008	0.064	0.003	0.253	0.006	BFGS	250	976
0.958	0.006	-0.014	0.008	0.066	0.003	0.257	0.006	Fisher(glm)	250	985
0.964	0.006	-0.016	0.008	0.065	0.003	0.254	0.006	Nelder-Mead	250	980
0.979	0.005	-0.017	0.008	0.066	0.003	0.256	0.006	Poisson(log)	250	1000
0.965	0.006	-0.018	0.008	0.064	0.003	0.253	0.006	squadP	250	1000
0.965	0.006	-0.022	0.008	0.054	0.003	0.231	0.005	EM	300	927
0.964	0.006	-0.018	0.007	0.054	0.003	0.232	0.005	BFGS	300	974
0.964	0.006	-0.018	0.007	0.054	0.003	0.231	0.005	Fisher(glm)	300	992
0.964	0.006	-0.019	0.007	0.054	0.003	0.232	0.005	Nelder-Mead	300	981
0.978	0.005	-0.018	0.007	0.054	0.003	0.231	0.005	Poisson(log)	300	1000
0.967	0.006	-0.018	0.007	0.052	0.003	0.228	0.005	squadP	300	1000
0.954	0.007	0.001	0.006	0.036	0.002	0.191	0.004	EM	400	943
0.952	0.007	0.002	0.006	0.037	0.002	0.192	0.004	BFGS	400	981
0.953	0.007	0.002	0.006	0.037	0.002	0.192	0.004	Fisher(glm)	400	998
0.952	0.007	0.002	0.006	0.037	0.002	0.192	0.004	Nelder-Mead	400	985
0.979	0.005	0.001	0.006	0.036	0.002	0.191	0.004	Poisson(log)	400	1000
0.954	0.007	0.002	0.006	0.036	0.002	0.190	0.004	squadP	400	1000
0.937	0.008	-0.010	0.006	0.034	0.002	0.184	0.004	EM	500	967
0.937	0.008	-0.010	0.006	0.034	0.002	0.184	0.004	BFGS	500	990
0.937	0.008	-0.009	0.006	0.034	0.002	0.184	0.004	Fisher(glm)	500	997
0.937	0.008	-0.010	0.006	0.034	0.002	0.184	0.004	Nelder-Mead	500	991
0.971	0.005	-0.009	0.006	0.034	0.002	0.185	0.004	Poisson(log)	500	1000
0.937	0.008	-0.009	0.006	0.034	0.002	0.183	0.004	squadP	500	1000
0.941	0.007	-0.005	0.004	0.016	0.001	0.128	0.003	EM	1000	987
0.942	0.007	-0.005	0.004	0.016	0.001	0.128	0.003	BFGS	1000	992
0.941	0.007	-0.004	0.004	0.016	0.001	0.128	0.003	Fisher(glm)	1000	998
0.943	0.007	-0.005	0.004	0.016	0.001	0.128	0.003	Nelder-Mead	1000	995
0.972	0.005	-0.004	0.004	0.016	0.001	0.128	0.003	Poisson(log)	1000	1000
0.943	0.007	-0.004	0.004	0.016	0.001	0.128	0.003	squadP	1000	1000
0.946	0.007	0.000	0.001	0.001	0.000	0.038	0.001	EM	10000	1000
0.946	0.007	0.000	0.001	0.001	0.000	0.038	0.001	BFGS	10000	994
0.946	0.007	0.000	0.001	0.001	0.000	0.038	0.001	Fisher(glm)	10000	1000
0.948	0.007	0.001	0.001	0.001	0.000	0.038	0.001	Nelder-Mead	10000	1000
0.980	0.004	0.001	0.001	0.001	0.000	0.038	0.001	Poisson(log)	10000	1000
0.946	0.007	0.000	0.001	0.001	0.000	0.038	0.001	squadP	10000	1000

Table 44: Performance measurments of scenarios 43→48 for coef.4

Covarage		Bias		MSE		EmpSE				
cover.	mcse	bias	mcse	mse	mcse	empse	mcse	model	s.size	converged
0.963	0.006	-0.013	0.002	0.005	0	0.073	0.002	EM	250	904
0.951	0.007	-0.004	0.002	0.006	0	0.077	0.002	BFGS	250	976
0.931	0.008	-0.003	0.002	0.006	0	0.078	0.002	Fisher(glm)	250	985
0.948	0.007	-0.004	0.002	0.006	0	0.078	0.002	Nelder-Mead	250	980
0.981	0.004	-0.001	0.003	0.006	0	0.079	0.002	Poisson(log)	250	1000
0.961	0.006	-0.005	0.002	0.006	0	0.075	0.002	squadP	250	1000
0.962	0.006	-0.012	0.002	0.005	0	0.066	0.002	EM	300	927
0.956	0.007	-0.006	0.002	0.005	0	0.070	0.002	BFGS	300	974
0.927	0.008	-0.004	0.002	0.005	0	0.071	0.002	Fisher(glm)	300	992
0.952	0.007	-0.005	0.002	0.005	0	0.071	0.002	Nelder-Mead	300	981
0.973	0.005	-0.003	0.002	0.005	0	0.071	0.002	Poisson(log)	300	1000
0.965	0.006	-0.006	0.002	0.005	0	0.068	0.002	squadP	300	1000
0.958	0.007	-0.008	0.002	0.003	0	0.058	0.001	EM	400	943
0.944	0.007	-0.005	0.002	0.004	0	0.060	0.001	BFGS	400	981
0.928	0.008	-0.003	0.002	0.004	0	0.061	0.001	Fisher(glm)	400	998
0.943	0.007	-0.004	0.002	0.004	0	0.060	0.001	Nelder-Mead	400	985
0.979	0.005	-0.003	0.002	0.004	0	0.061	0.001	Poisson(log)	400	1000
0.955	0.007	-0.005	0.002	0.003	0	0.059	0.001	squadP	400	1000
0.942	0.008	-0.008	0.002	0.003	0	0.055	0.001	EM	500	967
0.936	0.008	-0.005	0.002	0.003	0	0.056	0.001	BFGS	500	990
0.922	0.009	-0.005	0.002	0.003	0	0.057	0.001	Fisher(glm)	500	997
0.935	0.008	-0.005	0.002	0.003	0	0.056	0.001	Nelder-Mead	500	991
0.977	0.005	-0.005	0.002	0.003	0	0.056	0.001	Poisson(log)	500	1000
0.944	0.007	-0.006	0.002	0.003	0	0.055	0.001	squadP	500	1000
0.949	0.007	-0.002	0.001	0.001	0	0.037	0.001	EM	1000	987
0.946	0.007	-0.001	0.001	0.001	0	0.037	0.001	BFGS	1000	992
0.940	0.008	-0.001	0.001	0.001	0	0.037	0.001	Fisher(glm)	1000	998
0.945	0.007	-0.001	0.001	0.001	0	0.037	0.001	Nelder-Mead	1000	995
0.979	0.005	-0.001	0.001	0.001	0	0.037	0.001	Poisson(log)	1000	1000
0.946	0.007	-0.001	0.001	0.001	0	0.037	0.001	squadP	1000	1000
0.951	0.007	0.000	0.000	0.000	0	0.012	0.000	EM	10000	1000
0.950	0.007	0.000	0.000	0.000	0	0.012	0.000	BFGS	10000	994
0.951	0.007	0.000	0.000	0.000	0	0.012	0.000	Fisher(glm)	10000	1000
0.950	0.007	0.000	0.000	0.000	0	0.012	0.000	Nelder-Mead	10000	1000
0.979	0.005	0.000	0.000	0.000	0	0.012	0.000	Poisson(log)	10000	1000
0.950	0.007	0.000	0.000	0.000	0	0.012	0.000	squadP	10000	1000

8.2 Monte Carlo simulation results: scenarios with 8 covariates

Summary of the main findings of simulation study which consists of 9 senarios with 8 variables and event probabilties 12%, 24%, and 48% is showed here in the folowing tables.

Table 45: Performance measurments of scenarios 55→63 for coef.1

Covarage		Bias		MSE					
cover	mcse	bias	mcse	mse	mcse	model	s.size	event	converged
0.963	0.006	-0.006	0.006	0.036	0.002	BFGS	500	12%	976
0.959	0.006	-0.007	0.006	0.037	0.002	Fisher(glm)	500	12%	989
0.980	0.004	0.070	0.005	0.028	0.001	Nelder-Mead	500	12%	984
0.973	0.005	-0.006	0.006	0.037	0.002	Poisson(log)	500	12%	1000
0.997	0.002	0.031	0.004	0.018	0.001	squadP	500	12%	996
0.953	0.007	0.005	0.004	0.018	0.001	BFGS	1000	12%	997
0.949	0.007	0.004	0.004	0.018	0.001	Fisher(glm)	1000	12%	1000
0.960	0.006	0.075	0.003	0.017	0.001	Nelder-Mead	1000	12%	999
0.968	0.006	0.004	0.004	0.018	0.001	Poisson(log)	1000	12%	1000
0.985	0.004	0.025	0.003	0.012	0.001	squadP	1000	12%	1000
0.956	0.006	0.001	0.001	0.002	0.000	BFGS	10000	12%	1000
0.957	0.006	0.001	0.001	0.002	0.000	Fisher(glm)	10000	12%	1000
0.711	0.014	0.054	0.002	0.005	0.000	Nelder-Mead	10000	12%	1000
0.976	0.005	0.001	0.001	0.002	0.000	Poisson(log)	10000	12%	1000
0.970	0.005	0.003	0.001	0.002	0.000	squadP	10000	12%	1000
0.950	0.007	-0.003	0.004	0.017	0.001	BFGS	500	24%	981
0.943	0.007	-0.002	0.004	0.017	0.001	Fisher(glm)	500	24%	992
0.967	0.006	-0.001	0.004	0.014	0.001	Nelder-Mead	500	24%	986
0.974	0.005	-0.002	0.004	0.017	0.001	Poisson(log)	500	24%	1000
0.996	0.002	0.012	0.003	0.009	0.000	squadP	500	24%	1000
0.938	0.008	0.001	0.003	0.009	0.000	BFGS	1000	24%	995
0.934	0.008	0.001	0.003	0.009	0.000	Fisher(glm)	1000	24%	1000
0.965	0.006	0.002	0.003	0.006	0.000	Nelder-Mead	1000	24%	998
0.965	0.006	0.001	0.003	0.009	0.000	Poisson(log)	1000	24%	1000
0.979	0.005	0.010	0.002	0.006	0.000	squadP	1000	24%	1000
0.965	0.006	0.001	0.001	0.001	0.000	BFGS	10000	24%	1000
0.964	0.006	0.000	0.001	0.001	0.000	Fisher(glm)	10000	24%	1000
0.979	0.005	0.002	0.001	0.001	0.000	Nelder-Mead	10000	24%	1000
0.988	0.003	0.000	0.001	0.001	0.000	Poisson(log)	10000	24%	1000
0.966	0.006	0.001	0.001	0.001	0.000	squadP	10000	24%	1000
0.942	0.008	0.002	0.003	0.007	0.000	BFGS	500	48%	811
0.933	0.008	0.000	0.003	0.007	0.000	Fisher(glm)	500	48%	974
0.943	0.008	-0.029	0.002	0.007	0.000	Nelder-Mead	500	48%	934
0.988	0.003	-0.001	0.003	0.007	0.000	Poisson(log)	500	48%	1000
0.998	0.001	0.011	0.002	0.002	0.000	squadP	500	48%	1000
0.950	0.007	0.001	0.002	0.003	0.000	BFGS	1000	48%	917
0.946	0.007	0.002	0.002	0.003	0.000	Fisher(glm)	1000	48%	994
0.903	0.009	-0.032	0.002	0.004	0.000	Nelder-Mead	1000	48%	978
0.988	0.003	0.002	0.002	0.003	0.000	Poisson(log)	1000	48%	1000
0.997	0.002	0.007	0.001	0.002	0.000	squadP	1000	48%	1000
0.961	0.007	0.001	0.001	0.000	0.000	BFGS	10000	48%	710
0.964	0.006	0.000	0.001	0.000	0.000	Fisher(glm)	10000	48%	1000
0.596	0.016	-0.020	0.001	0.002	0.000	Nelder-Mead	10000	48%	1000
0.996	0.002	0.000	0.001	0.000	0.000	Poisson(log)	10000	48%	1000
0.966	0.006	0.001	0.001	0.000	0.000	squadP	10000	48%	1000

8.2.1 Scenarios with event probability 24%

The results of scenarios from 58 $\rightarrow$ 60 (3 scenarios) with event probabilty 24% and different sample sizes $(500, 1000, 10000)$ are showed here.

Table 46: Performance measurments of scenarios 55→63 for coef.2

Covarage		Bias		MSE					
cover	mcse	bias	mcse	mse	mcse	model	s.size	event	converged
0.945	0.007	-0.004	0.023	0.523	0.027	BFGS	500	12%	976
0.942	0.007	-0.005	0.023	0.533	0.027	Fisher(glm)	500	12%	989
0.976	0.005	0.259	0.019	0.403	0.019	Nelder-Mead	500	12%	984
0.963	0.006	-0.007	0.023	0.530	0.026	Poisson(log)	500	12%	1000
0.987	0.004	0.084	0.017	0.301	0.016	squadP	500	12%	996
0.947	0.007	-0.005	0.016	0.240	0.011	BFGS	1000	12%	997
0.946	0.007	-0.006	0.016	0.240	0.011	Fisher(glm)	1000	12%	1000
0.964	0.006	0.238	0.013	0.223	0.010	Nelder-Mead	1000	12%	999
0.961	0.006	-0.007	0.016	0.241	0.011	Poisson(log)	1000	12%	1000
0.978	0.005	0.052	0.013	0.167	0.008	squadP	1000	12%	1000
0.963	0.006	-0.004	0.005	0.023	0.001	BFGS	10000	12%	1000
0.964	0.006	-0.004	0.005	0.023	0.001	Fisher(glm)	10000	12%	1000
0.777	0.013	0.163	0.006	0.058	0.002	Nelder-Mead	10000	12%	1000
0.975	0.005	-0.004	0.005	0.023	0.001	Poisson(log)	10000	12%	1000
0.973	0.005	0.004	0.004	0.020	0.001	squadP	10000	12%	1000
0.946	0.007	0.016	0.015	0.233	0.011	BFGS	500	24%	981
0.939	0.008	0.020	0.015	0.236	0.011	Fisher(glm)	500	24%	992
0.968	0.006	0.023	0.014	0.194	0.009	Nelder-Mead	500	24%	986
0.971	0.005	0.019	0.015	0.233	0.011	Poisson(log)	500	24%	1000
0.983	0.004	0.051	0.012	0.142	0.007	squadP	500	24%	1000
0.948	0.007	-0.016	0.011	0.115	0.005	BFGS	1000	24%	995
0.945	0.007	-0.016	0.011	0.116	0.005	Fisher(glm)	1000	24%	1000
0.969	0.005	-0.013	0.009	0.088	0.004	Nelder-Mead	1000	24%	998
0.975	0.005	-0.016	0.011	0.116	0.005	Poisson(log)	1000	24%	1000
0.980	0.004	0.011	0.009	0.083	0.004	squadP	1000	24%	1000
0.955	0.007	-0.001	0.003	0.010	0.000	BFGS	10000	24%	1000
0.955	0.007	-0.002	0.003	0.010	0.000	Fisher(glm)	10000	24%	1000
0.979	0.005	0.001	0.003	0.008	0.000	Nelder-Mead	10000	24%	1000
0.980	0.004	-0.002	0.003	0.010	0.000	Poisson(log)	10000	24%	1000
0.957	0.006	-0.002	0.003	0.010	0.000	squadP	10000	24%	1000
0.940	0.008	-0.016	0.010	0.086	0.004	BFGS	500	48%	811
0.919	0.009	-0.011	0.010	0.089	0.004	Fisher(glm)	500	48%	974
0.936	0.008	-0.073	0.010	0.094	0.004	Nelder-Mead	500	48%	934
0.991	0.003	-0.010	0.009	0.088	0.004	Poisson(log)	500	48%	1000
0.987	0.004	0.001	0.007	0.045	0.002	squadP	500	48%	1000
0.947	0.007	0.003	0.007	0.041	0.002	BFGS	1000	48%	917
0.940	0.008	0.008	0.006	0.041	0.002	Fisher(glm)	1000	48%	994
0.910	0.009	-0.069	0.007	0.054	0.002	Nelder-Mead	1000	48%	978
0.993	0.003	0.008	0.006	0.042	0.002	Poisson(log)	1000	48%	1000
0.980	0.004	0.015	0.005	0.026	0.001	squadP	1000	48%	1000
0.952	0.008	0.000	0.002	0.004	0.000	BFGS	10000	48%	710
0.948	0.007	0.000	0.002	0.004	0.000	Fisher(glm)	10000	48%	1000
0.522	0.016	0.019	0.005	0.022	0.001	Nelder-Mead	10000	48%	1000
0.996	0.002	0.000	0.002	0.004	0.000	Poisson(log)	10000	48%	1000
0.951	0.007	0.000	0.002	0.004	0.000	squadP	10000	48%	1000

Table 47: Performance measurments of scenarios 55→63 for coef.3

Covarage		Bias		MSE					
cover	mcse	bias	mcse	mse	mcse	model	s.size	event	converged
0.953	0.007	-0.008	0.003	0.006	0	BFGS	500	12%	976
0.945	0.007	-0.007	0.003	0.006	0	Fisher(glm)	500	12%	989
0.956	0.007	-0.013	0.003	0.006	0	Nelder-Mead	500	12%	984
0.968	0.006	-0.007	0.003	0.006	0	Poisson(log)	500	12%	1000
0.981	0.004	-0.022	0.002	0.005	0	squadP	500	12%	996
0.943	0.007	-0.004	0.002	0.003	0	BFGS	1000	12%	997
0.935	0.008	-0.003	0.002	0.003	0	Fisher(glm)	1000	12%	1000
0.942	0.007	-0.008	0.002	0.004	0	Nelder-Mead	1000	12%	999
0.962	0.006	-0.003	0.002	0.003	0	Poisson(log)	1000	12%	1000
0.967	0.006	-0.012	0.002	0.003	0	squadP	1000	12%	1000
0.944	0.007	-0.001	0.001	0.000	0	BFGS	10000	12%	1000
0.944	0.007	-0.001	0.001	0.000	0	Fisher(glm)	10000	12%	1000
0.937	0.008	-0.004	0.001	0.000	0	Nelder-Mead	10000	12%	1000
0.963	0.006	-0.001	0.001	0.000	0	Poisson(log)	10000	12%	1000
0.958	0.006	-0.002	0.001	0.000	0	squadP	10000	12%	1000
0.949	0.007	-0.005	0.002	0.003	0	BFGS	500	24%	981
0.943	0.007	-0.004	0.002	0.003	0	Fisher(glm)	500	24%	992
0.950	0.007	-0.005	0.002	0.003	0	Nelder-Mead	500	24%	986
0.979	0.005	-0.005	0.002	0.003	0	Poisson(log)	500	24%	1000
0.977	0.005	-0.009	0.002	0.002	0	squadP	500	24%	1000
0.951	0.007	-0.002	0.001	0.002	0	BFGS	1000	24%	995
0.948	0.007	-0.002	0.001	0.002	0	Fisher(glm)	1000	24%	1000
0.955	0.007	-0.003	0.001	0.002	0	Nelder-Mead	1000	24%	998
0.974	0.005	-0.002	0.001	0.002	0	Poisson(log)	1000	24%	1000
0.960	0.006	-0.004	0.001	0.001	0	squadP	1000	24%	1000
0.950	0.007	0.000	0.000	0.000	0	BFGS	10000	24%	1000
0.950	0.007	0.000	0.000	0.000	0	Fisher(glm)	10000	24%	1000
0.952	0.007	0.000	0.000	0.000	0	Nelder-Mead	10000	24%	1000
0.976	0.005	0.000	0.000	0.000	0	Poisson(log)	10000	24%	1000
0.951	0.007	0.000	0.000	0.000	0	squadP	10000	24%	1000
0.957	0.007	-0.004	0.001	0.001	0	BFGS	500	48%	811
0.939	0.008	-0.003	0.001	0.001	0	Fisher(glm)	500	48%	974
0.939	0.008	0.002	0.001	0.001	0	Nelder-Mead	500	48%	934
0.995	0.002	-0.003	0.001	0.001	0	Poisson(log)	500	48%	1000
0.980	0.004	-0.007	0.001	0.001	0	squadP	500	48%	1000
0.959	0.007	-0.001	0.001	0.001	0	BFGS	1000	48%	917
0.959	0.006	0.000	0.001	0.001	0	Fisher(glm)	1000	48%	994
0.946	0.007	0.005	0.001	0.001	0	Nelder-Mead	1000	48%	978
0.992	0.003	-0.001	0.001	0.001	0	Poisson(log)	1000	48%	1000
0.979	0.005	-0.003	0.001	0.000	0	squadP	1000	48%	1000
0.948	0.008	0.000	0.000	0.000	0	BFGS	10000	48%	710
0.943	0.007	0.000	0.000	0.000	0	Fisher(glm)	10000	48%	1000
0.860	0.011	0.004	0.000	0.000	0	Nelder-Mead	10000	48%	1000
0.995	0.002	0.000	0.000	0.000	0	Poisson(log)	10000	48%	1000
0.945	0.007	0.000	0.000	0.000	0	squadP	10000	48%	1000

Table 48: Performance measurments of scenarios 55→63 for coef.4

Covarage		Bias		MSE					
cover	mcse	bias	mcse	mse	mcse	model	s.size	event	converged
0.954	0.007	-0.052	0.014	0.186	0.009	BFGS	500	12%	976
0.942	0.007	-0.050	0.014	0.191	0.009	Fisher(glm)	500	12%	989
0.960	0.006	0.075	0.012	0.139	0.006	Nelder-Mead	500	12%	984
0.962	0.006	-0.046	0.014	0.192	0.009	Poisson(log)	500	12%	1000
0.992	0.003	-0.024	0.010	0.109	0.006	squadP	500	12%	996
0.950	0.007	-0.012	0.009	0.084	0.004	BFGS	1000	12%	997
0.956	0.006	-0.012	0.009	0.084	0.004	Fisher(glm)	1000	12%	1000
0.944	0.007	0.098	0.008	0.076	0.003	Nelder-Mead	1000	12%	999
0.969	0.005	-0.011	0.009	0.085	0.004	Poisson(log)	1000	12%	1000
0.977	0.005	0.000	0.008	0.060	0.003	squadP	1000	12%	1000
0.955	0.007	0.000	0.003	0.008	0.000	BFGS	10000	12%	1000
0.956	0.006	0.000	0.003	0.008	0.000	Fisher(glm)	10000	12%	1000
0.808	0.012	0.085	0.003	0.017	0.001	Nelder-Mead	10000	12%	1000
0.971	0.005	0.000	0.003	0.008	0.000	Poisson(log)	10000	12%	1000
0.962	0.006	0.002	0.003	0.008	0.000	squadP	10000	12%	1000
0.951	0.007	-0.030	0.009	0.087	0.004	BFGS	500	24%	981
0.931	0.008	-0.025	0.009	0.089	0.004	Fisher(glm)	500	24%	992
0.958	0.006	-0.025	0.009	0.079	0.004	Nelder-Mead	500	24%	986
0.974	0.005	-0.024	0.009	0.088	0.004	Poisson(log)	500	24%	1000
0.987	0.004	-0.020	0.007	0.054	0.003	squadP	500	24%	1000
0.954	0.007	-0.002	0.006	0.040	0.002	BFGS	1000	24%	995
0.949	0.007	-0.001	0.006	0.041	0.002	Fisher(glm)	1000	24%	1000
0.964	0.006	0.000	0.006	0.035	0.002	Nelder-Mead	1000	24%	998
0.982	0.004	-0.001	0.006	0.040	0.002	Poisson(log)	1000	24%	1000
0.976	0.005	0.005	0.006	0.031	0.001	squadP	1000	24%	1000
0.948	0.007	0.002	0.002	0.004	0.000	BFGS	10000	24%	1000
0.948	0.007	0.001	0.002	0.004	0.000	Fisher(glm)	10000	24%	1000
0.959	0.006	0.004	0.002	0.004	0.000	Nelder-Mead	10000	24%	1000
0.982	0.004	0.001	0.002	0.004	0.000	Poisson(log)	10000	24%	1000
0.948	0.007	0.002	0.002	0.004	0.000	squadP	10000	24%	1000
0.965	0.006	-0.011	0.006	0.028	0.002	BFGS	500	48%	811
0.931	0.008	-0.009	0.006	0.030	0.001	Fisher(glm)	500	48%	974
0.960	0.006	-0.052	0.006	0.032	0.002	Nelder-Mead	500	48%	934
0.998	0.001	-0.007	0.005	0.030	0.001	Poisson(log)	500	48%	1000
0.998	0.001	-0.006	0.004	0.014	0.001	squadP	500	48%	1000
0.959	0.007	-0.007	0.004	0.014	0.001	BFGS	1000	48%	917
0.936	0.008	-0.004	0.004	0.015	0.001	Fisher(glm)	1000	48%	994
0.933	0.008	-0.051	0.004	0.019	0.001	Nelder-Mead	1000	48%	978
0.998	0.001	-0.003	0.004	0.015	0.001	Poisson(log)	1000	48%	1000
0.993	0.003	-0.004	0.003	0.009	0.000	squadP	1000	48%	1000
0.952	0.008	-0.001	0.001	0.001	0.000	BFGS	10000	48%	710
0.954	0.007	0.000	0.001	0.001	0.000	Fisher(glm)	10000	48%	1000
0.714	0.014	-0.020	0.002	0.005	0.000	Nelder-Mead	10000	48%	1000
0.997	0.002	0.000	0.001	0.001	0.000	Poisson(log)	10000	48%	1000
0.960	0.006	0.000	0.001	0.001	0.000	squadP	10000	48%	1000

Table 49: Performance measurments of scenarios 55→63 for coef.5

Covarage		Bias		MSE					
cover	mcse	bias	mcse	mse	mcse	model	s.size	event	converged
0.951	0.007	-0.020	0.009	0.080	0.004	BFGS	500	12%	976
0.948	0.007	-0.020	0.009	0.080	0.004	Fisher(glm)	500	12%	989
0.950	0.007	-0.041	0.009	0.078	0.004	Nelder-Mead	500	12%	984
0.967	0.006	-0.021	0.009	0.079	0.004	Poisson(log)	500	12%	1000
0.962	0.006	-0.048	0.008	0.068	0.003	squadP	500	12%	996
0.968	0.006	-0.013	0.006	0.033	0.001	BFGS	1000	12%	997
0.968	0.006	-0.013	0.006	0.033	0.001	Fisher(glm)	1000	12%	1000
0.962	0.006	-0.034	0.006	0.033	0.002	Nelder-Mead	1000	12%	999
0.979	0.005	-0.013	0.006	0.033	0.001	Poisson(log)	1000	12%	1000
0.979	0.005	-0.029	0.005	0.029	0.001	squadP	1000	12%	1000
0.954	0.007	-0.001	0.002	0.004	0.000	BFGS	10000	12%	1000
0.953	0.007	-0.001	0.002	0.004	0.000	Fisher(glm)	10000	12%	1000
0.933	0.008	-0.015	0.002	0.004	0.000	Nelder-Mead	10000	12%	1000
0.968	0.006	-0.001	0.002	0.004	0.000	Poisson(log)	10000	12%	1000
0.958	0.006	-0.002	0.002	0.003	0.000	squadP	10000	12%	1000
0.953	0.007	-0.009	0.006	0.032	0.001	BFGS	500	24%	981
0.951	0.007	-0.010	0.006	0.033	0.001	Fisher(glm)	500	24%	992
0.954	0.007	-0.010	0.006	0.032	0.001	Nelder-Mead	500	24%	986
0.981	0.004	-0.010	0.006	0.033	0.001	Poisson(log)	500	24%	1000
0.967	0.006	-0.020	0.005	0.029	0.001	squadP	500	24%	1000
0.959	0.006	-0.007	0.004	0.015	0.001	BFGS	1000	24%	995
0.957	0.006	-0.008	0.004	0.015	0.001	Fisher(glm)	1000	24%	1000
0.957	0.006	-0.009	0.004	0.015	0.001	Nelder-Mead	1000	24%	998
0.978	0.005	-0.008	0.004	0.015	0.001	Poisson(log)	1000	24%	1000
0.970	0.005	-0.014	0.004	0.014	0.001	squadP	1000	24%	1000
0.938	0.008	0.000	0.001	0.002	0.000	BFGS	10000	24%	1000
0.937	0.008	0.000	0.001	0.002	0.000	Fisher(glm)	10000	24%	1000
0.938	0.008	-0.001	0.001	0.002	0.000	Nelder-Mead	10000	24%	1000
0.976	0.005	0.000	0.001	0.002	0.000	Poisson(log)	10000	24%	1000
0.938	0.008	0.000	0.001	0.002	0.000	squadP	10000	24%	1000
0.967	0.006	-0.006	0.004	0.010	0.001	BFGS	500	48%	811
0.954	0.007	-0.001	0.003	0.011	0.000	Fisher(glm)	500	48%	974
0.949	0.007	0.012	0.003	0.011	0.001	Nelder-Mead	500	48%	934
0.996	0.002	-0.003	0.003	0.011	0.001	Poisson(log)	500	48%	1000
0.976	0.005	-0.013	0.003	0.010	0.000	squadP	500	48%	1000
0.953	0.007	-0.002	0.002	0.006	0.000	BFGS	1000	48%	917
0.950	0.007	-0.001	0.002	0.006	0.000	Fisher(glm)	1000	48%	994
0.929	0.008	0.016	0.002	0.006	0.000	Nelder-Mead	1000	48%	978
0.995	0.002	-0.001	0.002	0.005	0.000	Poisson(log)	1000	48%	1000
0.962	0.006	-0.005	0.002	0.005	0.000	squadP	1000	48%	1000
0.948	0.008	0.000	0.001	0.001	0.000	BFGS	10000	48%	710
0.947	0.007	0.001	0.001	0.001	0.000	Fisher(glm)	10000	48%	1000
0.868	0.011	0.010	0.001	0.001	0.000	Nelder-Mead	10000	48%	1000
0.992	0.003	0.001	0.001	0.001	0.000	Poisson(log)	10000	48%	1000
0.947	0.007	0.001	0.001	0.001	0.000	squadP	10000	48%	1000

Table 50: Performance measurments of scenarios 55→63 for coef.6

Covarage		Bias		MSE					
cover	mcse	bias	mcse	mse	mcse	model	s.size	event	converged
0.955	0.007	-0.105	0.021	0.453	0.119	BFGS	500	12%	976
0.951	0.007	-0.127	0.032	1.044	0.459	Fisher(glm)	500	12%	989
0.968	0.006	0.073	0.013	0.174	0.009	Nelder-Mead	500	12%	984
0.975	0.005	-0.125	0.032	1.044	0.460	Poisson(log)	500	12%	1000
0.979	0.005	-0.052	0.014	0.194	0.011	squadP	500	12%	996
0.955	0.007	-0.030	0.011	0.111	0.006	BFGS	1000	12%	997
0.955	0.007	-0.030	0.010	0.111	0.006	Fisher(glm)	1000	12%	1000
0.953	0.007	0.109	0.009	0.087	0.004	Nelder-Mead	1000	12%	999
0.970	0.005	-0.030	0.010	0.111	0.006	Poisson(log)	1000	12%	1000
0.970	0.005	-0.011	0.010	0.095	0.006	squadP	1000	12%	1000
0.960	0.006	-0.002	0.003	0.010	0.000	BFGS	10000	12%	1000
0.960	0.006	-0.003	0.003	0.010	0.000	Fisher(glm)	10000	12%	1000
0.781	0.013	0.098	0.003	0.022	0.001	Nelder-Mead	10000	12%	1000
0.973	0.005	-0.002	0.003	0.010	0.000	Poisson(log)	10000	12%	1000
0.965	0.006	0.000	0.003	0.010	0.000	squadP	10000	12%	1000
0.962	0.006	-0.024	0.010	0.099	0.005	BFGS	500	24%	981
0.955	0.007	-0.022	0.010	0.100	0.005	Fisher(glm)	500	24%	992
0.970	0.005	-0.019	0.009	0.087	0.005	Nelder-Mead	500	24%	986
0.980	0.004	-0.021	0.010	0.099	0.005	Poisson(log)	500	24%	1000
0.981	0.004	-0.018	0.009	0.078	0.005	squadP	500	24%	1000
0.953	0.007	-0.012	0.006	0.042	0.002	BFGS	1000	24%	995
0.952	0.007	-0.011	0.006	0.042	0.002	Fisher(glm)	1000	24%	1000
0.971	0.005	-0.009	0.006	0.035	0.002	Nelder-Mead	1000	24%	998
0.982	0.004	-0.011	0.006	0.042	0.002	Poisson(log)	1000	24%	1000
0.970	0.005	-0.004	0.006	0.036	0.002	squadP	1000	24%	1000
0.948	0.007	0.000	0.002	0.005	0.000	BFGS	10000	24%	1000
0.948	0.007	-0.001	0.002	0.005	0.000	Fisher(glm)	10000	24%	1000
0.961	0.006	0.001	0.002	0.004	0.000	Nelder-Mead	10000	24%	1000
0.977	0.005	0.000	0.002	0.004	0.000	Poisson(log)	10000	24%	1000
0.949	0.007	0.000	0.002	0.004	0.000	squadP	10000	24%	1000
0.957	0.007	-0.020	0.007	0.035	0.002	BFGS	500	48%	811
0.941	0.008	-0.016	0.006	0.037	0.002	Fisher(glm)	500	48%	974
0.961	0.006	-0.053	0.006	0.038	0.002	Nelder-Mead	500	48%	934
0.991	0.003	-0.016	0.006	0.037	0.002	Poisson(log)	500	48%	1000
0.984	0.004	-0.019	0.005	0.026	0.001	squadP	500	48%	1000
0.941	0.008	0.006	0.004	0.018	0.001	BFGS	1000	48%	917
0.937	0.008	0.008	0.004	0.018	0.001	Fisher(glm)	1000	48%	994
0.942	0.007	-0.035	0.004	0.020	0.001	Nelder-Mead	1000	48%	978
0.991	0.003	0.008	0.004	0.018	0.001	Poisson(log)	1000	48%	1000
0.965	0.006	0.008	0.004	0.014	0.001	squadP	1000	48%	1000
0.963	0.007	0.001	0.002	0.002	0.000	BFGS	10000	48%	710
0.964	0.006	0.001	0.001	0.002	0.000	Fisher(glm)	10000	48%	1000
0.684	0.015	-0.013	0.002	0.006	0.000	Nelder-Mead	10000	48%	1000
0.993	0.003	0.001	0.001	0.002	0.000	Poisson(log)	10000	48%	1000
0.965	0.006	0.001	0.001	0.002	0.000	squadP	10000	48%	1000

Table 51: Performance measurments of scenarios 55→63 for coef.7

Covarage		Bias		MSE					
cover	mcse	bias	mcse	mse	mcse	model	s.size	event	converged
0.960	0.006	0.001	0.004	0.014	0.001	BFGS	500	12%	976
0.953	0.007	0.000	0.004	0.014	0.001	Fisher(glm)	500	12%	989
0.984	0.004	-0.051	0.003	0.011	0.000	Nelder-Mead	500	12%	984
0.975	0.005	0.001	0.004	0.014	0.001	Poisson(log)	500	12%	1000
0.995	0.002	-0.020	0.002	0.006	0.000	squadP	500	12%	996
0.948	0.007	-0.004	0.003	0.007	0.000	BFGS	1000	12%	997
0.949	0.007	-0.003	0.003	0.007	0.000	Fisher(glm)	1000	12%	1000
0.973	0.005	-0.051	0.002	0.007	0.000	Nelder-Mead	1000	12%	999
0.964	0.006	-0.003	0.003	0.007	0.000	Poisson(log)	1000	12%	1000
0.986	0.004	-0.016	0.002	0.004	0.000	squadP	1000	12%	1000
0.945	0.007	0.000	0.001	0.001	0.000	BFGS	10000	12%	1000
0.945	0.007	0.000	0.001	0.001	0.000	Fisher(glm)	10000	12%	1000
0.659	0.015	-0.036	0.001	0.002	0.000	Nelder-Mead	10000	12%	1000
0.967	0.006	0.000	0.001	0.001	0.000	Poisson(log)	10000	12%	1000
0.969	0.005	-0.002	0.001	0.001	0.000	squadP	10000	12%	1000
0.957	0.006	0.001	0.003	0.006	0.000	BFGS	500	24%	981
0.954	0.007	0.000	0.003	0.006	0.000	Fisher(glm)	500	24%	992
0.977	0.005	-0.002	0.002	0.005	0.000	Nelder-Mead	500	24%	986
0.979	0.005	0.000	0.003	0.006	0.000	Poisson(log)	500	24%	1000
0.997	0.002	-0.009	0.002	0.003	0.000	squadP	500	24%	1000
0.944	0.007	0.000	0.002	0.003	0.000	BFGS	1000	24%	995
0.939	0.008	0.000	0.002	0.003	0.000	Fisher(glm)	1000	24%	1000
0.981	0.004	0.000	0.001	0.002	0.000	Nelder-Mead	1000	24%	998
0.978	0.005	0.000	0.002	0.003	0.000	Poisson(log)	1000	24%	1000
0.988	0.003	-0.006	0.001	0.002	0.000	squadP	1000	24%	1000
0.963	0.006	0.000	0.001	0.000	0.000	BFGS	10000	24%	1000
0.963	0.006	0.000	0.001	0.000	0.000	Fisher(glm)	10000	24%	1000
0.987	0.004	-0.001	0.000	0.000	0.000	Nelder-Mead	10000	24%	1000
0.986	0.004	0.000	0.001	0.000	0.000	Poisson(log)	10000	24%	1000
0.966	0.006	0.000	0.001	0.000	0.000	squadP	10000	24%	1000
0.940	0.008	0.000	0.002	0.002	0.000	BFGS	500	48%	811
0.924	0.008	0.000	0.002	0.003	0.000	Fisher(glm)	500	48%	974
0.935	0.008	0.024	0.001	0.003	0.000	Nelder-Mead	500	48%	934
0.990	0.003	0.000	0.002	0.002	0.000	Poisson(log)	500	48%	1000
0.999	0.001	-0.006	0.001	0.001	0.000	squadP	500	48%	1000
0.947	0.007	0.000	0.001	0.001	0.000	BFGS	1000	48%	917
0.950	0.007	-0.001	0.001	0.001	0.000	Fisher(glm)	1000	48%	994
0.879	0.010	0.025	0.001	0.002	0.000	Nelder-Mead	1000	48%	978
0.996	0.002	-0.001	0.001	0.001	0.000	Poisson(log)	1000	48%	1000
0.995	0.002	-0.004	0.001	0.001	0.000	squadP	1000	48%	1000
0.965	0.007	0.000	0.000	0.000	0.000	BFGS	10000	48%	710
0.956	0.006	0.000	0.000	0.000	0.000	Fisher(glm)	10000	48%	1000
0.540	0.016	0.018	0.001	0.001	0.000	Nelder-Mead	10000	48%	1000
0.998	0.001	0.000	0.000	0.000	0.000	Poisson(log)	10000	48%	1000
0.964	0.006	0.000	0.000	0.000	0.000	squadP	10000	48%	1000

Table 52: Performance measurments of scenarios 55→63 for coef.8

Covarage		Bias		MSE					
cover	mcse	bias	mcse	mse	mcse	model	s.size	event	converged
0.949	0.007	-0.001	0.002	0.005	0	BFGS	500	12%	976
0.945	0.007	-0.002	0.002	0.005	0	Fisher(glm)	500	12%	989
0.977	0.005	-0.029	0.002	0.004	0	Nelder-Mead	500	12%	984
0.962	0.006	-0.001	0.002	0.005	0	Poisson(log)	500	12%	1000
0.990	0.003	-0.007	0.002	0.003	0	squadP	500	12%	996
0.943	0.007	-0.001	0.002	0.002	0	BFGS	1000	12%	997
0.942	0.007	0.000	0.002	0.002	0	Fisher(glm)	1000	12%	1000
0.971	0.005	-0.027	0.001	0.002	0	Nelder-Mead	1000	12%	999
0.959	0.006	-0.001	0.002	0.002	0	Poisson(log)	1000	12%	1000
0.976	0.005	-0.004	0.001	0.002	0	squadP	1000	12%	1000
0.943	0.007	-0.001	0.000	0.000	0	BFGS	10000	12%	1000
0.942	0.007	-0.001	0.000	0.000	0	Fisher(glm)	10000	12%	1000
0.671	0.015	-0.021	0.001	0.001	0	Nelder-Mead	10000	12%	1000
0.959	0.006	-0.001	0.000	0.000	0	Poisson(log)	10000	12%	1000
0.954	0.007	-0.001	0.000	0.000	0	squadP	10000	12%	1000
0.951	0.007	-0.001	0.001	0.002	0	BFGS	500	24%	981
0.954	0.007	-0.001	0.001	0.002	0	Fisher(glm)	500	24%	992
0.978	0.005	-0.002	0.001	0.001	0	Nelder-Mead	500	24%	986
0.971	0.005	-0.001	0.001	0.002	0	Poisson(log)	500	24%	1000
0.983	0.004	-0.003	0.001	0.001	0	squadP	500	24%	1000
0.963	0.006	0.001	0.001	0.001	0	BFGS	1000	24%	995
0.959	0.006	0.001	0.001	0.001	0	Fisher(glm)	1000	24%	1000
0.984	0.004	0.001	0.001	0.001	0	Nelder-Mead	1000	24%	998
0.979	0.005	0.001	0.001	0.001	0	Poisson(log)	1000	24%	1000
0.980	0.004	-0.001	0.001	0.001	0	squadP	1000	24%	1000
0.958	0.006	0.000	0.000	0.000	0	BFGS	10000	24%	1000
0.957	0.006	0.000	0.000	0.000	0	Fisher(glm)	10000	24%	1000
0.975	0.005	0.000	0.000	0.000	0	Nelder-Mead	10000	24%	1000
0.984	0.004	0.000	0.000	0.000	0	Poisson(log)	10000	24%	1000
0.959	0.006	0.000	0.000	0.000	0	squadP	10000	24%	1000
0.945	0.008	-0.001	0.001	0.001	0	BFGS	500	48%	811
0.923	0.009	0.000	0.001	0.001	0	Fisher(glm)	500	48%	974
0.943	0.008	0.018	0.001	0.001	0	Nelder-Mead	500	48%	934
0.994	0.002	0.000	0.001	0.001	0	Poisson(log)	500	48%	1000
0.992	0.003	-0.002	0.001	0.000	0	squadP	500	48%	1000
0.943	0.008	0.000	0.001	0.000	0	BFGS	1000	48%	917
0.945	0.007	0.000	0.001	0.000	0	Fisher(glm)	1000	48%	994
0.851	0.011	0.020	0.001	0.001	0	Nelder-Mead	1000	48%	978
0.993	0.003	0.000	0.001	0.000	0	Poisson(log)	1000	48%	1000
0.981	0.004	-0.001	0.001	0.000	0	squadP	1000	48%	1000
0.961	0.007	0.000	0.000	0.000	0	BFGS	10000	48%	710
0.959	0.006	0.000	0.000	0.000	0	Fisher(glm)	10000	48%	1000
0.343	0.015	0.019	0.000	0.001	0	Nelder-Mead	10000	48%	1000
0.998	0.001	0.000	0.000	0.000	0	Poisson(log)	10000	48%	1000
0.965	0.006	0.000	0.000	0.000	0	squadP	10000	48%	1000

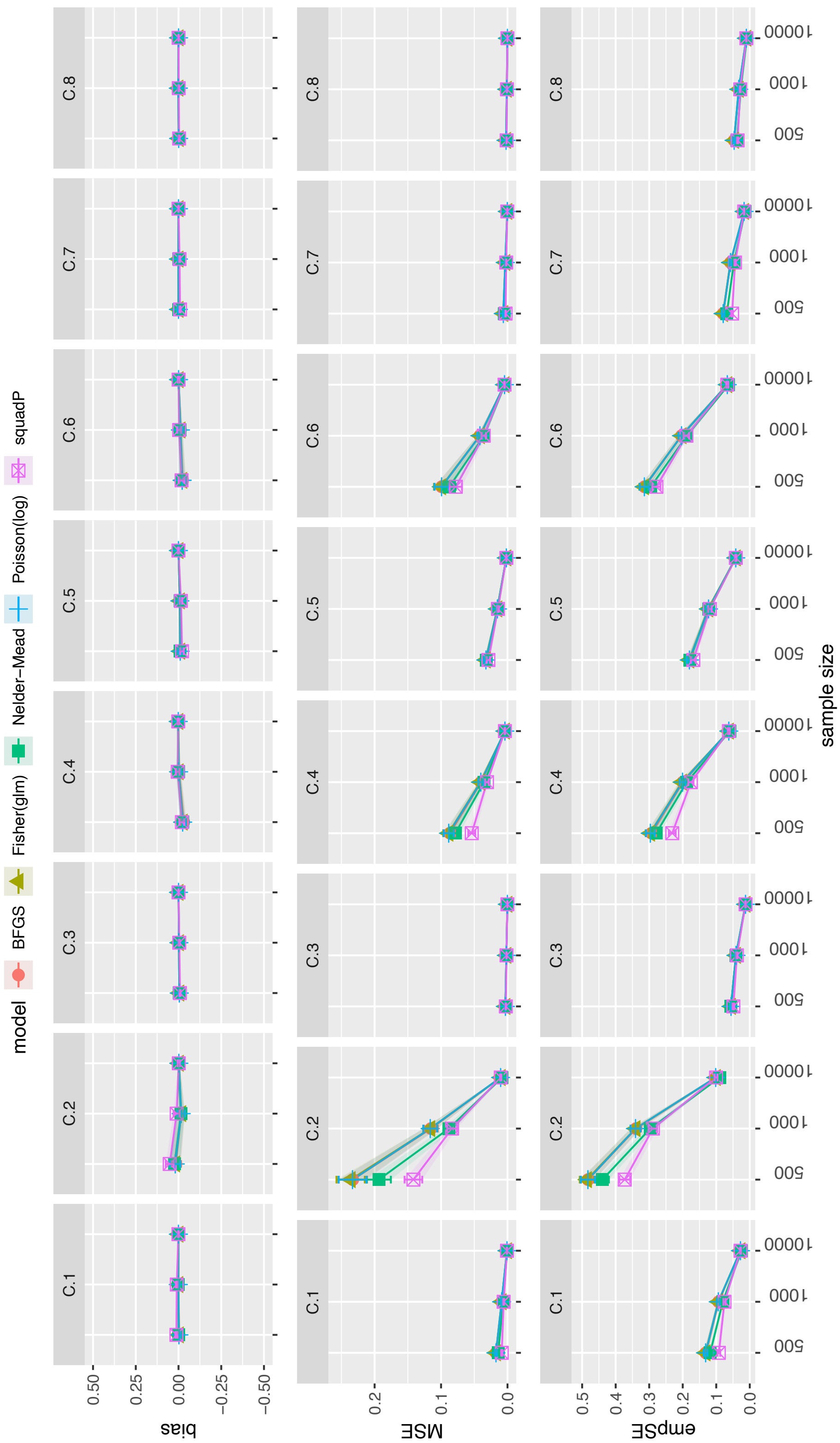

Figure 38: Performance measurments comparison from the five statistical methods. Absolute bias, MSE, empSE (y-axis) of scenarios (58→60) with 24% event propability and 8 covariates. On x-axis are the sample sizes.

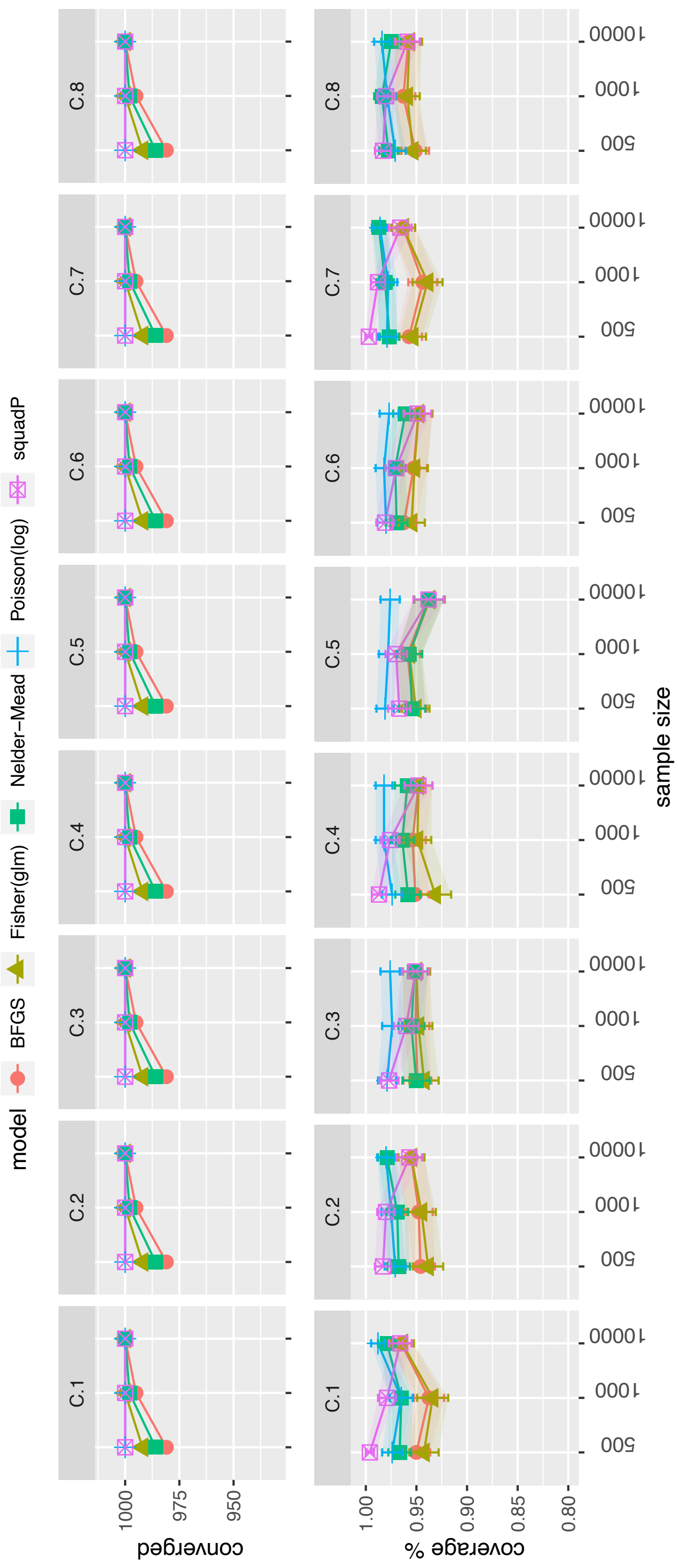

Figure 39: Performance measurments comparison from the five statistical methods. Convergence rate and coverage propabilty (y-axix) of scenarios (58→60) with 24% event propability and 8 covariates. On x-axis are the sample sizes.

8.3 Monte Carlo simulation results: scenarios under model misspecifications

8.3.1 Scenarios with event probability 3% and 2 covariates

The performance measurments (with Monte Carlo standerd error) such as bias, coverage prababilty, MSE, EmpSE and convergence for the intercept, coef.1, and coef.2 are shwon in figures 40, 41, and tables 53 for the intercept, 54 for coef.1, 55 for coef.2.

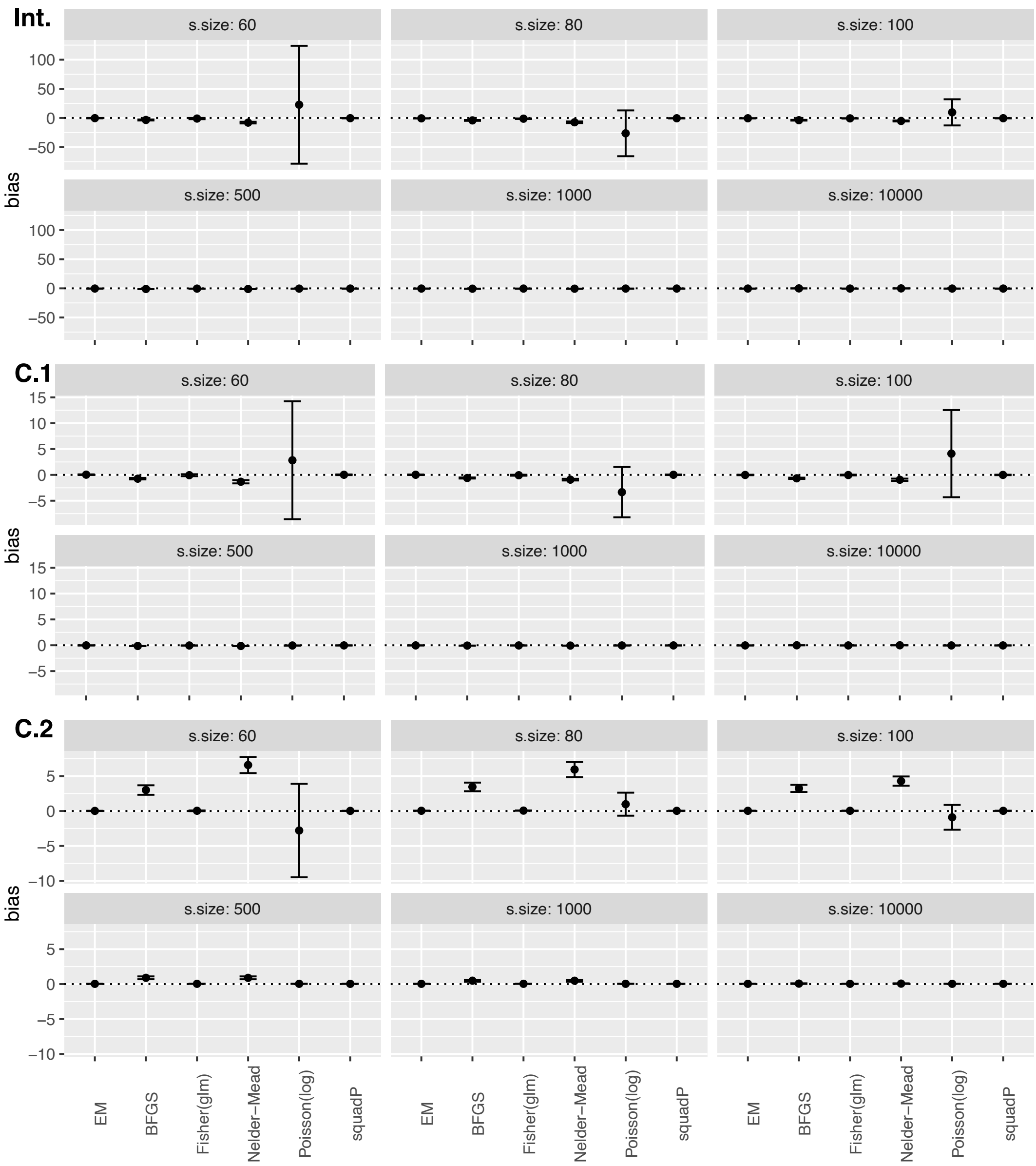

Figure 40: Absolute bias for scenarios 67→72 of the estimated log(RR) from the six methods under model misspecifications. Scenarios are with event probability 3% and sample sizes 60, 80, 100, 500, 1000, and 10000. y-axis: biases for coeffecients int, C.1, and C.2. x-axis: The six statistical methods compared for each scenario.

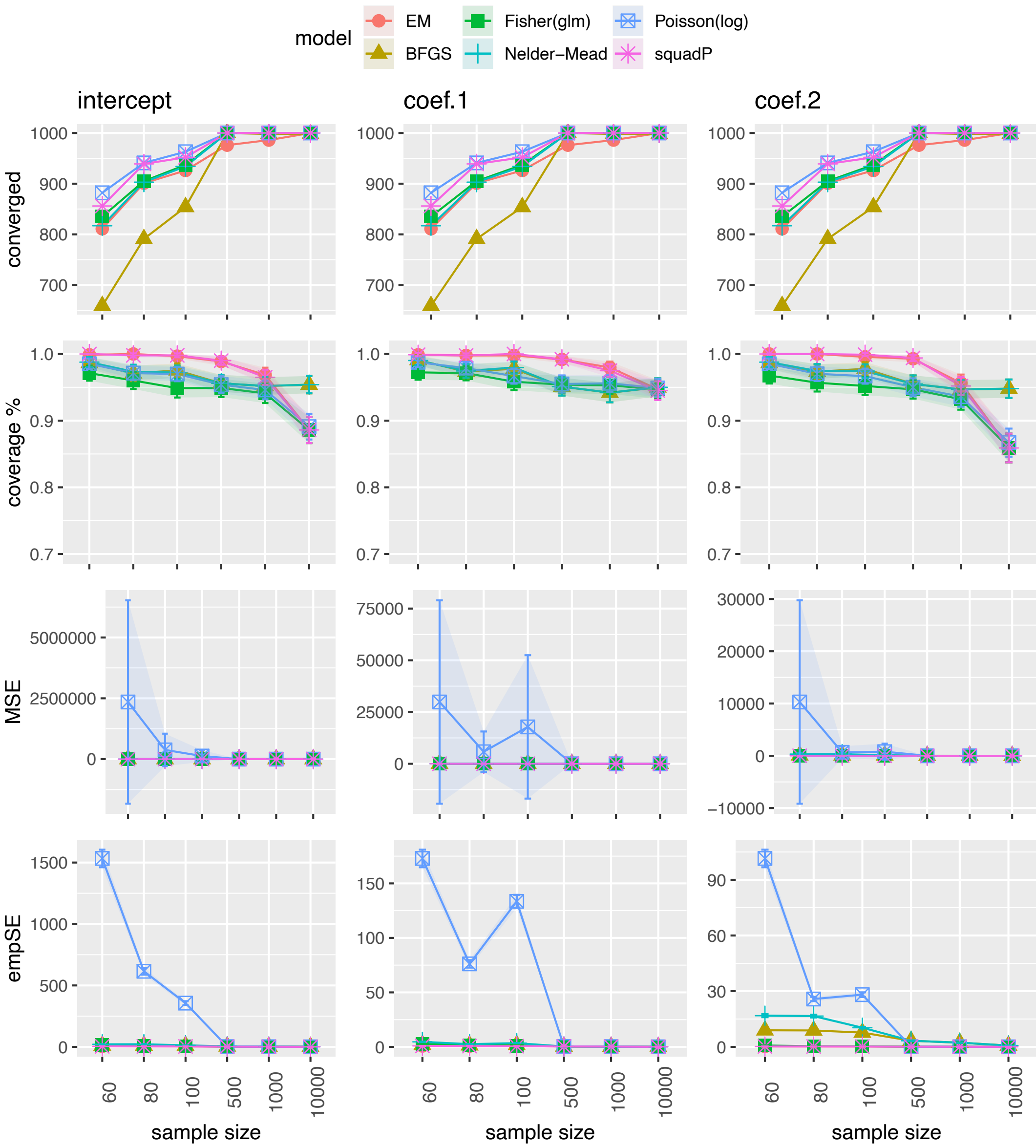

Figure 41: Performance measurments of scenarios 67→72 of the estimated log(RR) from the six methods under model misspecifications and event probability 3%. Measurments are convergence rate, coverage prababilty, MSE, and EmpSE (on y-axis) for the intercept, coef.1, and coef.2 using the six statistical methods (legend in the top of the figure). Each type of shapes with shaded earia (confidence interval) is represnting a different method. The measurments are done for 6 scenarios that are derived from six different sample sizes (60,80,100,500,1000,10000) on x-axis.

Table 53: Performance measurments of scenarios 67→72 for intercept

Covarage		Bias		MSE		EmpSE				
cover.	mcse	bias	mcse	mse	mcse	empse	mcse	model	s.size	converge
1.00	0.00	-0.29	0.15	18.38	1.36	4.28	0.11	EM	60	811
0.99	0.00	-3.46	0.44	136.79	10.73	11.18	0.31	BFGS	60	659
0.97	0.01	-1.00	0.52	231.13	127.07	15.18	0.37	Fisher(glm)	60	835
0.99	0.00	-7.97	0.73	496.8	119.2	20.83	0.52	Nelder-Mead	60	817
0.98	0.00	22.70	51.63	2348881	2132414	1533	36.53	Poisson(log)	60	882
1.00	0.00	-0.38	0.15	18.98	1.43	4.34	0.10	squadP	60	856
1.00	0.00	-0.64	0.13	14.98	1.09	3.82	0.09	EM	80	901
0.97	0.01	-4.16	0.40	140.40	12.08	11.10	0.28	BFGS	80	791
0.96	0.01	-1.24	0.25	57.65	13.71	7.50	0.18	Fisher(glm)	80	905
0.97	0.00	-7.38	0.71	510.8	210.6	21.38	0.50	Nelder-Mead	80	903
0.97	0.00	-26.32	20.06	378857	340369	615.3	14.19	Poisson(log)	80	941
1.00	0.00	-0.50	0.13	15.89	1.14	3.96	0.09	squadP	80	939
1.00	0.00	-0.50	0.11	12.00	0.75	3.43	0.08	EM	100	926
0.98	0.00	-3.85	0.33	105.61	7.10	9.53	0.23	BFGS	100	854
0.95	0.01	-0.67	0.18	31.80	3.47	5.60	0.13	Fisher(glm)	100	937
0.97	0.00	-5.17	0.41	185.5	23.64	12.61	0.29	Nelder-Mead	100	934
0.97	0.01	9.66	11.47	126732	123707	356	8.12	Poisson(log)	100	963
1.00	0.00	-0.44	0.11	12.45	0.76	3.50	0.08	squadP	100	952
0.99	0.00	-0.34	0.06	3.17	0.12	1.75	0.04	EM	500	976
0.96	0.01	-1.08	0.13	17.49	0.96	4.04	0.09	BFGS	500	1000
0.95	0.01	-0.44	0.06	4.37	0.22	2.04	0.05	Fisher(glm)	500	1000
0.96	0.01	-1.08	0.13	17.48	0.96	4.04	0.09	Nelder-Mead	500	1000
0.95	0.01	-0.44	0.06	4.42	0.22	2.06	0.05	Poisson(log)	500	1000
0.99	0.00	-0.36	0.06	3.30	0.12	1.78	0.04	squadP	500	1000
0.97	0.01	-0.35	0.04	1.72	0.07	1.27	0.03	EM	1000	986
0.95	0.01	-0.59	0.09	8.01	0.43	2.77	0.06	BFGS	1000	998
0.94	0.01	-0.39	0.04	2.03	0.10	1.37	0.03	Fisher(glm)	1000	1000
0.95	0.01	-0.58	0.09	8.01	0.43	2.77	0.06	Nelder-Mead	1000	1000
0.95	0.01	-0.39	0.04	2.04	0.10	1.37	0.03	Poisson(log)	1000	1000
0.96	0.01	-0.37	0.04	1.80	0.07	1.29	0.03	squadP	1000	1000
0.89	0.01	-0.36	0.01	0.30	0.01	0.42	0.01	EM	10000	1000
0.95	0.01	-0.09	0.03	0.78	0.04	0.88	0.02	BFGS	10000	998
0.89	0.01	-0.36	0.01	0.30	0.01	0.42	0.01	Fisher(glm)	10000	1000
0.95	0.01	-0.09	0.03	0.78	0.04	0.88	0.02	Nelder-Mead	10000	1000
0.89	0.01	-0.36	0.01	0.30	0.01	0.42	0.01	Poisson(log)	10000	1000
0.89	0.01	-0.36	0.01	0.30	0.01	0.42	0.01	squadP	10000	1000

Table 54: Performance measurments of scenarios 67→72 for coef.1

Covarage		Bias		MSE		EmpSE				
cover.	mcse	bias	mcse	mse	mcse	empse	mcse	model	s.size	converge
1.00	0.00	0.04	0.03	0.70	0.07	0.84	0.02	EM	60	811
0.99	0.00	-0.74	0.08	4.21	1.65	1.92	0.05	BFGS	60	659
0.97	0.01	-0.06	0.10	7.66	4.37	2.77	0.07	Fisher(glm)	60	835
0.99	0.00	-1.33	0.16	22.96	9.84	4.61	0.11	Nelder-Mead	60	817
0.99	0.00	2.83	5.82	29860	25065	172.8	4.12	Poisson(log)	60	882
1.00	0.00	0.02	0.03	0.80	0.10	0.90	0.02	squadP	60	856
1.00	0.00	0.02	0.02	0.51	0.03	0.71	0.02	EM	80	901
0.97	0.01	-0.61	0.05	2.51	0.54	1.46	0.04	BFGS	80	791
0.97	0.01	-0.07	0.05	2.06	0.49	1.43	0.03	Fisher(glm)	80	905
0.98	0.00	-0.92	0.08	7.21	2.63	2.52	0.06	Nelder-Mead	80	903
0.98	0.00	-3.35	2.48	5804	5000	76.16	1.76	Poisson(log)	80	941
1.00	0.00	0.02	0.02	0.54	0.04	0.74	0.02	squadP	80	939
1.00	0.00	-0.02	0.02	0.44	0.02	0.67	0.02	EM	100	926
0.98	0.00	-0.66	0.06	3.66	1.15	1.80	0.04	BFGS	100	854
0.96	0.01	-0.03	0.04	1.23	0.12	1.11	0.03	Fisher(glm)	100	937
0.98	0.00	-0.94	0.11	12.59	6.96	3.42	0.08	Nelder-Mead	100	934
0.97	0.01	4.11	4.30	17823	17673	133.5	3.04	Poisson(log)	100	963
1.00	0.00	-0.01	0.02	0.44	0.02	0.66	0.02	squadP	100	952
0.99	0.00	-0.02	0.01	0.12	0.00	0.34	0.01	EM	500	976
0.95	0.01	-0.14	0.01	0.16	0.02	0.37	0.01	BFGS	500	1000
0.95	0.01	-0.04	0.01	0.16	0.01	0.40	0.01	Fisher(glm)	500	1000
0.95	0.01	-0.14	0.01	0.16	0.02	0.37	0.01	Nelder-Mead	500	1000
0.96	0.01	-0.05	0.01	0.16	0.01	0.40	0.01	Poisson(log)	500	1000
0.99	0.00	-0.03	0.01	0.12	0.00	0.35	0.01	squadP	500	1000
0.98	0.00	-0.03	0.01	0.06	0.00	0.25	0.01	EM	1000	986
0.94	0.01	-0.06	0.01	0.04	0.00	0.20	0.00	BFGS	1000	998
0.95	0.01	-0.04	0.01	0.07	0.00	0.27	0.01	Fisher(glm)	1000	1000
0.94	0.01	-0.06	0.01	0.04	0.00	0.20	0.00	Nelder-Mead	1000	1000
0.96	0.01	-0.04	0.01	0.07	0.00	0.27	0.01	Poisson(log)	1000	1000
0.98	0.00	-0.03	0.01	0.06	0.00	0.25	0.01	squadP	1000	1000
0.95	0.01	-0.03	0.00	0.01	0.00	0.08	0.00	EM	10000	1000
0.95	0.01	0.00	0.00	0.00	0.00	0.06	0.00	BFGS	10000	998
0.95	0.01	-0.03	0.00	0.01	0.00	0.08	0.00	Fisher(glm)	10000	1000
0.95	0.01	0.00	0.00	0.00	0.00	0.06	0.00	Nelder-Mead	10000	1000
0.95	0.01	-0.03	0.00	0.01	0.00	0.08	0.00	Poisson(log)	10000	1000
0.94	0.01	-0.03	0.00	0.01	0.00	0.08	0.00	squadP	10000	1000

Table 55: Performance measurments of scenarios 67→72 for coef.2

Covarage		Bias		MSE		EmpSE				
cover.	mcse	bias	mcse	mse	mcse	empse	mcse	model	s.size	converge
1.00	0.00	0.00	0.01	0.05	0.00	0.84	0.02	EM	60	811
0.99	0.00	2.98	0.35	88.31	6.59	1.92	0.05	BFGS	60	659
0.97	0.01	0.01	0.02	0.54	0.27	2.77	0.07	Fisher(glm)	60	835
0.99	0.00	6.56	0.59	324.8	86.63	4.61	0.11	Nelder-Mead	60	817
0.98	0.00	-2.81	3.42	10298	9924	172.8	4.12	Poisson(log)	60	882
1.00	0.00	0.00	0.01	0.05	0.00	0.90	0.02	squadP	60	856
1.00	0.00	0.02	0.01	0.04	0.00	0.71	0.02	EM	80	901
0.97	0.01	3.43	0.31	89.74	7.14	1.46	0.04	BFGS	80	791
0.96	0.01	0.04	0.01	0.13	0.02	1.43	0.03	Fisher(glm)	80	905
0.98	0.00	5.92	0.55	309.6	120.7	2.52	0.06	Nelder-Mead	80	903
0.97	0.01	0.96	0.84	664.6	598.1	76.16	1.76	Poisson(log)	80	941
1.00	0.00	0.01	0.01	0.04	0.00	0.74	0.02	squadP	80	939
1.00	0.00	0.01	0.01	0.03	0.00	0.67	0.02	EM	100	926
0.98	0.00	3.22	0.26	69.35	4.57	1.80	0.04	BFGS	100	854
0.95	0.01	0.01	0.01	0.09	0.02	1.11	0.03	Fisher(glm)	100	937
0.97	0.00	4.26	0.34	124.7	19.59	3.42	0.08	Nelder-Mead	100	934
0.97	0.01	-0.92	0.91	790.9	782.5	133.5	3.04	Poisson(log)	100	963
1.00	0.00	0.01	0.01	0.03	0.00	0.66	0.02	squadP	100	952
0.99	0.00	0.02	0.00	0.01	0.00	0.34	0.01	EM	500	976
0.96	0.01	0.88	0.10	11.62	0.62	0.37	0.01	BFGS	500	1000
0.95	0.01	0.02	0.00	0.01	0.00	0.40	0.01	Fisher(glm)	500	1000
0.96	0.01	0.88	0.10	11.62	0.62	0.37	0.01	Nelder-Mead	500	1000
0.95	0.01	0.02	0.00	0.01	0.00	0.40	0.01	Poisson(log)	500	1000
0.99	0.00	0.02	0.00	0.01	0.00	0.35	0.01	squadP	500	1000
0.96	0.01	0.02	0.00	0.00	0.00	0.25	0.01	EM	1000	986
0.95	0.01	0.47	0.07	5.31	0.28	0.20	0.00	BFGS	1000	998
0.93	0.01	0.02	0.00	0.00	0.00	0.27	0.01	Fisher(glm)	1000	1000
0.95	0.01	0.46	0.07	5.30	0.28	0.20	0.00	Nelder-Mead	1000	1000
0.94	0.01	0.02	0.00	0.00	0.00	0.27	0.01	Poisson(log)	1000	1000
0.95	0.01	0.02	0.00	0.00	0.00	0.25	0.01	squadP	1000	1000
0.86	0.01	0.02	0.00	0.00	0.00	0.08	0.00	EM	10000	1000
0.95	0.01	0.05	0.02	0.52	0.02	0.06	0.00	BFGS	10000	998
0.86	0.01	0.02	0.00	0.00	0.00	0.08	0.00	Fisher(glm)	10000	1000
0.95	0.01	0.05	0.02	0.52	0.02	0.06	0.00	Nelder-Mead	10000	1000
0.87	0.01	0.02	0.00	0.00	0.00	0.08	0.00	Poisson(log)	10000	1000
0.86	0.01	0.02	0.00	0.00	0.00	0.08	0.00	squadP	10000	1000

8.3.2 Scenarios with event probability 6% and 2 covariates

The performance measurments (with Monte Carlo standerd error) such as bias, coverage prababilty, MSE, EmpSE and convergence for the intercept, coef.1, and coef.2 are shwon in figures 42, 43, and tables 56 for the intecept, 57 for coef.1, and 58 for coef.2.

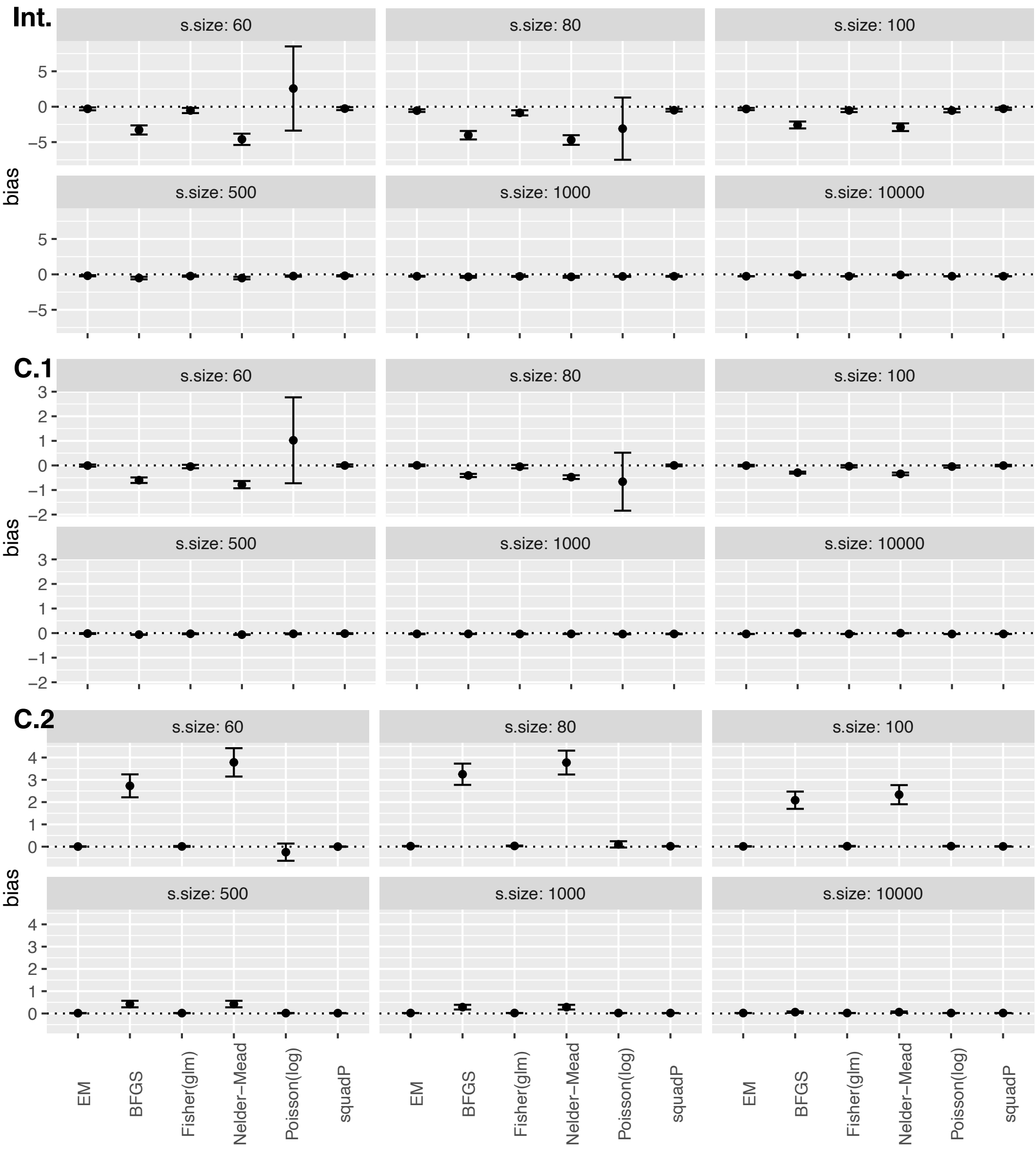

Figure 42: Absolute bias for scenarios 73→78 of the estimated log(RR) from the six methods under model misspecifications. Scenarios are with event probability 6% and sample sizes 60, 80, 100, 500, 1000, and 10000. y-axis: biases for coeffecients int, C.1, and C.2. x-axis: The six statistical methods compared for each scenario.

Table 56: Performance measurments of scenarios 73→78 for intercept

Covarage		Bias		MSE		EmpSE				
cover.	mcse	bias	mcse	mse	mcse	empse	mcse	model	s.size	converged
1.00	0.00	-0.30	0.10	10.05	0.58	3.16	0.07	EM	60	902
0.96	0.01	-3.29	0.33	107.27	7.49	9.83	0.23	BFGS	60	892
0.96	0.01	-0.53	0.19	32.70	4.41	5.70	0.13	Fisher(glm)	60	935
0.96	0.01	-4.61	0.41	175.86	19.53	12.44	0.29	Nelder-Mead	60	941
0.98	0.00	2.57	3.03	9027.89	8340.60	95.03	2.14	Poisson(log)	60	982
1.00	0.00	-0.27	0.11	11.00	0.62	3.31	0.07	squadP	60	974
1.00	0.00	-0.55	0.10	8.70	0.46	2.90	0.07	EM	80	939
0.95	0.01	-4.03	0.31	103.60	6.48	9.35	0.22	BFGS	80	936
0.94	0.01	-0.87	0.18	33.62	12.29	5.74	0.13	Fisher(glm)	80	975
0.95	0.01	-4.70	0.35	137.47	15.34	10.74	0.24	Nelder-Mead	80	966
0.97	0.01	-3.10	2.24	4996.80	4958.91	70.66	1.58	Poisson(log)	80	994
1.00	0.00	-0.50	0.10	9.12	0.46	2.98	0.07	squadP	80	991
1.00	0.00	-0.32	0.08	6.88	0.46	2.60	0.06	EM	100	975
0.96	0.01	-2.58	0.25	66.22	5.98	7.72	0.18	BFGS	100	964
0.95	0.01	-0.52	0.12	15.04	1.18	3.84	0.09	Fisher(glm)	100	984
0.96	0.01	-2.90	0.28	83.52	8.05	8.67	0.20	Nelder-Mead	100	983
0.96	0.01	-0.55	0.12	15.85	1.33	3.94	0.09	Poisson(log)	100	998
1.00	0.00	-0.29	0.08	6.86	0.38	2.60	0.06	squadP	100	992
0.98	0.00	-0.19	0.04	1.86	0.07	1.35	0.03	EM	500	991
0.94	0.01	-0.52	0.09	8.80	0.43	2.92	0.06	BFGS	500	997
0.95	0.01	-0.24	0.05	2.23	0.10	1.47	0.03	Fisher(glm)	500	1000
0.94	0.01	-0.52	0.09	8.80	0.43	2.92	0.06	Nelder-Mead	500	997
0.96	0.01	-0.24	0.05	2.24	0.10	1.48	0.03	Poisson(log)	500	1000
0.98	0.00	-0.20	0.04	1.90	0.07	1.36	0.03	squadP	500	1000
0.95	0.01	-0.26	0.03	1.05	0.04	0.99	0.02	EM	1000	994
0.95	0.01	-0.35	0.07	4.51	0.21	2.10	0.05	BFGS	1000	1000
0.93	0.01	-0.29	0.03	1.16	0.05	1.04	0.02	Fisher(glm)	1000	1000
0.95	0.01	-0.35	0.07	4.51	0.21	2.10	0.05	Nelder-Mead	1000	1000
0.94	0.01	-0.29	0.03	1.17	0.05	1.04	0.02	Poisson(log)	1000	1000
0.94	0.01	-0.27	0.03	1.08	0.04	1.00	0.02	squadP	1000	1000
0.89	0.01	-0.26	0.01	0.17	0.01	0.31	0.01	EM	10000	1000
0.95	0.01	-0.08	0.02	0.43	0.02	0.65	0.01	BFGS	10000	989
0.89	0.01	-0.26	0.01	0.17	0.01	0.31	0.01	Fisher(glm)	10000	1000
0.95	0.01	-0.08	0.02	0.42	0.02	0.65	0.01	Nelder-Mead	10000	1000
0.90	0.01	-0.27	0.01	0.17	0.01	0.31	0.01	Poisson(log)	10000	1000
0.88	0.01	-0.26	0.01	0.17	0.01	0.31	0.01	squadP	10000	1000

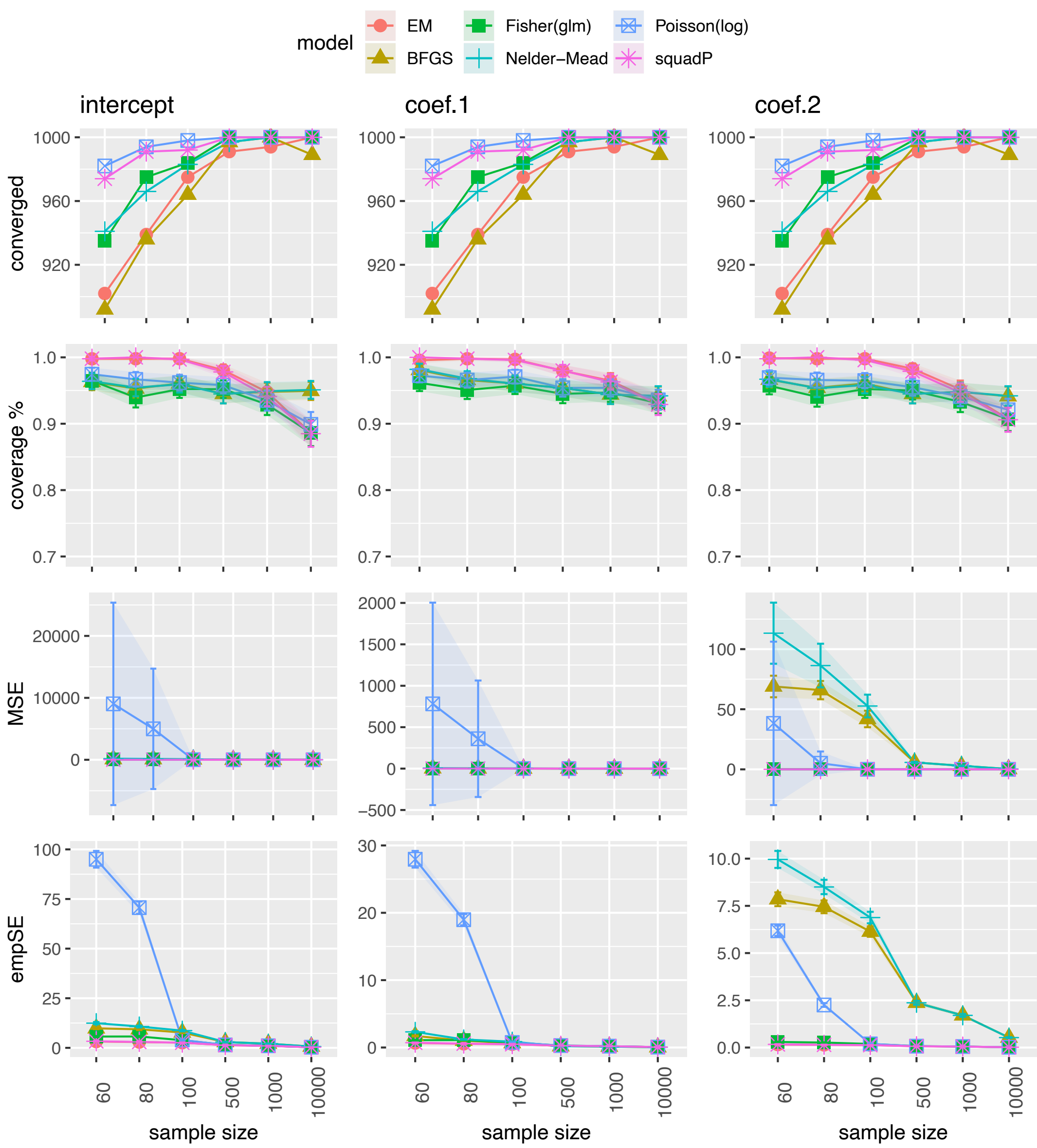

Figure 43: Performance measurments of scenarios 73→78 of the estimated log(RR) from the six methods under model misspecifications and event probability 6%. Measurments are convergence rate, coverage prababilty, MSE, and EmpSE (on y-axis) for the intercept, coef.1, and coef.2 using the six statistical methods (legend in the top of the figure). Each type of shapes with shaded earia (confidence interval) is represnting a different method. The measurments are done for 6 scenarios that are derived from six different sample sizes (60,80,100,500,1000,10000) on x-axis.

Table 57: Performance measurments of scenarios 73→78 for coef.1

Covarage		Bias		MSE		EmpSE				
cover.	mcse	bias	mcse	mse	mcse	empse	mcse	model	s.size	converged
1.00	0.00	-0.01	0.02	0.45	0.03	0.67	0.02	EM	60	902
0.98	0.00	-0.60	0.06	3.31	1.26	1.72	0.04	BFGS	60	892
0.96	0.01	-0.04	0.04	1.19	0.13	1.09	0.03	Fisher(glm)	60	935
0.98	0.00	-0.78	0.08	5.98	1.89	2.32	0.05	Nelder-Mead	60	941
0.97	0.00	1.02	0.89	781.43	623.51	27.95	0.63	Poisson(log)	60	982
1.00	0.00	0.00	0.02	0.50	0.05	0.71	0.02	squadP	60	974
1.00	0.00	0.00	0.02	0.30	0.01	0.55	0.01	EM	80	939
0.97	0.01	-0.41	0.03	1.20	0.31	1.01	0.02	BFGS	80	936
0.95	0.01	-0.05	0.04	1.19	0.44	1.09	0.03	Fisher(glm)	80	975
0.97	0.01	-0.48	0.04	1.64	0.38	1.19	0.03	Nelder-Mead	80	966
0.97	0.01	-0.66	0.60	360.08	358.70	18.97	0.43	Poisson(log)	80	994
1.00	0.00	0.00	0.02	0.33	0.03	0.58	0.01	squadP	80	991
1.00	0.00	-0.01	0.02	0.24	0.01	0.49	0.01	EM	100	975
0.96	0.01	-0.29	0.02	0.51	0.07	0.65	0.01	BFGS	100	964
0.96	0.01	-0.04	0.02	0.55	0.04	0.74	0.02	Fisher(glm)	100	984
0.96	0.01	-0.34	0.03	0.86	0.21	0.86	0.02	Nelder-Mead	100	983
0.97	0.00	-0.04	0.02	0.56	0.04	0.75	0.02	Poisson(log)	100	998
1.00	0.00	0.00	0.02	0.24	0.01	0.49	0.01	squadP	100	992
0.98	0.00	-0.02	0.01	0.07	0.00	0.26	0.01	EM	500	991
0.95	0.01	-0.06	0.01	0.04	0.00	0.19	0.00	BFGS	500	997
0.94	0.01	-0.03	0.01	0.08	0.00	0.28	0.01	Fisher(glm)	500	1000
0.95	0.01	-0.06	0.01	0.04	0.00	0.19	0.00	Nelder-Mead	500	997
0.95	0.01	-0.03	0.01	0.08	0.00	0.29	0.01	Poisson(log)	500	1000
0.98	0.00	-0.02	0.01	0.07	0.00	0.26	0.01	squadP	500	1000
0.96	0.01	-0.03	0.01	0.04	0.00	0.19	0.00	EM	1000	994
0.94	0.01	-0.03	0.00	0.02	0.00	0.13	0.00	BFGS	1000	1000
0.95	0.01	-0.04	0.01	0.04	0.00	0.20	0.00	Fisher(glm)	1000	1000
0.94	0.01	-0.03	0.00	0.02	0.00	0.13	0.00	Nelder-Mead	1000	1000
0.95	0.01	-0.04	0.01	0.04	0.00	0.20	0.00	Poisson(log)	1000	1000
0.96	0.01	-0.03	0.01	0.04	0.00	0.19	0.00	squadP	1000	1000
0.93	0.01	-0.04	0.00	0.00	0.00	0.06	0.00	EM	10000	1000
0.94	0.01	0.00	0.00	0.00	0.00	0.04	0.00	BFGS	10000	989
0.93	0.01	-0.04	0.00	0.00	0.00	0.06	0.00	Fisher(glm)	10000	1000
0.94	0.01	0.00	0.00	0.00	0.00	0.04	0.00	Nelder-Mead	10000	1000
0.94	0.01	-0.04	0.00	0.00	0.00	0.06	0.00	Poisson(log)	10000	1000
0.93	0.01	-0.04	0.00	0.00	0.00	0.06	0.00	squadP	10000	1000

Table 58: Performance measurments of scenarios 73→78 for coef.2

Covarage		Bias		MSE		EmpSE				
cover.	mcse	bias	mcse	mse	mcse	empse	mcse	model	s.size	converged
1.00	0.00	-0.01	0.00	0.03	0.00	0.67	0.02	EM	60	902
0.97	0.01	2.72	0.26	68.95	4.58	1.72	0.04	BFGS	60	892
0.96	0.01	0.00	0.01	0.09	0.02	1.09	0.03	Fisher(glm)	60	935
0.97	0.01	3.77	0.32	113.29	12.97	2.32	0.05	Nelder-Mead	60	941
0.97	0.00	-0.26	0.20	38.24	34.69	27.95	0.63	Poisson(log)	60	982
1.00	0.00	-0.01	0.00	0.03	0.00	0.71	0.02	squadP	60	974
1.00	0.00	0.01	0.00	0.02	0.00	0.55	0.01	EM	80	939
0.95	0.01	3.24	0.24	65.93	3.89	1.01	0.02	BFGS	80	936
0.94	0.01	0.02	0.01	0.07	0.02	1.09	0.03	Fisher(glm)	80	975
0.95	0.01	3.76	0.27	86.35	9.27	1.19	0.03	Nelder-Mead	80	966
0.97	0.01	0.09	0.07	5.07	5.00	18.97	0.43	Poisson(log)	80	994
1.00	0.00	0.01	0.00	0.02	0.00	0.58	0.01	squadP	80	991
1.00	0.00	0.00	0.00	0.02	0.00	0.49	0.01	EM	100	975
0.96	0.01	2.07	0.20	41.92	3.51	0.65	0.01	BFGS	100	964
0.95	0.01	0.01	0.01	0.04	0.00	0.74	0.02	Fisher(glm)	100	984
0.96	0.01	2.32	0.22	52.69	4.82	0.86	0.02	Nelder-Mead	100	983
0.96	0.01	0.01	0.01	0.04	0.00	0.75	0.02	Poisson(log)	100	998
1.00	0.00	0.00	0.00	0.02	0.00	0.49	0.01	squadP	100	992
0.98	0.00	0.00	0.00	0.00	0.00	0.26	0.01	EM	500	991
0.94	0.01	0.41	0.07	5.78	0.28	0.19	0.00	BFGS	500	997
0.95	0.01	0.01	0.00	0.00	0.00	0.28	0.01	Fisher(glm)	500	1000
0.94	0.01	0.41	0.07	5.78	0.28	0.19	0.00	Nelder-Mead	500	997
0.96	0.01	0.01	0.00	0.00	0.00	0.29	0.01	Poisson(log)	500	1000
0.98	0.00	0.00	0.00	0.00	0.00	0.26	0.01	squadP	500	1000
0.95	0.01	0.01	0.00	0.00	0.00	0.19	0.00	EM	1000	994
0.95	0.01	0.27	0.05	2.97	0.14	0.13	0.00	BFGS	1000	1000
0.93	0.01	0.01	0.00	0.00	0.00	0.20	0.00	Fisher(glm)	1000	1000
0.95	0.01	0.27	0.05	2.97	0.14	0.13	0.00	Nelder-Mead	1000	1000
0.94	0.01	0.01	0.00	0.00	0.00	0.20	0.00	Poisson(log)	1000	1000
0.95	0.01	0.01	0.00	0.00	0.00	0.19	0.00	squadP	1000	1000
0.91	0.01	0.01	0.00	0.00	0.00	0.06	0.00	EM	10000	1000
0.94	0.01	0.05	0.02	0.28	0.01	0.04	0.00	BFGS	10000	989
0.91	0.01	0.01	0.00	0.00	0.00	0.06	0.00	Fisher(glm)	10000	1000
0.94	0.01	0.05	0.02	0.28	0.01	0.04	0.00	Nelder-Mead	10000	1000
0.92	0.01	0.01	0.00	0.00	0.00	0.06	0.00	Poisson(log)	10000	1000
0.91	0.01	0.01	0.00	0.00	0.00	0.06	0.00	squadP	10000	1000

8.3.3 Scenarios with event probability 12% and 2 covariates

The performance measurments (with Monte Carlo standerd error) such as bias, coverage prabability, MSE, EmpSE and convergence for the intercept, coef.1, and coef.2 are shwon in figures 44, 45, and tables 59, 60, and 61 for the intercept, coef.1, and coef.2.

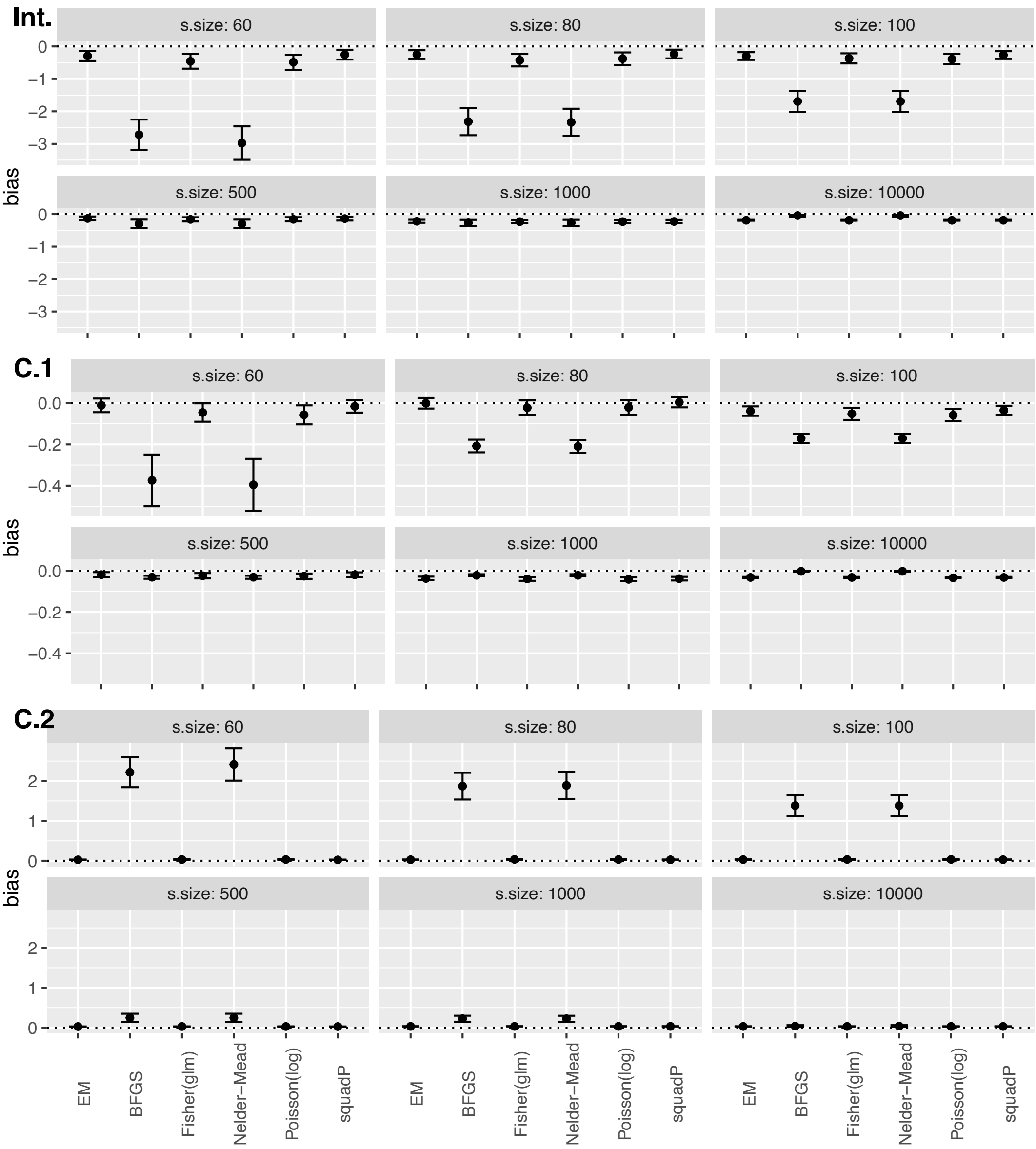

Figure 44: Absolute bias for scenarios 79→84 of the estimated log(RR) from the six methods under model misspecifications. Scenarios are with event probability 12% and sample sizes 60, 80, 100, 500, 1000, and 10000. y-axis: biases for coeffecients int, C.1, and C.2. x-axis: The six statistical methods compared for each scenario.

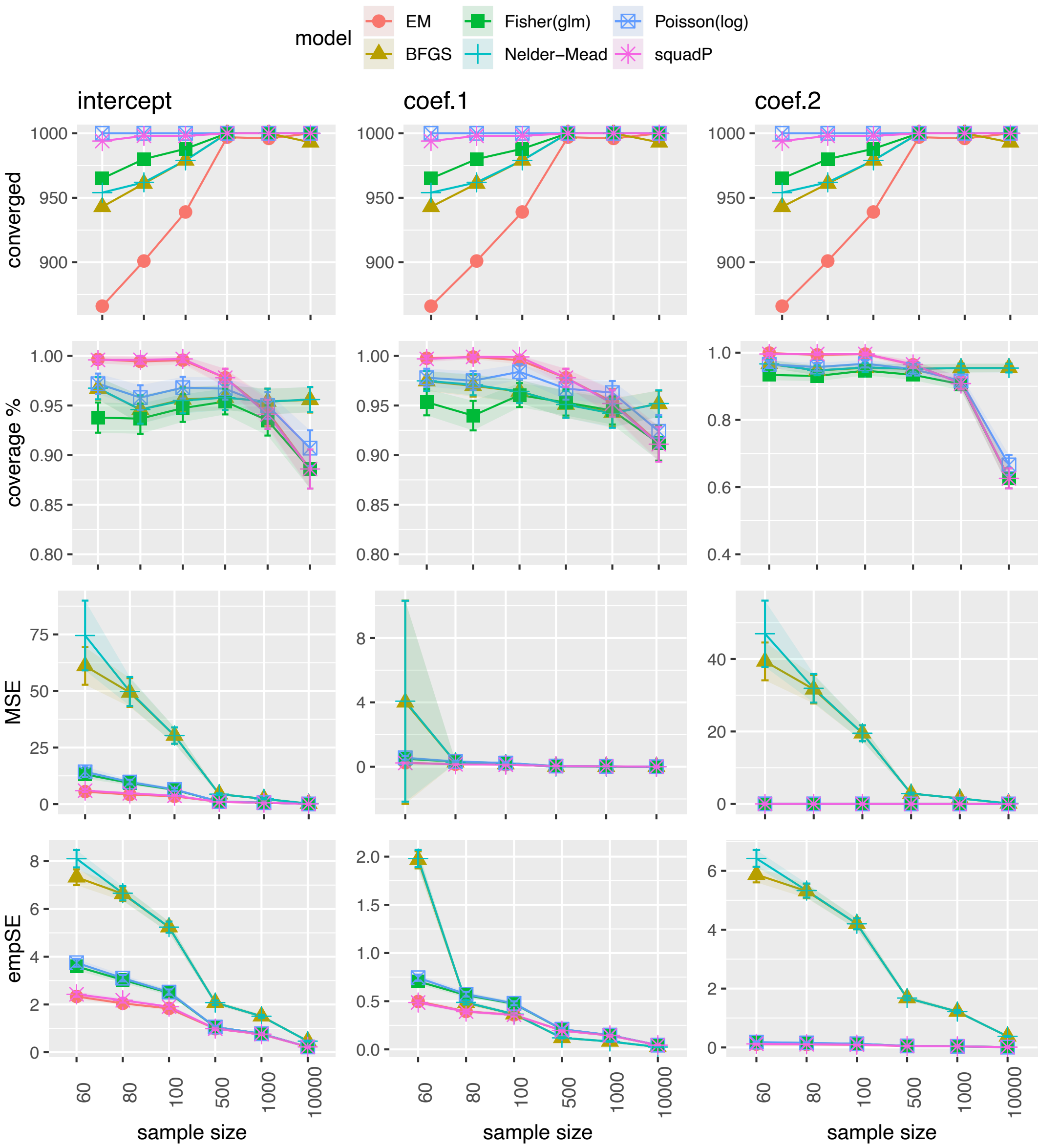

Figure 45: Performance measurments of scenarios 79→84 of the estimated log(RR) from the six methods under model misspecifications and event probability 12%. Measurments are convergence rate, coverage prababilty, MSE, and EmpSE (on y-axis) for the intercept, coef.1, and coef.2 using the six statistical methods (legend in the top of the figure). Each type of shapes with shaded earia (confidence interval) is represnting a different method. The measurments are done for 6 scenarios that are derived from six different sample sizes (60,80,100,500,1000,10000) on x-axis.

Table 59: Performance measurments of scenarios 79→84 for intercept

Covarage		Bias		MSE		EmpSE				
cover.	mcse	bias	mcse	mse	mcse	empse	mcse	model	s.size	converged
1.00	0.00	-0.29	0.08	5.54	0.39	2.34	0.06	EM	60	866
0.97	0.01	-2.72	0.24	61.04	4.23	7.33	0.17	BFGS	60	943
0.94	0.01	-0.46	0.12	13.12	1.00	3.59	0.08	Fisher(glm)	60	965
0.97	0.01	-2.98	0.26	74.51	7.86	8.11	0.19	Nelder-Mead	60	954
0.97	0.00	-0.49	0.12	14.28	1.21	3.75	0.08	Poisson(log)	60	1000
1.00	0.00	-0.25	0.08	5.94	0.29	2.43	0.05	squadP	60	994
0.99	0.00	-0.25	0.07	4.25	0.19	2.05	0.05	EM	80	901
0.95	0.01	-2.32	0.21	49.33	3.24	6.63	0.15	BFGS	80	961
0.94	0.01	-0.43	0.10	9.36	0.52	3.03	0.07	Fisher(glm)	80	980
0.95	0.01	-2.34	0.22	49.77	3.26	6.66	0.15	Nelder-Mead	80	962
0.96	0.01	-0.38	0.10	9.76	0.54	3.10	0.07	Poisson(log)	80	1000
1.00	0.00	-0.23	0.07	4.79	0.20	2.18	0.05	squadP	80	998
1.00	0.00	-0.30	0.06	3.46	0.14	1.84	0.04	EM	100	939
0.96	0.01	-1.70	0.17	30.28	1.84	5.24	0.12	BFGS	100	979
0.95	0.01	-0.37	0.08	6.33	0.31	2.49	0.06	Fisher(glm)	100	988
0.96	0.01	-1.70	0.17	30.29	1.84	5.24	0.12	Nelder-Mead	100	979
0.97	0.01	-0.39	0.08	6.49	0.32	2.52	0.06	Poisson(log)	100	1000
1.00	0.00	-0.27	0.06	3.71	0.15	1.91	0.04	squadP	100	998
0.98	0.00	-0.13	0.03	0.99	0.04	0.98	0.02	EM	500	997
0.96	0.01	-0.30	0.07	4.41	0.21	2.08	0.05	BFGS	500	1000
0.95	0.01	-0.16	0.03	1.14	0.05	1.05	0.02	Fisher(glm)	500	1000
0.96	0.01	-0.30	0.07	4.41	0.21	2.08	0.05	Nelder-Mead	500	1000
0.97	0.01	-0.16	0.03	1.14	0.05	1.06	0.02	Poisson(log)	500	1000
0.98	0.00	-0.14	0.03	0.99	0.04	0.99	0.02	squadP	500	1000
0.94	0.01	-0.22	0.02	0.61	0.03	0.75	0.02	EM	1000	996
0.95	0.01	-0.27	0.05	2.34	0.12	1.51	0.03	BFGS	1000	1000
0.94	0.01	-0.23	0.02	0.65	0.03	0.77	0.02	Fisher(glm)	1000	1000
0.95	0.01	-0.27	0.05	2.34	0.12	1.51	0.03	Nelder-Mead	1000	1000
0.95	0.01	-0.23	0.03	0.66	0.03	0.78	0.02	Poisson(log)	1000	1000
0.94	0.01	-0.23	0.02	0.62	0.03	0.76	0.02	squadP	1000	1000
0.89	0.01	-0.19	0.01	0.09	0.00	0.22	0.00	EM	10000	1000
0.96	0.01	-0.04	0.01	0.22	0.01	0.47	0.01	BFGS	10000	993
0.89	0.01	-0.19	0.01	0.09	0.00	0.22	0.00	Fisher(glm)	10000	1000
0.96	0.01	-0.05	0.01	0.22	0.01	0.47	0.01	Nelder-Mead	10000	1000
0.91	0.01	-0.19	0.01	0.09	0.00	0.22	0.00	Poisson(log)	10000	1000
0.89	0.01	-0.19	0.01	0.09	0.00	0.22	0.00	squadP	10000	1000

Table 60: Performance measurments of scenarios 79→84 for coef.1

Covarage		Bias		MSE		EmpSE				
cover.	mcse	bias	mcse	mse	mcse	empse	mcse	model	s.size	converged
1.00	0.00	-0.01	0.02	0.25	0.02	0.50	0.01	EM	60	866
0.98	0.00	-0.37	0.06	4.00	3.22	1.97	0.04	BFGS	60	943
0.95	0.01	-0.04	0.02	0.50	0.05	0.71	0.02	Fisher(glm)	60	965
0.98	0.00	-0.40	0.06	4.07	3.18	1.98	0.04	Nelder-Mead	60	954
0.98	0.00	-0.06	0.02	0.56	0.06	0.74	0.02	Poisson(log)	60	1000
1.00	0.00	-0.01	0.01	0.24	0.01	0.49	0.01	squadP	60	994
1.00	0.00	0.00	0.01	0.16	0.01	0.39	0.01	EM	80	901
0.97	0.01	-0.21	0.02	0.28	0.04	0.48	0.01	BFGS	80	961
0.94	0.01	-0.02	0.02	0.32	0.01	0.56	0.01	Fisher(glm)	80	980
0.97	0.00	-0.21	0.02	0.28	0.04	0.49	0.01	Nelder-Mead	80	962
0.98	0.00	-0.02	0.02	0.33	0.01	0.57	0.01	Poisson(log)	80	1000
1.00	0.00	0.00	0.01	0.15	0.01	0.39	0.01	squadP	80	998
1.00	0.00	-0.04	0.01	0.13	0.00	0.36	0.01	EM	100	939
0.96	0.01	-0.17	0.01	0.16	0.01	0.36	0.01	BFGS	100	979
0.96	0.01	-0.05	0.01	0.23	0.01	0.47	0.01	Fisher(glm)	100	988
0.96	0.01	-0.17	0.01	0.16	0.01	0.36	0.01	Nelder-Mead	100	979
0.98	0.00	-0.06	0.01	0.23	0.01	0.48	0.01	Poisson(log)	100	1000
1.00	0.00	-0.04	0.01	0.13	0.00	0.36	0.01	squadP	100	998
0.98	0.00	-0.02	0.01	0.04	0.00	0.20	0.00	EM	500	997
0.95	0.01	-0.03	0.00	0.01	0.00	0.12	0.00	BFGS	500	1000
0.95	0.01	-0.02	0.01	0.04	0.00	0.21	0.00	Fisher(glm)	500	1000
0.95	0.01	-0.03	0.00	0.01	0.00	0.12	0.00	Nelder-Mead	500	1000
0.97	0.01	-0.03	0.01	0.04	0.00	0.21	0.00	Poisson(log)	500	1000
0.98	0.00	-0.02	0.01	0.04	0.00	0.20	0.00	squadP	500	1000
0.95	0.01	-0.04	0.00	0.02	0.00	0.14	0.00	EM	1000	996
0.94	0.01	-0.02	0.00	0.01	0.00	0.08	0.00	BFGS	1000	1000
0.94	0.01	-0.04	0.00	0.02	0.00	0.15	0.00	Fisher(glm)	1000	1000
0.94	0.01	-0.02	0.00	0.01	0.00	0.08	0.00	Nelder-Mead	1000	1000
0.96	0.01	-0.04	0.00	0.02	0.00	0.15	0.00	Poisson(log)	1000	1000
0.95	0.01	-0.04	0.00	0.02	0.00	0.14	0.00	squadP	1000	1000
0.91	0.01	-0.03	0.00	0.00	0.00	0.04	0.00	EM	10000	1000
0.95	0.01	0.00	0.00	0.00	0.00	0.02	0.00	BFGS	10000	993
0.91	0.01	-0.03	0.00	0.00	0.00	0.04	0.00	Fisher(glm)	10000	1000
0.95	0.01	0.00	0.00	0.00	0.00	0.02	0.00	Nelder-Mead	10000	1000
0.92	0.01	-0.03	0.00	0.00	0.00	0.04	0.00	Poisson(log)	10000	1000
0.91	0.01	-0.03	0.00	0.00	0.00	0.04	0.00	squadP	10000	1000

Table 61: Performance measurments of scenarios 79→84 for coef.2

Covarage		Bias		MSE		EmpSE				
cover.	mcse	bias	mcse	mse	mcse	empse	mcse	model	s.size	converged
1.00	0.00	0.01	0.00	0.01	0.00	0.50	0.01	EM	60	866
0.97	0.01	2.21	0.19	39.33	2.66	1.97	0.04	BFGS	60	943
0.93	0.01	0.02	0.01	0.03	0.00	0.71	0.02	Fisher(glm)	60	965
0.97	0.01	2.41	0.21	46.97	4.66	1.98	0.04	Nelder-Mead	60	954
0.96	0.01	0.02	0.01	0.03	0.00	0.74	0.02	Poisson(log)	60	1000
1.00	0.00	0.01	0.00	0.01	0.00	0.49	0.01	squadP	60	994
0.99	0.00	0.02	0.00	0.01	0.00	0.39	0.01	EM	80	901
0.95	0.01	1.86	0.17	31.64	2.00	0.48	0.01	BFGS	80	961
0.93	0.01	0.02	0.00	0.02	0.00	0.56	0.01	Fisher(glm)	80	980
0.95	0.01	1.88	0.17	31.92	2.01	0.49	0.01	Nelder-Mead	80	962
0.96	0.01	0.02	0.00	0.02	0.00	0.57	0.01	Poisson(log)	80	1000
1.00	0.00	0.02	0.00	0.01	0.00	0.39	0.01	squadP	80	998
1.00	0.00	0.02	0.00	0.01	0.00	0.36	0.01	EM	100	939
0.96	0.01	1.37	0.13	19.51	1.12	0.36	0.01	BFGS	100	979
0.95	0.01	0.02	0.00	0.02	0.00	0.47	0.01	Fisher(glm)	100	988
0.96	0.01	1.37	0.13	19.52	1.12	0.36	0.01	Nelder-Mead	100	979
0.97	0.01	0.02	0.00	0.02	0.00	0.48	0.01	Poisson(log)	100	1000
1.00	0.00	0.02	0.00	0.01	0.00	0.36	0.01	squadP	100	998
0.96	0.01	0.02	0.00	0.00	0.00	0.20	0.00	EM	500	997
0.95	0.01	0.23	0.05	2.87	0.14	0.12	0.00	BFGS	500	1000
0.93	0.01	0.02	0.00	0.00	0.00	0.21	0.00	Fisher(glm)	500	1000
0.95	0.01	0.23	0.05	2.87	0.14	0.12	0.00	Nelder-Mead	500	1000
0.95	0.01	0.02	0.00	0.00	0.00	0.21	0.00	Poisson(log)	500	1000
0.96	0.01	0.02	0.00	0.00	0.00	0.20	0.00	squadP	500	1000
0.91	0.01	0.02	0.00	0.00	0.00	0.14	0.00	EM	1000	996
0.95	0.01	0.21	0.04	1.53	0.08	0.08	0.00	BFGS	1000	1000
0.91	0.01	0.02	0.00	0.00	0.00	0.15	0.00	Fisher(glm)	1000	1000
0.95	0.01	0.21	0.04	1.53	0.08	0.08	0.00	Nelder-Mead	1000	1000
0.92	0.01	0.02	0.00	0.00	0.00	0.15	0.00	Poisson(log)	1000	1000
0.91	0.01	0.02	0.00	0.00	0.00	0.14	0.00	squadP	1000	1000
0.63	0.01	0.02	0.00	0.00	0.00	0.04	0.00	EM	10000	1000
0.96	0.01	0.03	0.01	0.14	0.01	0.02	0.00	BFGS	10000	993
0.63	0.01	0.02	0.00	0.00	0.00	0.04	0.00	Fisher(glm)	10000	1000
0.95	0.01	0.03	0.01	0.14	0.01	0.02	0.00	Nelder-Mead	10000	1000
0.67	0.01	0.02	0.00	0.00	0.00	0.04	0.00	Poisson(log)	10000	1000
0.63	0.01	0.02	0.00	0.00	0.00	0.04	0.00	squadP	10000	1000

8.3.4 Scenarios with event probability 24% and 2 covariates

The performance measurments (with Monte Carlo standerd error) such as bias, coverage prababilty, MSE, EmpSE and convergence for the intercept, coef.1, and coef.2 are shwon in figures 46, 47, and tables 62, 63, and 64 for the intercept, coef.1, and coef.2.

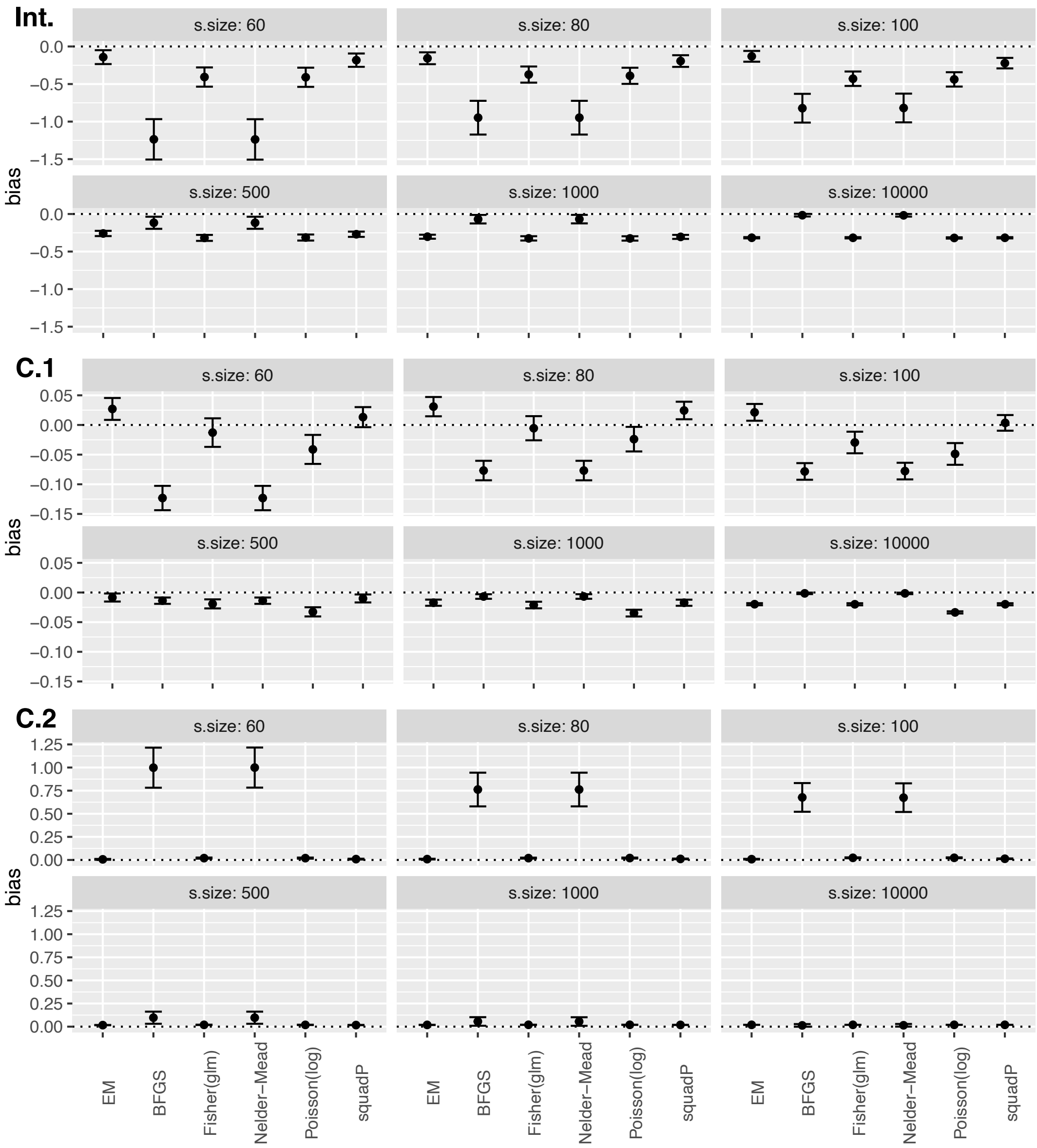

Figure 46: Absolute bias for scenarios 85→90 of the estimated log(RR) from the six methods under model misspecifications. Scenarios are with event probability 24% and sample sizes 60, 80, 100, 500, 1000, and 10000. y-axis: biases for coeffecients intercept, C.1, and C.2. x-axis: The six statistical methods compared for each scenario.

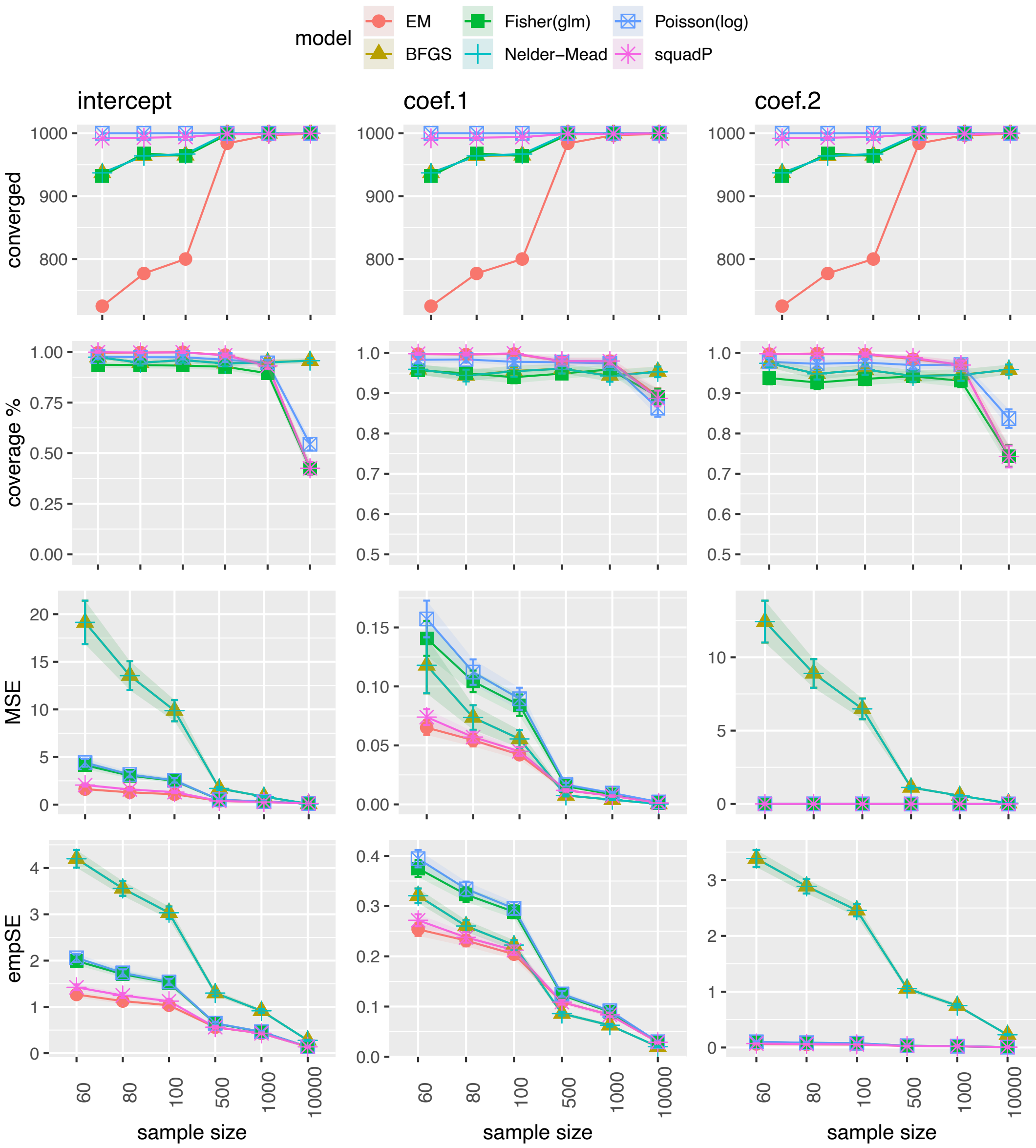

Figure 47: Performance measurments of scenarios 85→90 of the estimated log(RR) from the six methods under model misspecifications and event probability 24%. Measurments are convergence rate, coverage prabability, MSE, and EmpSE (on y-axis) for the intercept, coef.1, and coef.2 using the six statistical methods (legend in the top of the figure). Each type of shapes with shaded earia (confidence interval) is represnting a different method. The measurments are done for 6 scenarios that are derived from six different sample sizes (60,80,100,500,1000,10000) on x-axis.

Table 62: Performance measurments of scenarios 85→90 for intercept

Covarage		Bias		MSE		EmpSE				
cover.	mcse	bias	mcse	mse	mcse	empse	mcse	model	s.size	converged
0.996	0.002	-0.142	0.047	1.626	0.080	1.268	0.033	EM	60	725
0.974	0.005	-1.236	0.137	19.134	1.164	4.198	0.097	BFGS	60	937
0.937	0.008	-0.407	0.065	4.136	0.221	1.994	0.046	Fisher(glm)	60	932
0.974	0.005	-1.237	0.137	19.144	1.165	4.199	0.097	Nelder-Mead	60	937
0.977	0.005	-0.410	0.065	4.407	0.219	2.060	0.046	Poisson(log)	60	1000
0.999	0.001	-0.182	0.045	2.056	0.084	1.423	0.032	squadP	60	992
0.997	0.002	-0.157	0.040	1.291	0.059	1.126	0.029	EM	80	777
0.947	0.007	-0.948	0.115	13.553	0.776	3.559	0.081	BFGS	80	964
0.935	0.008	-0.374	0.055	3.050	0.143	1.707	0.039	Fisher(glm)	80	968
0.947	0.007	-0.948	0.115	13.548	0.776	3.558	0.081	Nelder-Mead	80	964
0.974	0.005	-0.390	0.055	3.161	0.152	1.735	0.039	Poisson(log)	80	1000
0.997	0.002	-0.194	0.040	1.592	0.058	1.247	0.028	squadP	80	993
0.999	0.001	-0.131	0.037	1.089	0.047	1.036	0.026	EM	100	800
0.960	0.006	-0.822	0.098	9.874	0.569	3.035	0.069	BFGS	100	965
0.933	0.008	-0.429	0.049	2.503	0.125	1.524	0.035	Fisher(glm)	100	964
0.960	0.006	-0.819	0.098	9.866	0.568	3.034	0.069	Nelder-Mead	100	967
0.974	0.005	-0.438	0.049	2.558	0.126	1.539	0.034	Poisson(log)	100	1000
0.998	0.001	-0.222	0.036	1.313	0.047	1.125	0.025	squadP	100	994
0.985	0.004	-0.259	0.018	0.381	0.014	0.560	0.013	EM	500	984
0.944	0.007	-0.117	0.041	1.702	0.081	1.300	0.029	BFGS	500	1000
0.927	0.008	-0.318	0.020	0.511	0.024	0.640	0.014	Fisher(glm)	500	998
0.943	0.007	-0.117	0.041	1.701	0.081	1.300	0.029	Nelder-Mead	500	1000
0.962	0.006	-0.313	0.020	0.517	0.024	0.647	0.014	Poisson(log)	500	1000
0.984	0.004	-0.270	0.018	0.389	0.014	0.563	0.013	squadP	500	999
0.935	0.008	-0.302	0.014	0.275	0.010	0.429	0.010	EM	1000	997
0.950	0.007	-0.069	0.029	0.842	0.037	0.916	0.020	BFGS	1000	1000
0.895	0.010	-0.324	0.015	0.319	0.013	0.463	0.010	Fisher(glm)	1000	1000
0.948	0.007	-0.068	0.029	0.844	0.037	0.916	0.021	Nelder-Mead	1000	1000
0.946	0.007	-0.325	0.015	0.322	0.014	0.465	0.010	Poisson(log)	1000	1000
0.931	0.008	-0.305	0.014	0.278	0.010	0.431	0.010	squadP	1000	999
0.425	0.016	-0.316	0.004	0.119	0.003	0.138	0.003	EM	10000	999
0.957	0.006	-0.016	0.009	0.078	0.004	0.279	0.006	BFGS	10000	1000
0.425	0.016	-0.317	0.004	0.119	0.003	0.139	0.003	Fisher(glm)	10000	1000
0.957	0.006	-0.018	0.009	0.078	0.004	0.279	0.006	Nelder-Mead	10000	1000
0.544	0.016	-0.319	0.004	0.121	0.003	0.140	0.003	Poisson(log)	10000	1000
0.425	0.016	-0.317	0.004	0.119	0.003	0.138	0.003	squadP	10000	1000

Table 63: Performance measurments of scenarios 85→90 for coef.1

Covarage		Bias		MSE		EmpSE				
cover.	mcse	bias	mcse	mse	mcse	empse	mcse	model	s.size	converged
0.997	0.002	0.027	0.009	0.065	0.003	0.254	0.007	EM	60	725
0.959	0.006	-0.123	0.010	0.118	0.012	0.321	0.007	BFGS	60	937
0.957	0.007	-0.013	0.012	0.141	0.008	0.375	0.009	Fisher(glm)	60	932
0.959	0.006	-0.123	0.010	0.118	0.012	0.321	0.007	Nelder-Mead	60	937
0.983	0.004	-0.041	0.012	0.157	0.008	0.395	0.009	Poisson(log)	60	1000
0.998	0.001	0.013	0.009	0.074	0.003	0.272	0.006	squadP	60	992
0.996	0.002	0.031	0.008	0.054	0.003	0.231	0.006	EM	80	777
0.944	0.007	-0.077	0.008	0.074	0.005	0.260	0.006	BFGS	80	964
0.949	0.007	-0.006	0.010	0.104	0.005	0.323	0.007	Fisher(glm)	80	968
0.944	0.007	-0.077	0.008	0.074	0.005	0.260	0.006	Nelder-Mead	80	964
0.984	0.004	-0.024	0.011	0.112	0.005	0.334	0.007	Poisson(log)	80	1000
0.997	0.002	0.024	0.008	0.057	0.002	0.238	0.005	squadP	80	993
0.998	0.002	0.021	0.007	0.042	0.002	0.204	0.005	EM	100	800
0.955	0.007	-0.078	0.007	0.056	0.004	0.222	0.005	BFGS	100	965
0.940	0.008	-0.030	0.009	0.084	0.005	0.289	0.007	Fisher(glm)	100	964
0.954	0.007	-0.078	0.007	0.056	0.004	0.223	0.005	Nelder-Mead	100	967
0.978	0.005	-0.049	0.009	0.090	0.005	0.295	0.007	Poisson(log)	100	1000
0.999	0.001	0.004	0.007	0.045	0.002	0.212	0.005	squadP	100	994
0.980	0.004	-0.008	0.003	0.012	0.001	0.109	0.002	EM	500	984
0.961	0.006	-0.014	0.003	0.008	0.000	0.086	0.002	BFGS	500	1000
0.949	0.007	-0.019	0.004	0.015	0.001	0.123	0.003	Fisher(glm)	500	998
0.961	0.006	-0.014	0.003	0.008	0.000	0.086	0.002	Nelder-Mead	500	1000
0.977	0.005	-0.033	0.004	0.017	0.001	0.125	0.003	Poisson(log)	500	1000
0.980	0.004	-0.010	0.003	0.012	0.001	0.109	0.002	squadP	500	999
0.980	0.004	-0.017	0.003	0.007	0.000	0.084	0.002	EM	1000	997
0.944	0.007	-0.007	0.002	0.004	0.000	0.063	0.001	BFGS	1000	1000
0.959	0.006	-0.021	0.003	0.008	0.000	0.090	0.002	Fisher(glm)	1000	1000
0.943	0.007	-0.007	0.002	0.004	0.000	0.063	0.001	Nelder-Mead	1000	1000
0.974	0.005	-0.035	0.003	0.010	0.000	0.092	0.002	Poisson(log)	1000	1000
0.980	0.004	-0.017	0.003	0.007	0.000	0.084	0.002	squadP	1000	999
0.893	0.010	-0.020	0.001	0.001	0.000	0.029	0.001	EM	10000	999
0.954	0.007	-0.001	0.001	0.000	0.000	0.020	0.000	BFGS	10000	1000
0.892	0.010	-0.020	0.001	0.001	0.000	0.029	0.001	Fisher(glm)	10000	1000
0.953	0.007	-0.002	0.001	0.000	0.000	0.020	0.000	Nelder-Mead	10000	1000
0.863	0.011	-0.034	0.001	0.002	0.000	0.030	0.001	Poisson(log)	10000	1000
0.887	0.010	-0.020	0.001	0.001	0.000	0.029	0.001	squadP	10000	1000

Table 64: Performance measurments of scenarios 85→90 for coef.2

Covarage		Bias		MSE		EmpSE				
cover.	mcse	bias	mcse	mse	mcse	empse	mcse	model	s.size	converged
0.997	0.002	-0.003	0.002	0.004	0.000	0.254	0.007	EM	60	725
0.974	0.005	0.988	0.111	12.426	0.729	0.321	0.007	BFGS	60	937
0.938	0.008	0.010	0.003	0.010	0.001	0.375	0.009	Fisher(glm)	60	932
0.974	0.005	0.989	0.111	12.432	0.729	0.321	0.007	Nelder-Mead	60	937
0.978	0.005	0.010	0.003	0.010	0.001	0.395	0.009	Poisson(log)	60	1000
0.998	0.001	-0.001	0.002	0.005	0.000	0.272	0.006	squadP	60	992
0.999	0.001	-0.001	0.002	0.003	0.000	0.231	0.006	EM	80	777
0.948	0.007	0.752	0.093	8.901	0.497	0.260	0.006	BFGS	80	964
0.927	0.008	0.010	0.003	0.007	0.000	0.323	0.007	Fisher(glm)	80	968
0.948	0.007	0.752	0.093	8.898	0.497	0.260	0.006	Nelder-Mead	80	964
0.973	0.005	0.011	0.003	0.007	0.000	0.334	0.007	Poisson(log)	80	1000
0.997	0.002	0.001	0.002	0.004	0.000	0.238	0.005	squadP	80	993
0.996	0.002	-0.002	0.002	0.003	0.000	0.204	0.005	EM	100	800
0.959	0.006	0.667	0.079	6.487	0.361	0.222	0.005	BFGS	100	965
0.936	0.008	0.013	0.002	0.006	0.000	0.289	0.007	Fisher(glm)	100	964
0.959	0.006	0.664	0.079	6.481	0.360	0.223	0.005	Nelder-Mead	100	967
0.976	0.005	0.013	0.002	0.006	0.000	0.295	0.007	Poisson(log)	100	1000
0.997	0.002	0.003	0.002	0.003	0.000	0.212	0.005	squadP	100	994
0.985	0.004	0.006	0.001	0.001	0.000	0.109	0.002	EM	500	984
0.943	0.007	0.087	0.033	1.125	0.053	0.086	0.002	BFGS	500	1000
0.942	0.007	0.009	0.001	0.001	0.000	0.123	0.003	Fisher(glm)	500	998
0.943	0.007	0.087	0.033	1.125	0.053	0.086	0.002	Nelder-Mead	500	1000
0.970	0.005	0.009	0.001	0.001	0.000	0.125	0.003	Poisson(log)	500	1000
0.988	0.003	0.007	0.001	0.001	0.000	0.109	0.002	squadP	500	999
0.971	0.005	0.009	0.001	0.001	0.000	0.084	0.002	EM	1000	997
0.946	0.007	0.046	0.024	0.564	0.025	0.063	0.001	BFGS	1000	1000
0.931	0.008	0.010	0.001	0.001	0.000	0.090	0.002	Fisher(glm)	1000	1000
0.945	0.007	0.045	0.024	0.565	0.025	0.063	0.001	Nelder-Mead	1000	1000
0.971	0.005	0.010	0.001	0.001	0.000	0.092	0.002	Poisson(log)	1000	1000
0.970	0.005	0.009	0.001	0.001	0.000	0.084	0.002	squadP	1000	999
0.745	0.014	0.010	0.000	0.000	0.000	0.029	0.001	EM	10000	999
0.958	0.006	0.003	0.007	0.052	0.002	0.020	0.000	BFGS	10000	1000
0.744	0.014	0.010	0.000	0.000	0.000	0.029	0.001	Fisher(glm)	10000	1000
0.959	0.006	0.005	0.007	0.052	0.002	0.020	0.000	Nelder-Mead	10000	1000
0.837	0.012	0.010	0.000	0.000	0.000	0.030	0.001	Poisson(log)	10000	1000
0.743	0.014	0.010	0.000	0.000	0.000	0.029	0.001	squadP	10000	1000

8.3.5 Scenarios with event probability 48% and 2 covariates

The performance measurments (with Monte Carlo standerd error) such as bias, coverage prababilty, MSE, EmpSE and convergence for the intercept, coef.1, and coef.2 are shwon in figures 48, 49, and tables 65, 66, 67 for the intercept, coef.1, and coef.2.

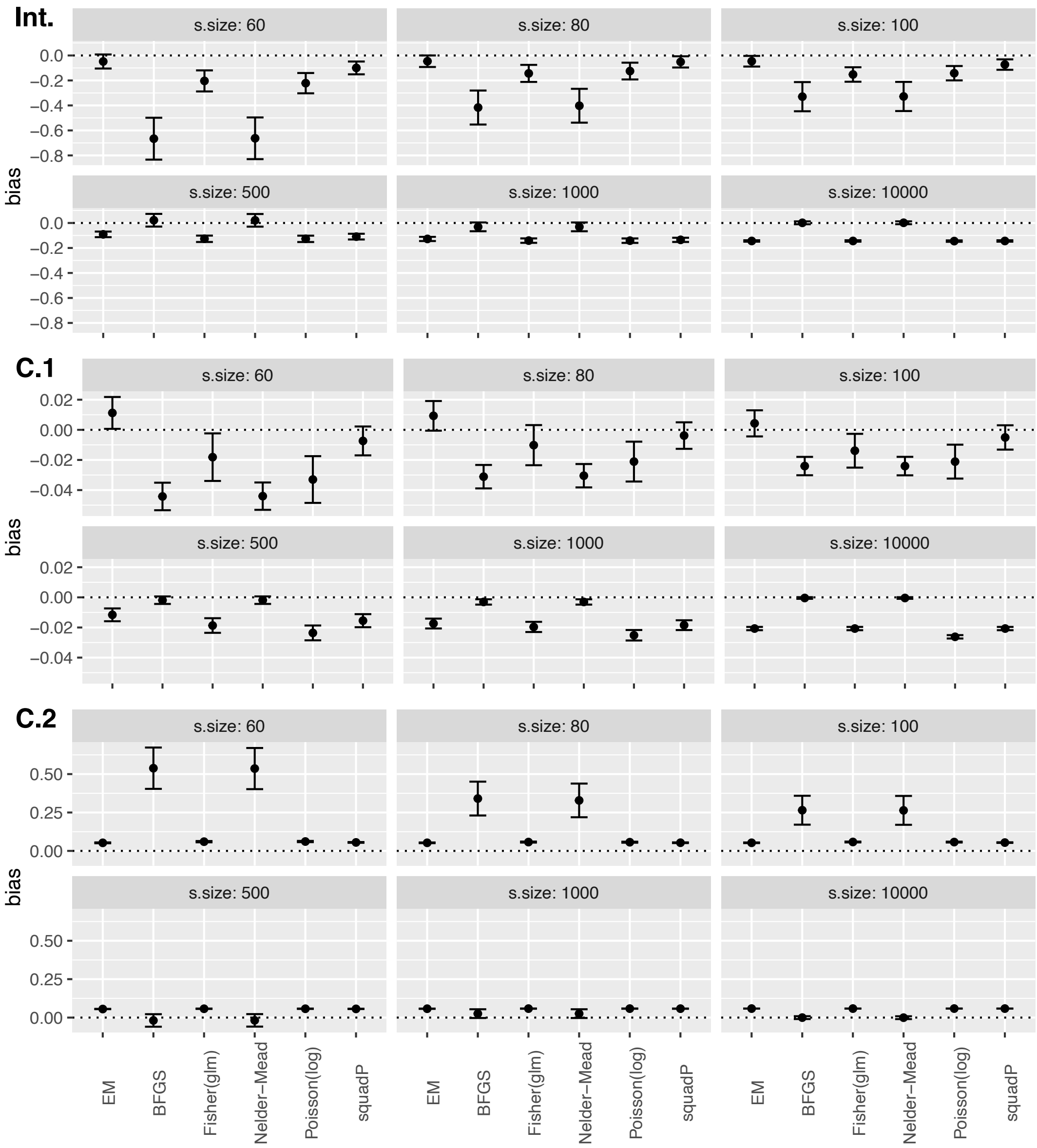

Figure 48: Absolute bias for scenarios 91→96 of the estimated log(RR) from the six methods under model misspecifications. Scenarios are with event probability 48% and sample sizes 60, 80, 100, 500, 1000, and 10000. y-axis: biases for coeffecients intercept, C.1, and C.2. x-axis: The six statistical methods compared for each scenario.

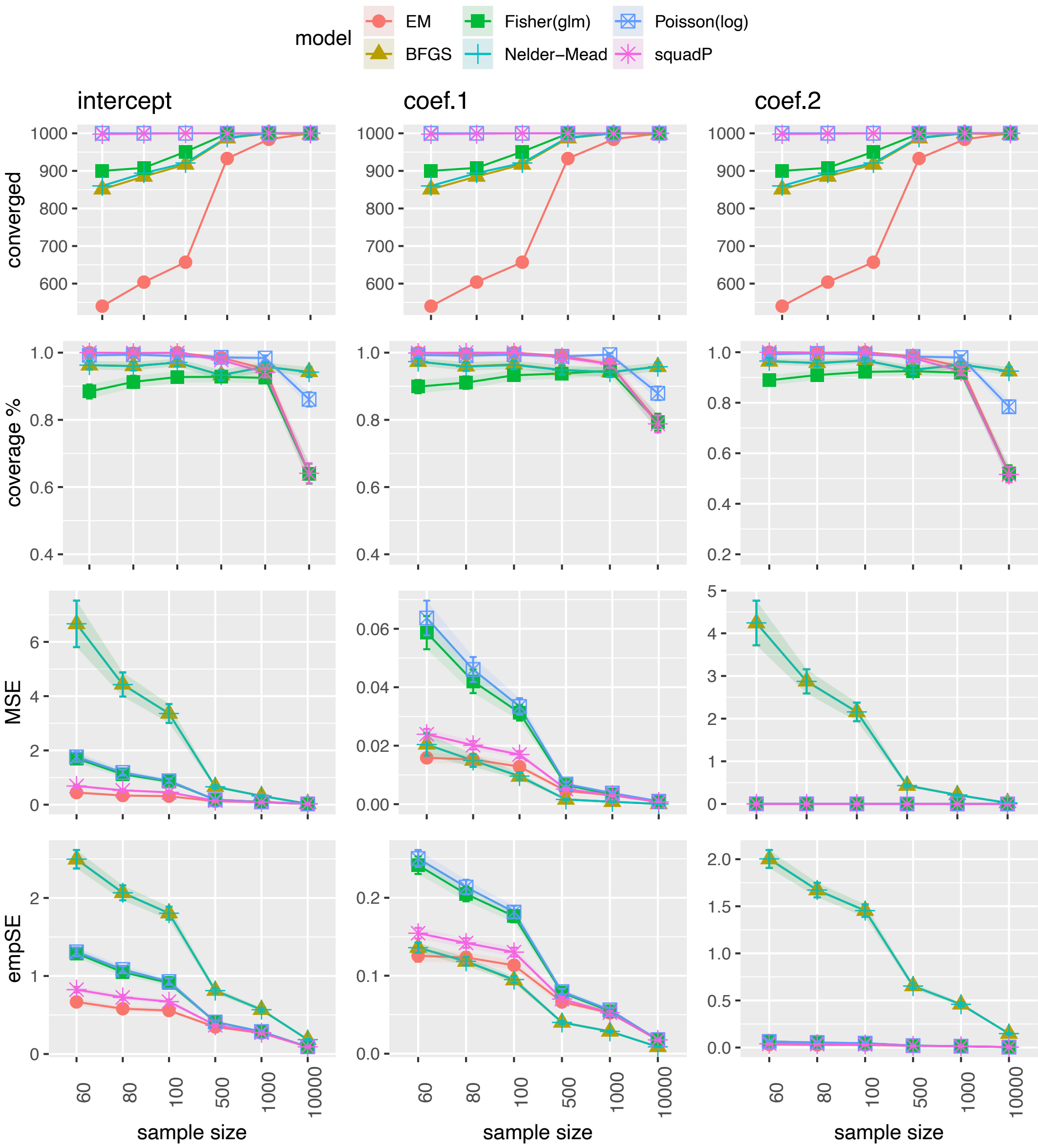

Figure 49: Performance measurments of scenarios 91→96 of the estimated log(RR) from the six methods under model misspecifications and event probability 48%. Measurments are convergence rate, coverage prababilty, MSE, and EmpSE (on y-axis) for the intercept, coef.1, and coef.2 using the six statistical methods (legend in the top of the figure). Each type of shapes with shaded earia (confidence interval) is represnting a different method. The measurments are done for 6 scenarios that are derived from six different sample sizes (60,80,100,500,1000,10000) on x-axis.

Table 65: Performance measurments of scenarios 91→96 for intercept

Covarage		Bias		MSE		EmpSE				
cover.	mcse	bias	mcse	mse	mcse	empse	mcse	model	s.size	converged
1.000	0.000	-0.048	0.029	0.447	0.031	0.667	0.020	EM	60	540
0.962	0.007	-0.666	0.086	6.661	0.438	2.495	0.061	BFGS	60	851
0.884	0.011	-0.204	0.043	1.704	0.090	1.290	0.030	Fisher(glm)	60	900
0.963	0.006	-0.663	0.085	6.666	0.436	2.497	0.060	Nelder-Mead	60	860
0.992	0.003	-0.222	0.041	1.762	0.091	1.309	0.029	Poisson(log)	60	1000
1.000	0.000	-0.100	0.026	0.689	0.028	0.825	0.018	squadP	60	998
0.998	0.002	-0.046	0.024	0.338	0.018	0.580	0.017	EM	80	604
0.960	0.007	-0.417	0.069	4.435	0.228	2.066	0.049	BFGS	80	885
0.913	0.009	-0.144	0.035	1.121	0.056	1.049	0.025	Fisher(glm)	80	908
0.960	0.007	-0.402	0.069	4.426	0.226	2.066	0.049	Nelder-Mead	80	894
0.994	0.002	-0.125	0.034	1.188	0.056	1.083	0.024	Poisson(log)	80	1000
0.999	0.001	-0.051	0.023	0.529	0.019	0.726	0.016	squadP	80	999
1.000	0.000	-0.046	0.022	0.312	0.015	0.557	0.015	EM	100	657
0.971	0.006	-0.330	0.060	3.362	0.178	1.805	0.042	BFGS	100	917
0.927	0.008	-0.152	0.030	0.853	0.038	0.911	0.021	Fisher(glm)	100	951
0.971	0.006	-0.328	0.059	3.360	0.177	1.804	0.042	Nelder-Mead	100	921
0.990	0.003	-0.142	0.029	0.881	0.040	0.929	0.021	Poisson(log)	100	1000
0.999	0.001	-0.073	0.021	0.455	0.016	0.671	0.015	squadP	100	1000
0.985	0.004	-0.091	0.011	0.129	0.005	0.347	0.008	EM	500	933
0.933	0.008	0.022	0.026	0.661	0.030	0.813	0.018	BFGS	500	987
0.928	0.008	-0.127	0.013	0.185	0.009	0.412	0.009	Fisher(glm)	500	999
0.933	0.008	0.021	0.026	0.660	0.030	0.812	0.018	Nelder-Mead	500	989
0.986	0.004	-0.127	0.013	0.184	0.008	0.410	0.009	Poisson(log)	500	1000
0.978	0.005	-0.109	0.012	0.148	0.005	0.369	0.008	squadP	500	1000
0.951	0.007	-0.128	0.009	0.089	0.004	0.269	0.006	EM	1000	984
0.958	0.006	-0.031	0.018	0.323	0.014	0.568	0.013	BFGS	1000	1000
0.925	0.008	-0.141	0.009	0.101	0.005	0.286	0.006	Fisher(glm)	1000	1000
0.958	0.006	-0.031	0.018	0.323	0.014	0.568	0.013	Nelder-Mead	1000	1000
0.984	0.004	-0.142	0.009	0.103	0.005	0.288	0.006	Poisson(log)	1000	1000
0.941	0.007	-0.135	0.009	0.093	0.004	0.273	0.006	squadP	1000	1000
0.640	0.015	-0.144	0.003	0.029	0.001	0.092	0.002	EM	10000	1000
0.943	0.007	0.001	0.006	0.034	0.001	0.184	0.004	BFGS	10000	997
0.640	0.015	-0.144	0.003	0.029	0.001	0.092	0.002	Fisher(glm)	10000	1000
0.942	0.007	0.002	0.006	0.034	0.001	0.184	0.004	Nelder-Mead	10000	1000
0.861	0.011	-0.145	0.003	0.030	0.001	0.093	0.002	Poisson(log)	10000	1000
0.641	0.015	-0.144	0.003	0.029	0.001	0.092	0.002	squadP	10000	1000

Table 66: Performance measurments of scenarios 91→96 for coef.1

Covarage		Bias		MSE		EmpSE				
cover.	mcse	bias	mcse	mse	mcse	empse	mcse	model	s.size	converged
1.000	0.000	0.011	0.005	0.016	0.001	0.126	0.004	EM	60	540
0.973	0.006	-0.044	0.005	0.020	0.002	0.136	0.003	BFGS	60	851
0.899	0.010	-0.018	0.008	0.059	0.003	0.242	0.006	Fisher(glm)	60	900
0.973	0.006	-0.044	0.005	0.020	0.002	0.136	0.003	Nelder-Mead	60	860
0.993	0.003	-0.033	0.008	0.064	0.003	0.250	0.006	Poisson(log)	60	1000
0.999	0.001	-0.007	0.005	0.024	0.001	0.155	0.003	squadP	60	998
0.997	0.002	0.009	0.005	0.015	0.001	0.123	0.004	EM	80	604
0.959	0.007	-0.031	0.004	0.015	0.001	0.118	0.003	BFGS	80	885
0.911	0.009	-0.010	0.007	0.042	0.002	0.205	0.005	Fisher(glm)	80	908
0.959	0.007	-0.030	0.004	0.015	0.001	0.118	0.003	Nelder-Mead	80	894
0.991	0.003	-0.021	0.007	0.046	0.002	0.214	0.005	Poisson(log)	80	1000
1.000	0.000	-0.004	0.004	0.020	0.001	0.142	0.003	squadP	80	999
1.000	0.000	0.004	0.004	0.013	0.001	0.113	0.003	EM	100	657
0.965	0.006	-0.024	0.003	0.009	0.001	0.094	0.002	BFGS	100	917
0.933	0.008	-0.014	0.006	0.031	0.001	0.176	0.004	Fisher(glm)	100	951
0.963	0.006	-0.024	0.003	0.010	0.001	0.095	0.002	Nelder-Mead	100	921
0.994	0.002	-0.021	0.006	0.033	0.001	0.182	0.004	Poisson(log)	100	1000
1.000	0.000	-0.005	0.004	0.017	0.001	0.130	0.003	squadP	100	1000
0.990	0.003	-0.012	0.002	0.005	0.000	0.066	0.002	EM	500	933
0.948	0.007	-0.002	0.001	0.002	0.000	0.040	0.001	BFGS	500	987
0.938	0.008	-0.019	0.002	0.006	0.000	0.078	0.002	Fisher(glm)	500	999
0.948	0.007	-0.002	0.001	0.002	0.000	0.040	0.001	Nelder-Mead	500	989
0.989	0.003	-0.024	0.003	0.007	0.000	0.080	0.002	Poisson(log)	500	1000
0.986	0.004	-0.015	0.002	0.005	0.000	0.070	0.002	squadP	500	1000
0.966	0.006	-0.017	0.002	0.003	0.000	0.052	0.001	EM	1000	984
0.942	0.007	-0.003	0.001	0.001	0.000	0.029	0.001	BFGS	1000	1000
0.946	0.007	-0.020	0.002	0.003	0.000	0.055	0.001	Fisher(glm)	1000	1000
0.942	0.007	-0.003	0.001	0.001	0.000	0.029	0.001	Nelder-Mead	1000	1000
0.994	0.002	-0.025	0.002	0.004	0.000	0.056	0.001	Poisson(log)	1000	1000
0.966	0.006	-0.018	0.002	0.003	0.000	0.053	0.001	squadP	1000	1000
0.793	0.013	-0.021	0.001	0.001	0.000	0.018	0.000	EM	10000	1000
0.958	0.006	0.000	0.000	0.000	0.000	0.009	0.000	BFGS	10000	997
0.793	0.013	-0.021	0.001	0.001	0.000	0.018	0.000	Fisher(glm)	10000	1000
0.958	0.006	0.000	0.000	0.000	0.000	0.009	0.000	Nelder-Mead	10000	1000
0.879	0.010	-0.026	0.001	0.001	0.000	0.018	0.000	Poisson(log)	10000	1000
0.788	0.013	-0.021	0.001	0.001	0.000	0.018	0.000	squadP	10000	1000

Table 67: Performance measurments of scenarios 91→96 for coef.2

Covarage		Bias		MSE		EmpSE				
cover.	mcse	bias	mcse	mse	mcse	empse	mcse	model	s.size	converged
1.000	0.000	0.003	0.001	0.001	0.000	0.126	0.004	EM	60	540
0.964	0.006	0.488	0.069	4.239	0.267	0.136	0.003	BFGS	60	851
0.889	0.010	0.010	0.002	0.004	0.000	0.242	0.006	Fisher(glm)	60	900
0.965	0.006	0.486	0.068	4.244	0.266	0.136	0.003	Nelder-Mead	60	860
0.992	0.003	0.011	0.002	0.004	0.000	0.250	0.006	Poisson(log)	60	1000
1.000	0.000	0.005	0.001	0.002	0.000	0.155	0.003	squadP	60	998
0.998	0.002	0.003	0.001	0.001	0.000	0.123	0.004	EM	80	604
0.958	0.007	0.291	0.056	2.876	0.145	0.118	0.003	BFGS	80	885
0.910	0.010	0.007	0.002	0.003	0.000	0.205	0.005	Fisher(glm)	80	908
0.956	0.007	0.279	0.056	2.872	0.144	0.118	0.003	Nelder-Mead	80	894
0.995	0.002	0.007	0.002	0.003	0.000	0.214	0.005	Poisson(log)	80	1000
0.998	0.001	0.003	0.001	0.001	0.000	0.142	0.003	squadP	80	999
1.000	0.000	0.003	0.001	0.001	0.000	0.113	0.003	EM	100	657
0.967	0.006	0.215	0.048	2.157	0.111	0.094	0.002	BFGS	100	917
0.922	0.009	0.008	0.001	0.002	0.000	0.176	0.004	Fisher(glm)	100	951
0.967	0.006	0.214	0.048	2.158	0.111	0.095	0.002	Nelder-Mead	100	921
0.991	0.003	0.007	0.001	0.002	0.000	0.182	0.004	Poisson(log)	100	1000
0.998	0.001	0.004	0.001	0.001	0.000	0.130	0.003	squadP	100	1000
0.984	0.004	0.006	0.001	0.000	0.000	0.066	0.002	EM	500	933
0.930	0.008	-0.069	0.021	0.431	0.019	0.040	0.001	BFGS	500	987
0.925	0.008	0.007	0.001	0.000	0.000	0.078	0.002	Fisher(glm)	500	999
0.930	0.008	-0.068	0.021	0.431	0.019	0.040	0.001	Nelder-Mead	500	989
0.983	0.004	0.007	0.001	0.000	0.000	0.080	0.002	Poisson(log)	500	1000
0.977	0.005	0.007	0.001	0.000	0.000	0.070	0.002	squadP	500	1000
0.943	0.007	0.008	0.000	0.000	0.000	0.052	0.001	EM	1000	984
0.951	0.007	-0.024	0.015	0.211	0.009	0.029	0.001	BFGS	1000	1000
0.919	0.009	0.008	0.000	0.000	0.000	0.055	0.001	Fisher(glm)	1000	1000
0.951	0.007	-0.024	0.015	0.211	0.009	0.029	0.001	Nelder-Mead	1000	1000
0.979	0.005	0.008	0.000	0.000	0.000	0.056	0.001	Poisson(log)	1000	1000
0.925	0.008	0.008	0.000	0.000	0.000	0.053	0.001	squadP	1000	1000
0.521	0.016	0.009	0.000	0.000	0.000	0.018	0.000	EM	10000	1000
0.925	0.008	-0.050	0.005	0.024	0.001	0.009	0.000	BFGS	10000	997
0.521	0.016	0.009	0.000	0.000	0.000	0.018	0.000	Fisher(glm)	10000	1000
0.925	0.008	-0.051	0.005	0.024	0.001	0.009	0.000	Nelder-Mead	10000	1000
0.784	0.013	0.009	0.000	0.000	0.000	0.018	0.000	Poisson(log)	10000	1000
0.516	0.016	0.009	0.000	0.000	0.000	0.018	0.000	squadP	10000	1000

8.3.6 Scenarios with event probability 12% and 8 covariates

The performance measurments (with Monte Carlo standerd error) such as bias, coverage prababilty, MSE, EmpSE and convergence are shwon in figures 50, and 51.

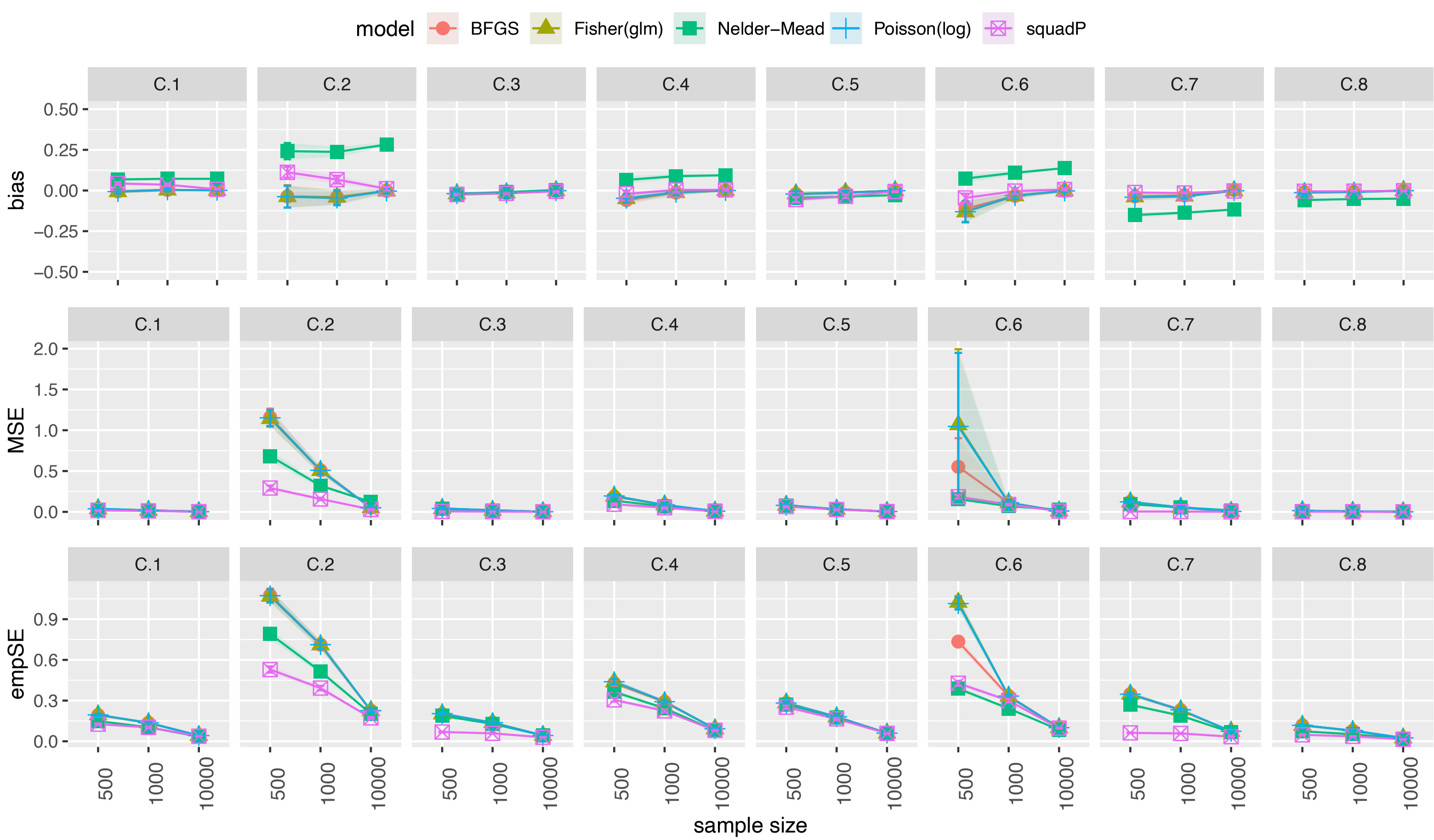

Figure 50: Absolute bias, MSE, and empSE of scenarios 97→99 of the estimated log(RR) from the five methods under model misspecifications and event probability 12%. Performance measurments for the eight coefficents using the six statistical methods (legend in the top of the figure) are on y-axis. The measurments are done for 3 scenarios that are derived from three different sample sizes (500,1000,10000) on x-axis. Each type of shapes with line is represnting a different method.

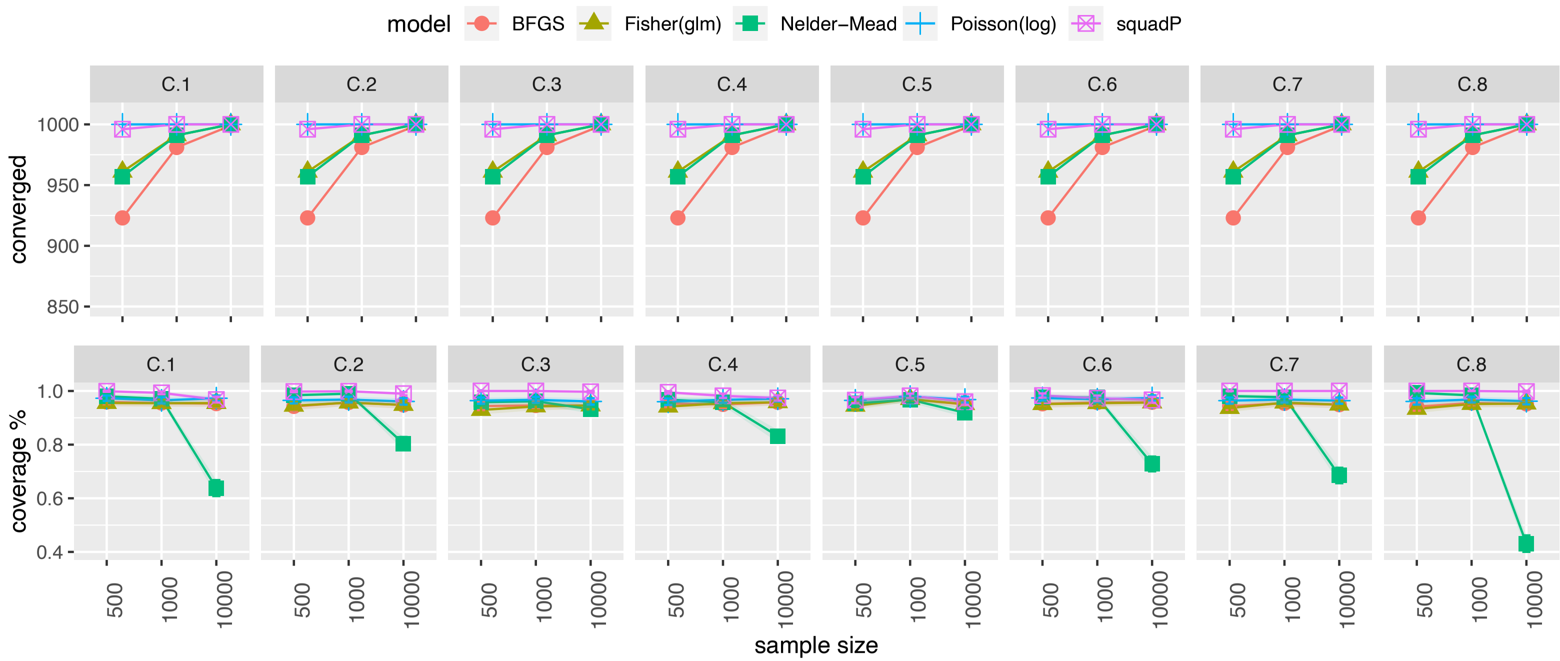

Figure 51: Covarege probabiltiy, and convergence of scenarios 97→99 of the estimated log(RR) from the five methods under model misspecifications and event probability 12%. Performance measurments for the eight coefficents using the six statistical methods (legend in the top of the figure) are on y-axis. The measurments are done for 3 scenarios that are derived from three different sample sizes (500,1000,10000) on x-axis. Each type of shapes with line is represnting a different method.

8.3.7 Scenarios with event probability 24% and 8 covariates

The performance measurments (with Monte Carlo standerd error) such as bias, coverage prababilty, MSE, EmpSE and convergence are shwon in figures 52, and 53.

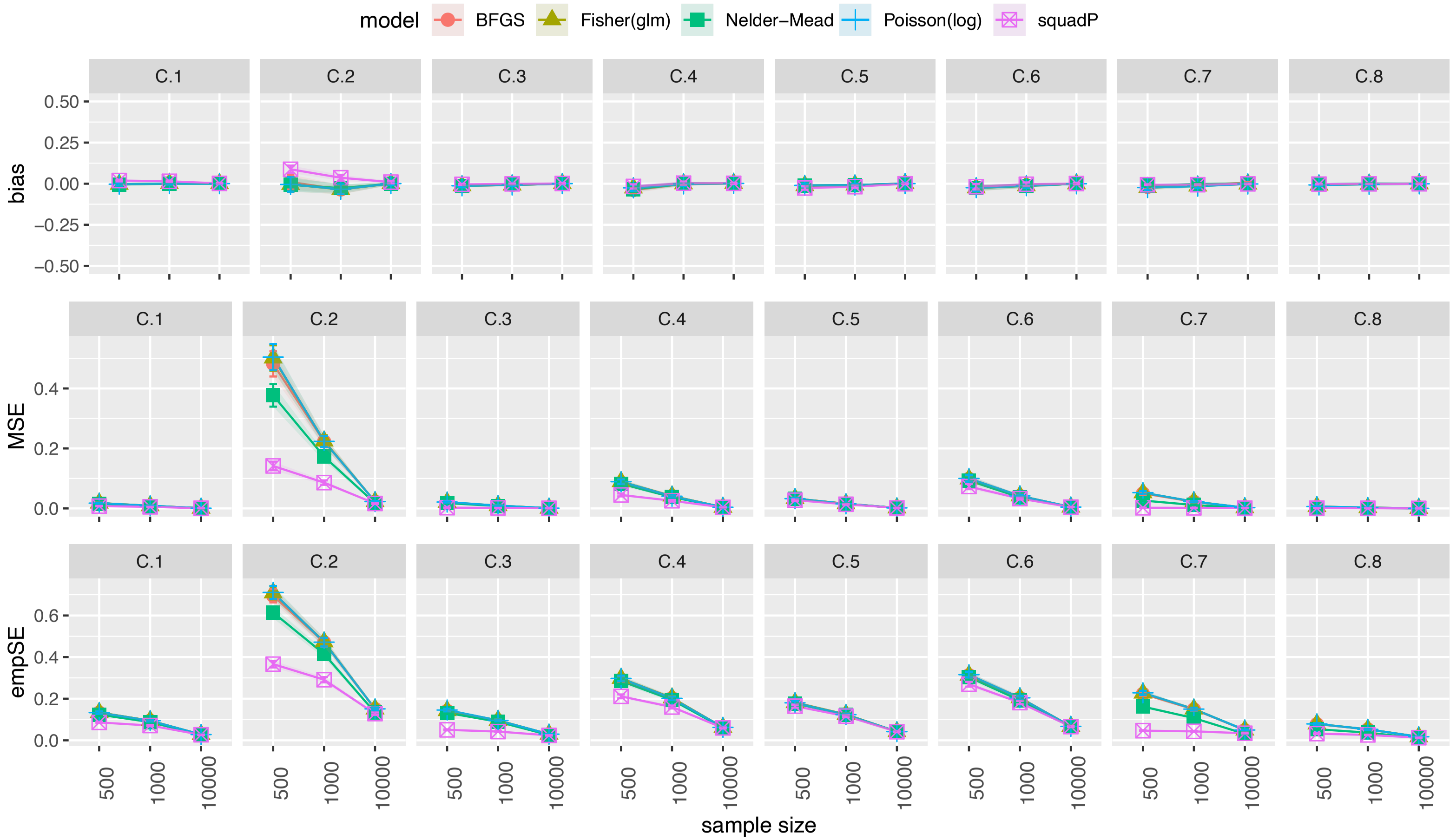

Figure 52: Absolute bias, MSE, and empSE of scenarios 100→102 of the estimated log(RR) from the five methods under model misspecifications and event probability 24%. Performance measurments for the eight coefficents using the six statistical methods (legend in the top of the figure) are on y-axis. The measurments are done for 3 scenarios that are derived from three different sample sizes (500,1000,10000) on x-axis. Each type of shapes with line is represnting a different method.

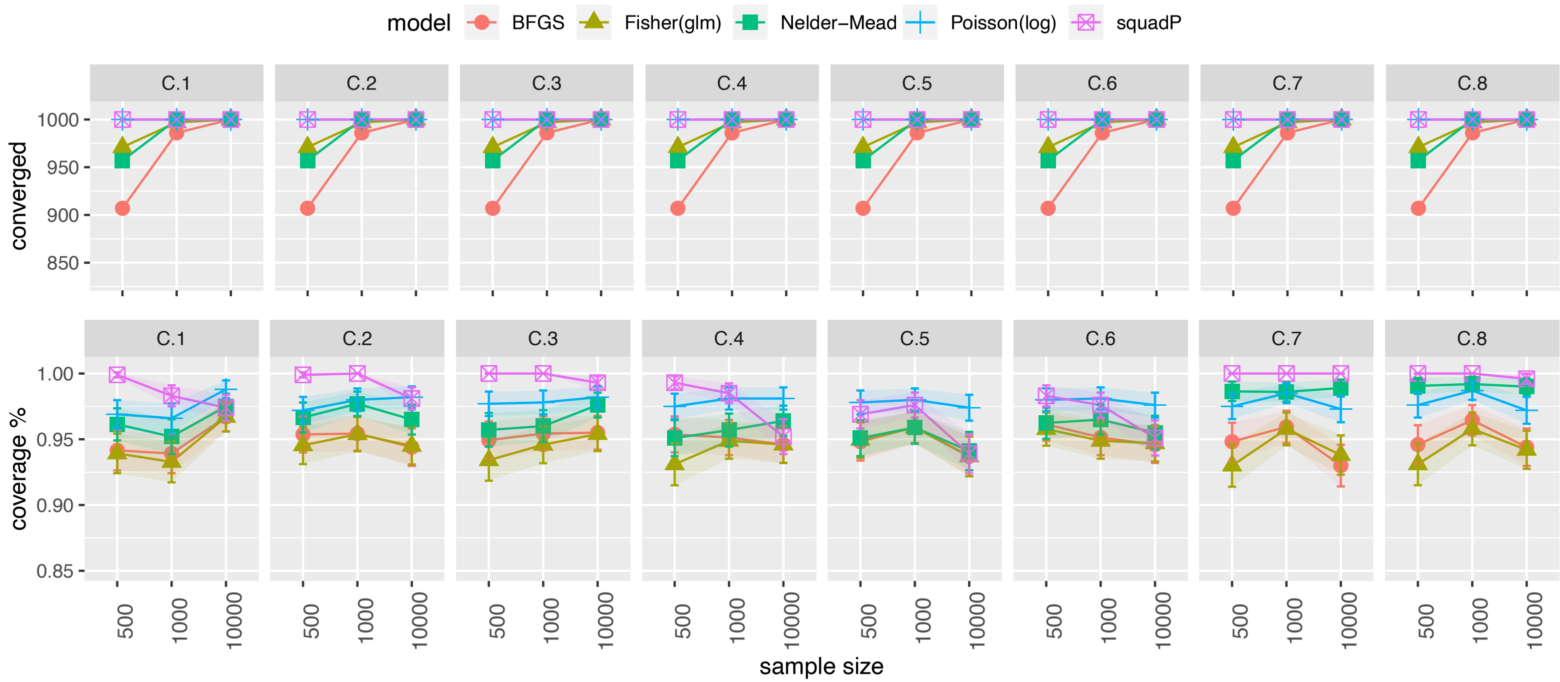

Figure 53: Covarege probabiltiy, and convergence of scenarios 100→102 of the estimated log(RR) from the five methods under model misspecifications and event probability 24%. Performance measurments for the eight coefficents using the six statistical methods (legend in the top of the figure) are on y-axis. The measurments are done for 3 scenarios that are derived from three different sample sizes (500,1000,10000) on x-axis. Each type of shapes with line is represnting a different method.

8.3.8 Scenarios with event probability 48% and 8 covariates

The performance measurments (with Monte Carlo standerd error) such as bias, coverage prababilty, MSE, EmpSE and convergence are shwon in figures 54, and 55.

Summary results of measurments for all scenarios with 8 covariates and event propabilities $12\%, 24\%, 48\%$ under model misspecifications are shown in tables 68, 69, 70, 71, 72, 73, 74, and 75 for coefficents C.1, C.2, C.3, C.4, C.5, C.6, C.7, and C.8.

Table 68: Performance measurments of scenarios 97→105 for coef.1

Covarage		Bias		MSE					
cover	mcse	bias	mcse	mse	mcse	model	s.size	event	converged
0.959	0.007	-0.005	0.006	0.037	0.002	BFGS	500	12%	923
0.955	0.007	-0.007	0.006	0.038	0.002	Fisher(glm)	500	12%	961
0.980	0.005	0.068	0.005	0.027	0.001	Nelder-Mead	500	12%	957
0.973	0.005	-0.007	0.006	0.038	0.002	Poisson(log)	500	12%	1000
0.999	0.001	0.043	0.004	0.018	0.001	squadP	500	12%	996
0.955	0.007	0.004	0.004	0.018	0.001	BFGS	1000	12%	981
0.955	0.007	0.003	0.004	0.018	0.001	Fisher(glm)	1000	12%	991
0.971	0.005	0.072	0.003	0.016	0.001	Nelder-Mead	1000	12%	991
0.965	0.006	0.004	0.004	0.019	0.001	Poisson(log)	1000	12%	1000
0.993	0.003	0.035	0.003	0.012	0.000	squadP	1000	12%	1000
0.953	0.007	0.001	0.001	0.002	0.000	BFGS	10000	12%	999
0.955	0.007	0.001	0.001	0.002	0.000	Fisher(glm)	10000	12%	1000
0.638	0.015	0.072	0.001	0.006	0.000	Nelder-Mead	10000	12%	1000
0.973	0.005	0.001	0.001	0.002	0.000	Poisson(log)	10000	12%	1000
0.968	0.006	0.008	0.001	0.001	0.000	squadP	10000	12%	1000
0.942	0.008	-0.004	0.004	0.018	0.001	BFGS	500	24%	907
0.939	0.008	-0.004	0.004	0.018	0.001	Fisher(glm)	500	24%	971
0.961	0.006	-0.005	0.004	0.015	0.001	Nelder-Mead	500	24%	957
0.969	0.005	-0.003	0.004	0.018	0.001	Poisson(log)	500	24%	1000
0.999	0.001	0.019	0.003	0.008	0.000	squadP	500	24%	1000
0.939	0.008	0.001	0.003	0.009	0.000	BFGS	1000	24%	986
0.933	0.008	0.001	0.003	0.009	0.000	Fisher(glm)	1000	24%	997
0.952	0.007	0.000	0.003	0.008	0.000	Nelder-Mead	1000	24%	1000
0.966	0.006	0.001	0.003	0.009	0.000	Poisson(log)	1000	24%	1000
0.983	0.004	0.014	0.002	0.005	0.000	squadP	1000	24%	1000
0.967	0.006	0.001	0.001	0.001	0.000	BFGS	10000	24%	1000
0.967	0.006	0.000	0.001	0.001	0.000	Fisher(glm)	10000	24%	1000
0.975	0.005	0.000	0.001	0.001	0.000	Nelder-Mead	10000	24%	1000
0.988	0.003	0.000	0.001	0.001	0.000	Poisson(log)	10000	24%	1000
0.974	0.005	0.002	0.001	0.001	0.000	squadP	10000	24%	1000
0.935	0.010	0.004	0.004	0.007	0.000	BFGS	500	48%	555
0.925	0.009	-0.001	0.003	0.007	0.000	Fisher(glm)	500	48%	915
0.953	0.008	-0.018	0.003	0.006	0.000	Nelder-Mead	500	48%	779
0.991	0.003	-0.001	0.003	0.007	0.000	Poisson(log)	500	48%	1000
0.999	0.001	0.014	0.001	0.002	0.000	squadP	500	48%	1000
0.962	0.007	0.003	0.002	0.003	0.000	BFGS	1000	48%	658
0.946	0.007	0.001	0.002	0.003	0.000	Fisher(glm)	1000	48%	983
0.948	0.007	-0.017	0.002	0.003	0.000	Nelder-Mead	1000	48%	924
0.990	0.003	0.001	0.002	0.003	0.000	Poisson(log)	1000	48%	1000
0.998	0.001	0.010	0.001	0.002	0.000	squadP	1000	48%	1000
0.966	0.007	0.002	0.001	0.000	0.000	BFGS	10000	48%	671
0.961	0.006	0.000	0.001	0.000	0.000	Fisher(glm)	10000	48%	1000
0.649	0.015	-0.023	0.001	0.001	0.000	Nelder-Mead	10000	48%	977
0.996	0.002	0.000	0.001	0.000	0.000	Poisson(log)	10000	48%	1000
0.972	0.005	0.002	0.001	0.000	0.000	squadP	10000	48%	1000

Table 69: Performance measurments of scenarios 97→105 for coef.2

Covarage		Bias		MSE					
cover	mcse	bias	mcse	mse	mcse	model	s.size	event	converged
0.943	0.008	-0.036	0.035	1.159	0.054	BFGS	500	12%	923
0.945	0.007	-0.038	0.034	1.143	0.051	Fisher(glm)	500	12%	961
0.984	0.004	0.242	0.026	0.683	0.033	Nelder-Mead	500	12%	957
0.965	0.006	-0.038	0.034	1.152	0.051	Poisson(log)	500	12%	1000
0.998	0.001	0.112	0.017	0.292	0.016	squadP	500	12%	996
0.957	0.006	-0.041	0.023	0.509	0.023	BFGS	1000	12%	981
0.957	0.006	-0.041	0.023	0.506	0.023	Fisher(glm)	1000	12%	991
0.990	0.003	0.237	0.016	0.321	0.014	Nelder-Mead	1000	12%	991
0.968	0.006	-0.047	0.023	0.509	0.023	Poisson(log)	1000	12%	1000
0.999	0.001	0.067	0.012	0.158	0.008	squadP	1000	12%	1000
0.948	0.007	-0.002	0.007	0.051	0.002	BFGS	10000	12%	999
0.948	0.007	-0.004	0.007	0.051	0.002	Fisher(glm)	10000	12%	1000
0.804	0.013	0.282	0.006	0.121	0.004	Nelder-Mead	10000	12%	1000
0.961	0.006	-0.004	0.007	0.051	0.002	Poisson(log)	10000	12%	1000
0.990	0.003	0.011	0.005	0.030	0.001	squadP	10000	12%	1000
0.954	0.007	0.010	0.023	0.482	0.022	BFGS	500	24%	907
0.945	0.007	-0.005	0.023	0.502	0.022	Fisher(glm)	500	24%	971
0.967	0.006	-0.007	0.020	0.377	0.019	Nelder-Mead	500	24%	957
0.972	0.005	-0.005	0.022	0.505	0.022	Poisson(log)	500	24%	1000
0.999	0.001	0.087	0.012	0.142	0.007	squadP	500	24%	1000
0.954	0.007	-0.036	0.015	0.223	0.009	BFGS	1000	24%	986
0.954	0.007	-0.035	0.015	0.225	0.010	Fisher(glm)	1000	24%	997
0.977	0.005	-0.027	0.013	0.173	0.008	Nelder-Mead	1000	24%	1000
0.980	0.004	-0.034	0.015	0.224	0.009	Poisson(log)	1000	24%	1000
1.000	0.000	0.036	0.009	0.086	0.004	squadP	1000	24%	1000
0.944	0.007	0.002	0.005	0.023	0.001	BFGS	10000	24%	1000
0.945	0.007	0.000	0.005	0.023	0.001	Fisher(glm)	10000	24%	1000
0.965	0.006	0.000	0.004	0.018	0.001	Nelder-Mead	10000	24%	1000
0.982	0.004	0.000	0.005	0.023	0.001	Poisson(log)	10000	24%	1000
0.981	0.004	0.009	0.004	0.017	0.001	squadP	10000	24%	1000
0.953	0.009	-0.007	0.017	0.156	0.010	BFGS	500	48%	555
0.917	0.009	-0.016	0.014	0.184	0.009	Fisher(glm)	500	48%	915
0.973	0.006	-0.013	0.012	0.115	0.007	Nelder-Mead	500	48%	779
0.989	0.003	-0.010	0.013	0.177	0.008	Poisson(log)	500	48%	1000
1.000	0.000	0.017	0.007	0.043	0.002	squadP	500	48%	1000
0.959	0.008	0.005	0.011	0.079	0.004	BFGS	1000	48%	658
0.952	0.007	-0.005	0.009	0.082	0.004	Fisher(glm)	1000	48%	983
0.970	0.006	-0.023	0.008	0.063	0.004	Nelder-Mead	1000	48%	924
0.991	0.003	-0.003	0.009	0.084	0.004	Poisson(log)	1000	48%	1000
0.999	0.001	0.021	0.005	0.027	0.001	squadP	1000	48%	1000
0.946	0.009	0.005	0.003	0.008	0.000	BFGS	10000	48%	671
0.942	0.007	0.002	0.003	0.009	0.000	Fisher(glm)	10000	48%	1000
0.834	0.012	-0.032	0.004	0.016	0.001	Nelder-Mead	10000	48%	977
0.993	0.003	0.002	0.003	0.008	0.000	Poisson(log)	10000	48%	1000
0.991	0.003	0.004	0.002	0.005	0.000	squadP	10000	48%	1000

Table 70: Performance measurments of scenarios 97→105 for coef.3

Covarage		Bias		MSE					
cover	mcse	bias	mcse	mse	mcse	model	s.size	event	converged
0.944	0.008	-0.021	0.007	0.041	0.002	BFGS	500	12%	923
0.929	0.008	-0.021	0.007	0.041	0.002	Fisher(glm)	500	12%	961
0.958	0.006	-0.022	0.006	0.036	0.002	Nelder-Mead	500	12%	957
0.964	0.006	-0.019	0.006	0.042	0.002	Poisson(log)	500	12%	1000
1.000	0.000	-0.025	0.002	0.005	0.000	squadP	500	12%	996
0.946	0.007	-0.015	0.004	0.019	0.001	BFGS	1000	12%	981
0.942	0.007	-0.014	0.004	0.019	0.001	Fisher(glm)	1000	12%	991
0.962	0.006	-0.010	0.004	0.017	0.001	Nelder-Mead	1000	12%	991
0.967	0.006	-0.015	0.004	0.019	0.001	Poisson(log)	1000	12%	1000
1.000	0.000	-0.018	0.002	0.004	0.000	squadP	1000	12%	1000
0.946	0.007	-0.001	0.001	0.002	0.000	BFGS	10000	12%	999
0.946	0.007	-0.001	0.001	0.002	0.000	Fisher(glm)	10000	12%	1000
0.932	0.008	0.003	0.001	0.002	0.000	Nelder-Mead	10000	12%	1000
0.961	0.006	-0.001	0.001	0.002	0.000	Poisson(log)	10000	12%	1000
0.997	0.002	-0.005	0.001	0.001	0.000	squadP	10000	12%	1000
0.949	0.007	-0.012	0.005	0.020	0.001	BFGS	500	24%	907
0.934	0.008	-0.012	0.005	0.021	0.001	Fisher(glm)	500	24%	971
0.957	0.007	-0.013	0.004	0.018	0.001	Nelder-Mead	500	24%	957
0.977	0.005	-0.013	0.005	0.021	0.001	Poisson(log)	500	24%	1000
1.000	0.000	-0.004	0.002	0.003	0.000	squadP	500	24%	1000
0.954	0.007	-0.008	0.003	0.009	0.000	BFGS	1000	24%	986
0.946	0.007	-0.008	0.003	0.009	0.000	Fisher(glm)	1000	24%	997
0.960	0.006	-0.004	0.003	0.008	0.000	Nelder-Mead	1000	24%	1000
0.978	0.005	-0.008	0.003	0.009	0.000	Poisson(log)	1000	24%	1000
1.000	0.000	-0.001	0.001	0.002	0.000	squadP	1000	24%	1000
0.955	0.007	0.000	0.001	0.001	0.000	BFGS	10000	24%	1000
0.954	0.007	0.000	0.001	0.001	0.000	Fisher(glm)	10000	24%	1000
0.976	0.005	0.001	0.001	0.001	0.000	Nelder-Mead	10000	24%	1000
0.982	0.004	0.000	0.001	0.001	0.000	Poisson(log)	10000	24%	1000
0.993	0.003	0.001	0.001	0.001	0.000	squadP	10000	24%	1000
0.960	0.008	-0.005	0.003	0.006	0.000	BFGS	500	48%	555
0.930	0.008	-0.006	0.003	0.008	0.000	Fisher(glm)	500	48%	915
0.958	0.007	0.010	0.003	0.006	0.000	Nelder-Mead	500	48%	779
0.992	0.003	-0.004	0.003	0.007	0.000	Poisson(log)	500	48%	1000
1.000	0.000	-0.005	0.001	0.001	0.000	squadP	500	48%	1000
0.956	0.008	-0.002	0.002	0.003	0.000	BFGS	1000	48%	658
0.948	0.007	-0.004	0.002	0.004	0.000	Fisher(glm)	1000	48%	983
0.959	0.007	0.007	0.002	0.003	0.000	Nelder-Mead	1000	48%	924
0.993	0.003	-0.004	0.002	0.004	0.000	Poisson(log)	1000	48%	1000
1.000	0.000	-0.003	0.001	0.000	0.000	squadP	1000	48%	1000
0.946	0.009	0.000	0.001	0.000	0.000	BFGS	10000	48%	671
0.944	0.007	0.000	0.001	0.000	0.000	Fisher(glm)	10000	48%	1000
0.829	0.012	0.010	0.001	0.001	0.000	Nelder-Mead	10000	48%	977
0.990	0.003	0.000	0.001	0.000	0.000	Poisson(log)	10000	48%	1000
1.000	0.000	0.000	0.000	0.000	0.000	squadP	10000	48%	1000

Table 71: Performance measurments of scenarios 97→105 for coef.4

Covarage		Bias		MSE					
cover	mcse	bias	mcse	mse	mcse	model	s.size	event	converged
0.953	0.007	-0.059	0.014	0.187	0.009	BFGS	500	12%	923
0.942	0.008	-0.050	0.014	0.194	0.010	Fisher(glm)	500	12%	961
0.968	0.006	0.065	0.012	0.137	0.007	Nelder-Mead	500	12%	957
0.960	0.006	-0.048	0.014	0.195	0.009	Poisson(log)	500	12%	1000
0.995	0.002	-0.020	0.010	0.093	0.006	squadP	500	12%	996
0.950	0.007	-0.013	0.009	0.084	0.004	BFGS	1000	12%	981
0.955	0.007	-0.013	0.009	0.085	0.004	Fisher(glm)	1000	12%	991
0.958	0.006	0.088	0.008	0.067	0.003	Nelder-Mead	1000	12%	991
0.967	0.006	-0.011	0.009	0.086	0.004	Poisson(log)	1000	12%	1000
0.982	0.004	0.003	0.007	0.050	0.003	squadP	1000	12%	1000
0.958	0.006	0.000	0.003	0.008	0.000	BFGS	10000	12%	999
0.958	0.006	0.000	0.003	0.008	0.000	Fisher(glm)	10000	12%	1000
0.832	0.012	0.094	0.003	0.015	0.001	Nelder-Mead	10000	12%	1000
0.971	0.005	0.000	0.003	0.008	0.000	Poisson(log)	10000	12%	1000
0.974	0.005	0.003	0.003	0.007	0.000	squadP	10000	12%	1000
0.954	0.007	-0.039	0.010	0.086	0.004	BFGS	500	24%	907
0.931	0.008	-0.026	0.010	0.089	0.004	Fisher(glm)	500	24%	971
0.951	0.007	-0.034	0.009	0.082	0.004	Nelder-Mead	500	24%	957
0.975	0.005	-0.025	0.009	0.089	0.004	Poisson(log)	500	24%	1000
0.993	0.003	-0.017	0.007	0.045	0.002	squadP	500	24%	1000
0.951	0.007	-0.002	0.006	0.041	0.002	BFGS	1000	24%	986
0.949	0.007	-0.001	0.006	0.042	0.002	Fisher(glm)	1000	24%	997
0.957	0.006	-0.002	0.006	0.038	0.002	Nelder-Mead	1000	24%	1000
0.981	0.004	-0.001	0.006	0.041	0.002	Poisson(log)	1000	24%	1000
0.985	0.004	0.003	0.005	0.026	0.001	squadP	1000	24%	1000
0.946	0.007	0.002	0.002	0.004	0.000	BFGS	10000	24%	1000
0.946	0.007	0.001	0.002	0.004	0.000	Fisher(glm)	10000	24%	1000
0.964	0.006	0.002	0.002	0.004	0.000	Nelder-Mead	10000	24%	1000
0.981	0.004	0.001	0.002	0.004	0.000	Poisson(log)	10000	24%	1000
0.952	0.007	0.002	0.002	0.004	0.000	squadP	10000	24%	1000
0.973	0.007	-0.013	0.007	0.027	0.002	BFGS	500	48%	555
0.923	0.009	-0.009	0.006	0.031	0.002	Fisher(glm)	500	48%	915
0.969	0.006	-0.034	0.006	0.029	0.002	Nelder-Mead	500	48%	779
0.997	0.002	-0.008	0.005	0.030	0.001	Poisson(log)	500	48%	1000
0.998	0.001	-0.006	0.003	0.012	0.001	squadP	500	48%	1000
0.968	0.007	-0.009	0.005	0.014	0.001	BFGS	1000	48%	658
0.936	0.008	-0.004	0.004	0.015	0.001	Fisher(glm)	1000	48%	983
0.948	0.007	-0.028	0.004	0.015	0.001	Nelder-Mead	1000	48%	924
0.998	0.001	-0.003	0.004	0.015	0.001	Poisson(log)	1000	48%	1000
0.995	0.002	-0.005	0.003	0.008	0.000	squadP	1000	48%	1000
0.960	0.008	0.001	0.001	0.001	0.000	BFGS	10000	48%	671
0.954	0.007	0.000	0.001	0.001	0.000	Fisher(glm)	10000	48%	1000
0.782	0.013	-0.025	0.002	0.004	0.000	Nelder-Mead	10000	48%	977
0.994	0.002	0.000	0.001	0.001	0.000	Poisson(log)	10000	48%	1000
0.967	0.006	0.000	0.001	0.001	0.000	squadP	10000	48%	1000

Table 72: Performance measurments of scenarios 97→105 for coef.5

Covarage		Bias		MSE					
cover	mcse	bias	mcse	mse	mcse	model	s.size	event	converged
0.950	0.007	-0.022	0.009	0.080	0.004	BFGS	500	12%	923
0.945	0.007	-0.021	0.009	0.080	0.004	Fisher(glm)	500	12%	961
0.954	0.007	-0.045	0.009	0.076	0.004	Nelder-Mead	500	12%	957
0.965	0.006	-0.022	0.009	0.080	0.004	Poisson(log)	500	12%	1000
0.966	0.006	-0.056	0.008	0.066	0.003	squadP	500	12%	996
0.969	0.005	-0.013	0.006	0.032	0.001	BFGS	1000	12%	981
0.970	0.005	-0.013	0.006	0.032	0.001	Fisher(glm)	1000	12%	991
0.968	0.006	-0.037	0.006	0.032	0.001	Nelder-Mead	1000	12%	991
0.979	0.005	-0.013	0.006	0.033	0.001	Poisson(log)	1000	12%	1000
0.982	0.004	-0.036	0.005	0.029	0.001	squadP	1000	12%	1000
0.950	0.007	-0.001	0.002	0.004	0.000	BFGS	10000	12%	999
0.951	0.007	-0.001	0.002	0.004	0.000	Fisher(glm)	10000	12%	1000
0.919	0.009	-0.028	0.002	0.004	0.000	Nelder-Mead	10000	12%	1000
0.968	0.006	-0.001	0.002	0.004	0.000	Poisson(log)	10000	12%	1000
0.960	0.006	-0.006	0.002	0.003	0.000	squadP	10000	12%	1000
0.948	0.007	-0.013	0.006	0.033	0.002	BFGS	500	24%	907
0.950	0.007	-0.012	0.006	0.033	0.001	Fisher(glm)	500	24%	971
0.951	0.007	-0.011	0.006	0.032	0.001	Nelder-Mead	500	24%	957
0.978	0.005	-0.010	0.006	0.033	0.001	Poisson(log)	500	24%	1000
0.969	0.005	-0.026	0.005	0.028	0.001	squadP	500	24%	1000
0.959	0.006	-0.008	0.004	0.016	0.001	BFGS	1000	24%	986
0.959	0.006	-0.008	0.004	0.016	0.001	Fisher(glm)	1000	24%	997
0.959	0.006	-0.009	0.004	0.015	0.001	Nelder-Mead	1000	24%	1000
0.980	0.004	-0.008	0.004	0.015	0.001	Poisson(log)	1000	24%	1000
0.976	0.005	-0.018	0.004	0.014	0.001	squadP	1000	24%	1000
0.937	0.008	0.000	0.001	0.002	0.000	BFGS	10000	24%	1000
0.937	0.008	0.000	0.001	0.002	0.000	Fisher(glm)	10000	24%	1000
0.941	0.007	0.000	0.001	0.002	0.000	Nelder-Mead	10000	24%	1000
0.974	0.005	0.000	0.001	0.002	0.000	Poisson(log)	10000	24%	1000
0.939	0.008	-0.001	0.001	0.002	0.000	squadP	10000	24%	1000
0.973	0.007	-0.015	0.004	0.011	0.001	BFGS	500	48%	555
0.953	0.007	-0.002	0.003	0.011	0.001	Fisher(glm)	500	48%	915
0.969	0.006	0.002	0.004	0.010	0.001	Nelder-Mead	500	48%	779
0.995	0.002	-0.003	0.003	0.011	0.001	Poisson(log)	500	48%	1000
0.979	0.005	-0.017	0.003	0.010	0.000	squadP	500	48%	1000
0.954	0.008	-0.005	0.003	0.006	0.000	BFGS	1000	48%	658
0.948	0.007	0.000	0.002	0.006	0.000	Fisher(glm)	1000	48%	983
0.955	0.007	0.006	0.002	0.006	0.000	Nelder-Mead	1000	48%	924
0.995	0.002	-0.001	0.002	0.005	0.000	Poisson(log)	1000	48%	1000
0.961	0.006	-0.008	0.002	0.005	0.000	squadP	1000	48%	1000
0.949	0.008	0.000	0.001	0.001	0.000	BFGS	10000	48%	671
0.948	0.007	0.001	0.001	0.001	0.000	Fisher(glm)	10000	48%	1000
0.856	0.011	0.009	0.001	0.001	0.000	Nelder-Mead	10000	48%	977
0.992	0.003	0.001	0.001	0.001	0.000	Poisson(log)	10000	48%	1000
0.952	0.007	0.000	0.001	0.001	0.000	squadP	10000	48%	1000

Table 73: Performance measurments of scenarios 97→105 for coef.6

Covarage		Bias		MSE					
cover	mcse	bias	mcse	mse	mcse	model	s.size	event	converged
0.952	0.007	-0.112	0.024	0.552	0.179	BFGS	500	12%	923
0.951	0.007	-0.132	0.033	1.067	0.473	Fisher(glm)	500	12%	961
0.977	0.005	0.073	0.013	0.156	0.009	Nelder-Mead	500	12%	957
0.974	0.005	-0.130	0.032	1.047	0.460	Poisson(log)	500	12%	1000
0.983	0.004	-0.046	0.014	0.185	0.011	squadP	500	12%	996
0.957	0.006	-0.033	0.011	0.112	0.007	BFGS	1000	12%	981
0.955	0.007	-0.033	0.011	0.111	0.006	Fisher(glm)	1000	12%	991
0.976	0.005	0.109	0.008	0.070	0.003	Nelder-Mead	1000	12%	991
0.969	0.005	-0.031	0.011	0.112	0.007	Poisson(log)	1000	12%	1000
0.973	0.005	-0.003	0.009	0.090	0.005	squadP	1000	12%	1000
0.958	0.006	-0.002	0.003	0.010	0.000	BFGS	10000	12%	999
0.957	0.006	-0.003	0.003	0.010	0.000	Fisher(glm)	10000	12%	1000
0.728	0.014	0.138	0.003	0.026	0.001	Nelder-Mead	10000	12%	1000
0.974	0.005	-0.003	0.003	0.010	0.000	Poisson(log)	10000	12%	1000
0.967	0.006	0.006	0.003	0.009	0.000	squadP	10000	12%	1000
0.961	0.006	-0.026	0.010	0.099	0.005	BFGS	500	24%	907
0.958	0.006	-0.025	0.010	0.099	0.005	Fisher(glm)	500	24%	971
0.962	0.006	-0.026	0.010	0.093	0.005	Nelder-Mead	500	24%	957
0.980	0.004	-0.024	0.010	0.100	0.005	Poisson(log)	500	24%	1000
0.983	0.004	-0.018	0.009	0.073	0.005	squadP	500	24%	1000
0.951	0.007	-0.013	0.006	0.042	0.002	BFGS	1000	24%	986
0.949	0.007	-0.012	0.006	0.042	0.002	Fisher(glm)	1000	24%	997
0.965	0.006	-0.014	0.006	0.037	0.002	Nelder-Mead	1000	24%	1000
0.981	0.004	-0.012	0.006	0.042	0.002	Poisson(log)	1000	24%	1000
0.976	0.005	-0.004	0.006	0.033	0.002	squadP	1000	24%	1000
0.946	0.007	0.000	0.002	0.005	0.000	BFGS	10000	24%	1000
0.947	0.007	-0.001	0.002	0.005	0.000	Fisher(glm)	10000	24%	1000
0.955	0.007	0.000	0.002	0.004	0.000	Nelder-Mead	10000	24%	1000
0.976	0.005	-0.001	0.002	0.004	0.000	Poisson(log)	10000	24%	1000
0.951	0.007	0.001	0.002	0.004	0.000	squadP	10000	24%	1000
0.960	0.008	-0.028	0.008	0.034	0.002	BFGS	500	48%	555
0.940	0.008	-0.019	0.006	0.038	0.002	Fisher(glm)	500	48%	915
0.960	0.007	-0.043	0.007	0.037	0.002	Nelder-Mead	500	48%	779
0.992	0.003	-0.017	0.006	0.038	0.002	Poisson(log)	500	48%	1000
0.986	0.004	-0.019	0.005	0.025	0.001	squadP	500	48%	1000
0.950	0.009	0.002	0.005	0.019	0.001	BFGS	1000	48%	658
0.935	0.008	0.006	0.004	0.018	0.001	Fisher(glm)	1000	48%	983
0.947	0.007	-0.015	0.004	0.018	0.001	Nelder-Mead	1000	48%	924
0.991	0.003	0.007	0.004	0.018	0.001	Poisson(log)	1000	48%	1000
0.973	0.005	0.007	0.004	0.013	0.001	squadP	1000	48%	1000
0.969	0.007	0.001	0.002	0.002	0.000	BFGS	10000	48%	671
0.966	0.006	0.001	0.001	0.002	0.000	Fisher(glm)	10000	48%	1000
0.827	0.012	-0.024	0.002	0.004	0.000	Nelder-Mead	10000	48%	977
0.994	0.002	0.001	0.001	0.002	0.000	Poisson(log)	10000	48%	1000
0.971	0.005	0.002	0.001	0.002	0.000	squadP	10000	48%	1000

Table 74: Performance measurments of scenarios 97→105 for coef.7

Covarage		Bias		MSE					
cover	mcse	bias	mcse	mse	mcse	model	s.size	event	converged
0.944	0.008	-0.036	0.011	0.122	0.006	BFGS	500	12%	923
0.937	0.008	-0.039	0.011	0.122	0.006	Fisher(glm)	500	12%	961
0.981	0.004	-0.151	0.009	0.095	0.005	Nelder-Mead	500	12%	957
0.964	0.006	-0.041	0.011	0.122	0.006	Poisson(log)	500	12%	1000
1.000	0.000	-0.012	0.002	0.004	0.000	squadP	500	12%	996
0.954	0.007	-0.030	0.007	0.053	0.002	BFGS	1000	12%	981
0.957	0.006	-0.034	0.007	0.054	0.003	Fisher(glm)	1000	12%	991
0.977	0.005	-0.136	0.006	0.054	0.002	Nelder-Mead	1000	12%	991
0.968	0.006	-0.036	0.007	0.055	0.003	Poisson(log)	1000	12%	1000
1.000	0.000	-0.015	0.002	0.004	0.000	squadP	1000	12%	1000
0.949	0.007	0.004	0.002	0.005	0.000	BFGS	10000	12%	999
0.949	0.007	-0.001	0.002	0.005	0.000	Fisher(glm)	10000	12%	1000
0.685	0.015	-0.117	0.002	0.018	0.001	Nelder-Mead	10000	12%	1000
0.964	0.006	-0.002	0.002	0.005	0.000	Poisson(log)	10000	12%	1000
1.000	0.000	-0.003	0.001	0.001	0.000	squadP	10000	12%	1000
0.948	0.007	-0.020	0.007	0.050	0.002	BFGS	500	24%	907
0.930	0.008	-0.024	0.007	0.052	0.002	Fisher(glm)	500	24%	971
0.986	0.004	-0.011	0.005	0.026	0.001	Nelder-Mead	500	24%	957
0.975	0.005	-0.024	0.007	0.053	0.002	Poisson(log)	500	24%	1000
1.000	0.000	-0.006	0.001	0.002	0.000	squadP	500	24%	1000
0.959	0.006	-0.006	0.005	0.022	0.001	BFGS	1000	24%	986
0.958	0.006	-0.014	0.005	0.023	0.001	Fisher(glm)	1000	24%	997
0.986	0.004	-0.002	0.003	0.011	0.001	Nelder-Mead	1000	24%	1000
0.985	0.004	-0.015	0.005	0.023	0.001	Poisson(log)	1000	24%	1000
1.000	0.000	-0.006	0.001	0.002	0.000	squadP	1000	24%	1000
0.930	0.008	0.005	0.002	0.002	0.000	BFGS	10000	24%	1000
0.938	0.008	-0.002	0.002	0.002	0.000	Fisher(glm)	10000	24%	1000
0.989	0.003	0.001	0.001	0.001	0.000	Nelder-Mead	10000	24%	1000
0.973	0.005	-0.002	0.002	0.002	0.000	Poisson(log)	10000	24%	1000
1.000	0.000	-0.001	0.001	0.001	0.000	squadP	10000	24%	1000
0.951	0.009	-0.004	0.006	0.017	0.001	BFGS	500	48%	555
0.918	0.009	-0.008	0.005	0.019	0.001	Fisher(glm)	500	48%	915
0.955	0.007	0.071	0.003	0.014	0.001	Nelder-Mead	500	48%	779
0.990	0.003	-0.008	0.004	0.019	0.001	Poisson(log)	500	48%	1000
1.000	0.000	-0.004	0.001	0.001	0.000	squadP	500	48%	1000
0.959	0.008	0.003	0.003	0.007	0.000	BFGS	1000	48%	658
0.936	0.008	-0.007	0.003	0.008	0.000	Fisher(glm)	1000	48%	983
0.893	0.010	0.078	0.002	0.011	0.000	Nelder-Mead	1000	48%	924
0.991	0.003	-0.007	0.003	0.008	0.000	Poisson(log)	1000	48%	1000
1.000	0.000	-0.003	0.001	0.000	0.000	squadP	1000	48%	1000
0.931	0.010	0.011	0.001	0.001	0.000	BFGS	10000	48%	671
0.947	0.007	-0.001	0.001	0.001	0.000	Fisher(glm)	10000	48%	1000
0.178	0.012	0.087	0.001	0.009	0.000	Nelder-Mead	10000	48%	977
0.994	0.002	-0.001	0.001	0.001	0.000	Poisson(log)	10000	48%	1000
1.000	0.000	-0.001	0.000	0.000	0.000	squadP	10000	48%	1000

Table 75: Performance measurments of scenarios 97→105 for coef.8

Covarage		Bias		MSE					
cover	mcse	bias	mcse	mse	mcse	model	s.size	event	converged
0.941	0.008	-0.011	0.004	0.014	0.001	BFGS	500	12%	923
0.933	0.008	-0.012	0.004	0.014	0.001	Fisher(glm)	500	12%	961
0.993	0.003	-0.059	0.002	0.009	0.000	Nelder-Mead	500	12%	957
0.961	0.006	-0.013	0.004	0.014	0.001	Poisson(log)	500	12%	1000
1.000	0.000	-0.005	0.002	0.002	0.000	squadP	500	12%	996
0.955	0.007	-0.008	0.002	0.006	0.000	BFGS	1000	12%	981
0.952	0.007	-0.009	0.002	0.006	0.000	Fisher(glm)	1000	12%	991
0.984	0.004	-0.052	0.002	0.005	0.000	Nelder-Mead	1000	12%	991
0.968	0.006	-0.009	0.002	0.006	0.000	Poisson(log)	1000	12%	1000
1.000	0.000	-0.004	0.001	0.001	0.000	squadP	1000	12%	1000
0.952	0.007	0.000	0.001	0.001	0.000	BFGS	10000	12%	999
0.953	0.007	-0.001	0.001	0.001	0.000	Fisher(glm)	10000	12%	1000
0.431	0.016	-0.050	0.001	0.003	0.000	Nelder-Mead	10000	12%	1000
0.962	0.006	-0.001	0.001	0.001	0.000	Poisson(log)	10000	12%	1000
0.998	0.001	-0.002	0.000	0.000	0.000	squadP	10000	12%	1000
0.946	0.008	-0.007	0.003	0.006	0.000	BFGS	500	24%	907
0.931	0.008	-0.008	0.003	0.006	0.000	Fisher(glm)	500	24%	971
0.991	0.003	-0.003	0.002	0.003	0.000	Nelder-Mead	500	24%	957
0.976	0.005	-0.008	0.002	0.006	0.000	Poisson(log)	500	24%	1000
1.000	0.000	-0.003	0.001	0.001	0.000	squadP	500	24%	1000
0.965	0.006	-0.001	0.002	0.003	0.000	BFGS	1000	24%	986
0.958	0.006	-0.003	0.002	0.003	0.000	Fisher(glm)	1000	24%	997
0.992	0.003	0.001	0.001	0.001	0.000	Nelder-Mead	1000	24%	1000
0.987	0.004	-0.004	0.002	0.003	0.000	Poisson(log)	1000	24%	1000
1.000	0.000	-0.001	0.001	0.001	0.000	squadP	1000	24%	1000
0.944	0.007	0.001	0.001	0.000	0.000	BFGS	10000	24%	1000
0.942	0.007	-0.001	0.001	0.000	0.000	Fisher(glm)	10000	24%	1000
0.990	0.003	0.000	0.000	0.000	0.000	Nelder-Mead	10000	24%	1000
0.972	0.005	-0.001	0.001	0.000	0.000	Poisson(log)	10000	24%	1000
0.996	0.002	-0.001	0.000	0.000	0.000	squadP	10000	24%	1000
0.957	0.009	-0.002	0.002	0.002	0.000	BFGS	500	48%	555
0.927	0.009	-0.002	0.002	0.002	0.000	Fisher(glm)	500	48%	915
0.978	0.005	0.033	0.001	0.002	0.000	Nelder-Mead	500	48%	779
0.998	0.001	-0.003	0.002	0.002	0.000	Poisson(log)	500	48%	1000
1.000	0.000	-0.002	0.001	0.000	0.000	squadP	500	48%	1000
0.960	0.008	0.002	0.001	0.001	0.000	BFGS	1000	48%	658
0.946	0.007	-0.002	0.001	0.001	0.000	Fisher(glm)	1000	48%	983
0.879	0.011	0.035	0.001	0.002	0.000	Nelder-Mead	1000	48%	924
0.997	0.002	-0.001	0.001	0.001	0.000	Poisson(log)	1000	48%	1000
1.000	0.000	0.000	0.000	0.000	0.000	squadP	1000	48%	1000
0.927	0.010	0.004	0.000	0.000	0.000	BFGS	10000	48%	671
0.951	0.007	0.000	0.000	0.000	0.000	Fisher(glm)	10000	48%	1000
0.116	0.010	0.037	0.000	0.002	0.000	Nelder-Mead	10000	48%	977
0.995	0.002	0.000	0.000	0.000	0.000	Poisson(log)	10000	48%	1000
0.999	0.001	0.000	0.000	0.000	0.000	squadP	10000	48%	1000

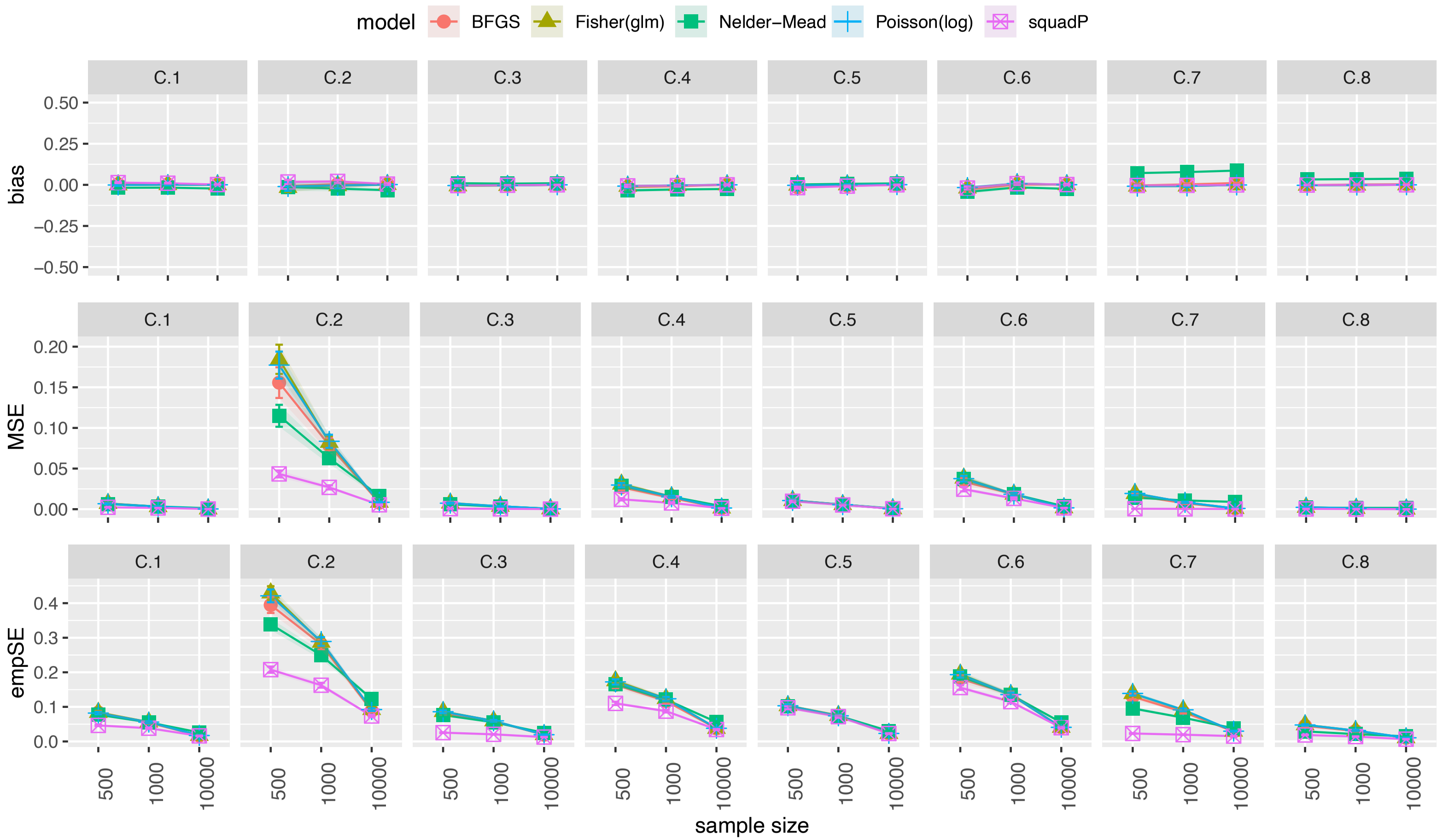

Figure 54: Absolute bias, MSE, and empSE of scenarios 103→105 of the estimated log(RR) from the five methods under model misspecifications and event probability 48%. Performance measurments for the eight coefficents using the six statistical methods (legend in the top of the figure) are on y-axis. The measurments are done for 3 scenarios that are derived from three different sample sizes (500,1000,10000) on x-axis. Each type of shapes with line is represnting a different method.

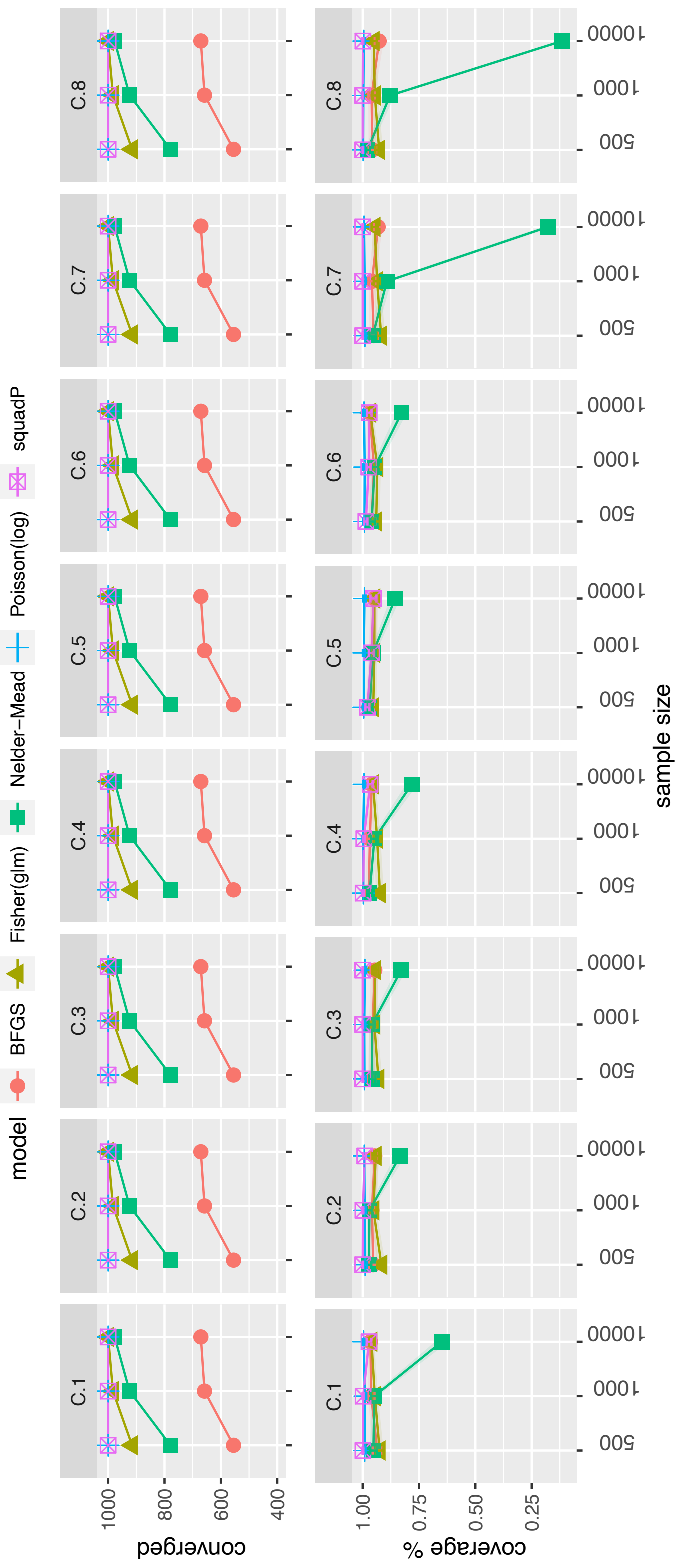

Figure 55: Covarege probabiltiy, and convergence of scenarios 103→105 of the estimated log(RR) from the five methods under model misspecifications and event probability 48%. Performance measurments for the eight coefficents using the six statistical methods (legend in the top of the figure) are on y-axis. The measurments are done for 3 scenarios that are derived from three different sample sizes (500,1000,10000) on x-axis. Each type of shapes with line is representing a different method.

8.4 Simulation results: comparison of large sample size (1 million)

(model correctly spcified and model mispecified) with event rates of 12%, 24%, and 48%.

Table 76: Bias, and MSE of scenarios 64→66 (sample size of 1 million) for coefficents c.1 and c.2 under correct model pecifications

Bias		MSE				
bias	mcse	mse	mcse	model	event %	n.coef
0.100	0.000	0.010	0.000	BFGS	12	c.1
0.100	0.000	0.010	0.000	Fisher(glm)	12	c.1
0.024	0.005	0.010	0.001	Nelder-Mead	12	c.1
0.100	0.000	0.010	0.000	Poisson(log)	12	c.1
0.120	0.000	0.014	0.000	squadP	12	c.1
0.000	0.000	0.000	0.000	BFGS	24	c.1
0.000	0.000	0.000	0.000	Fisher(glm)	24	c.1
-0.172	0.003	0.033	0.001	Nelder-Mead	24	c.1
0.000	0.000	0.000	0.000	Poisson(log)	24	c.1
0.000	0.000	0.000	0.000	squadP	24	c.1
0.000	0.000	0.000	0.000	BFGS	48	c.1
0.000	0.000	0.000	0.000	Fisher(glm)	48	c.1
-0.002	0.000	0.000	0.000	Nelder-Mead	48	c.1
0.000	0.000	0.000	0.000	Poisson(log)	48	c.1
0.000	0.000	0.000	0.000	squadP	48	c.1
0.100	0.001	0.010	0.000	BFGS	12	c.2
0.100	0.001	0.010	0.000	Fisher(glm)	12	c.2
-0.232	0.029	0.331	0.017	Nelder-Mead	12	c.2
0.100	0.001	0.010	0.000	Poisson(log)	12	c.2
0.142	0.001	0.020	0.000	squadP	12	c.2
0.000	0.001	0.000	0.000	BFGS	24	c.2
0.000	0.000	0.000	0.000	Fisher(glm)	24	c.2
-0.644	0.015	0.492	0.013	Nelder-Mead	24	c.2
0.000	0.000	0.000	0.000	Poisson(log)	24	c.2
0.000	0.000	0.000	0.000	squadP	24	c.2
0.001	0.000	0.000	0.000	BFGS	48	c.2
0.000	0.000	0.000	0.000	Fisher(glm)	48	c.2
-0.002	0.001	0.000	0.000	Nelder-Mead	48	c.2
0.000	0.000	0.000	0.000	Poisson(log)	48	c.2

0.000	0.000	0.000	0.000	squadP	48	c.2

Table 77: Bias, and MSE of scenarios 64→66 (sample size of 1 million) for coefficents c.3 and c.4 under correct model pecifications

Bias		MSE				
bias	mcse	mse	mcse	model	event %	n.coef
0.020	0.000	0.000	0.000	BFGS	12	c.3
0.020	0.000	0.000	0.000	Fisher(glm)	12	c.3
0.035	0.001	0.002	0.000	Nelder-Mead	12	c.3
0.020	0.000	0.000	0.000	Poisson(log)	12	c.3
0.022	0.000	0.000	0.000	squadP	12	c.3
0.000	0.000	0.000	0.000	BFGS	24	c.3
0.000	0.000	0.000	0.000	Fisher(glm)	24	c.3
0.033	0.001	0.001	0.000	Nelder-Mead	24	c.3
0.000	0.000	0.000	0.000	Poisson(log)	24	c.3
0.000	0.000	0.000	0.000	squadP	24	c.3
0.000	0.000	0.000	0.000	BFGS	48	c.3
0.000	0.000	0.000	0.000	Fisher(glm)	48	c.3
0.001	0.000	0.000	0.000	Nelder-Mead	48	c.3
0.000	0.000	0.000	0.000	Poisson(log)	48	c.3
0.000	0.000	0.000	0.000	squadP	48	c.3
0.101	0.001	0.010	0.000	BFGS	12	c.4
0.101	0.001	0.010	0.000	Fisher(glm)	12	c.4
0.069	0.014	0.071	0.005	Nelder-Mead	12	c.4
0.101	0.001	0.010	0.000	Poisson(log)	12	c.4
0.136	0.001	0.019	0.000	squadP	12	c.4
0.000	0.000	0.000	0.000	BFGS	24	c.4
0.000	0.000	0.000	0.000	Fisher(glm)	24	c.4
-0.194	0.007	0.052	0.002	Nelder-Mead	24	c.4
0.000	0.000	0.000	0.000	Poisson(log)	24	c.4
0.000	0.000	0.000	0.000	squadP	24	c.4
0.000	0.000	0.000	0.000	BFGS	48	c.4
0.000	0.000	0.000	0.000	Fisher(glm)	48	c.4
0.000	0.001	0.000	0.000	Nelder-Mead	48	c.4
0.000	0.000	0.000	0.000	Poisson(log)	48	c.4
0.000	0.000	0.000	0.000	squadP	48	c.4

Table 78: Bias, and MSE of scenarios 64→66 (sample size of 1 million) for coefficents c.5 and c.6 under correct model pecifications

Bias		MSE				
bias	mcse	mse	mcse	model	event %	n.coef
0.000	0.000	0.000	0.000	BFGS	12	c.5
0.000	0.000	0.000	0.000	Fisher(glm)	12	c.5
0.018	0.005	0.010	0.001	Nelder-Mead	12	c.5
0.000	0.000	0.000	0.000	Poisson(log)	12	c.5
-0.002	0.000	0.000	0.000	squadP	12	c.5
0.000	0.000	0.000	0.000	BFGS	24	c.5
0.000	0.000	0.000	0.000	Fisher(glm)	24	c.5
0.091	0.002	0.010	0.000	Nelder-Mead	24	c.5
0.000	0.000	0.000	0.000	Poisson(log)	24	c.5
0.000	0.000	0.000	0.000	squadP	24	c.5
0.000	0.000	0.000	0.000	BFGS	48	c.5
0.000	0.000	0.000	0.000	Fisher(glm)	48	c.5
0.001	0.000	0.000	0.000	Nelder-Mead	48	c.5
0.000	0.000	0.000	0.000	Poisson(log)	48	c.5
0.000	0.000	0.000	0.000	squadP	48	c.5
0.101	0.001	0.010	0.000	BFGS	12	c.6
0.101	0.001	0.010	0.000	Fisher(glm)	12	c.6
-0.023	0.010	0.032	0.003	Nelder-Mead	12	c.6
0.101	0.001	0.010	0.000	Poisson(log)	12	c.6
0.124	0.001	0.015	0.000	squadP	12	c.6
0.000	0.000	0.000	0.000	BFGS	24	c.6
0.000	0.000	0.000	0.000	Fisher(glm)	24	c.6
-0.241	0.005	0.068	0.002	Nelder-Mead	24	c.6
0.000	0.000	0.000	0.000	Poisson(log)	24	c.6
0.000	0.000	0.000	0.000	squadP	24	c.6
0.000	0.000	0.000	0.000	BFGS	48	c.6
0.000	0.000	0.000	0.000	Fisher(glm)	48	c.6
-0.001	0.000	0.000	0.000	Nelder-Mead	48	c.6
0.000	0.000	0.000	0.000	Poisson(log)	48	c.6
0.000	0.000	0.000	0.000	squadP	48	c.6

Table 79: Bias, and MSE of scenarios 64→66 (sample size of 1 million) for coefficents c.7 and c.8 under correct model pecifications

Bias		MSE				
bias	mcse	mse	mcse	model	event %	n.coef
0.100	0.000	0.010	0.000	BFGS	12	c.7
0.100	0.000	0.010	0.000	Fisher(glm)	12	c.7
0.179	0.004	0.038	0.001	Nelder-Mead	12	c.7
0.100	0.000	0.010	0.000	Poisson(log)	12	c.7
0.096	0.000	0.009	0.000	squadP	12	c.7
0.000	0.000	0.000	0.000	BFGS	24	c.7
0.000	0.000	0.000	0.000	Fisher(glm)	24	c.7
0.148	0.002	0.024	0.000	Nelder-Mead	24	c.7
0.000	0.000	0.000	0.000	Poisson(log)	24	c.7
0.000	0.000	0.000	0.000	squadP	24	c.7
0.000	0.000	0.000	0.000	BFGS	48	c.7
0.000	0.000	0.000	0.000	Fisher(glm)	48	c.7
0.002	0.000	0.000	0.000	Nelder-Mead	48	c.7
0.000	0.000	0.000	0.000	Poisson(log)	48	c.7
0.000	0.000	0.000	0.000	squadP	48	c.7
0.100	0.000	0.010	0.000	BFGS	12	c.8
0.100	0.000	0.010	0.000	Fisher(glm)	12	c.8
0.149	0.002	0.024	0.001	Nelder-Mead	12	c.8
0.100	0.000	0.010	0.000	Poisson(log)	12	c.8
0.104	0.000	0.011	0.000	squadP	12	c.8
0.000	0.000	0.000	0.000	BFGS	24	c.8
0.000	0.000	0.000	0.000	Fisher(glm)	24	c.8
0.105	0.001	0.012	0.000	Nelder-Mead	24	c.8
0.000	0.000	0.000	0.000	Poisson(log)	24	c.8
0.000	0.000	0.000	0.000	squadP	24	c.8
0.000	0.000	0.000	0.000	BFGS	48	c.8
0.000	0.000	0.000	0.000	Fisher(glm)	48	c.8
0.002	0.000	0.000	0.000	Nelder-Mead	48	c.8
0.000	0.000	0.000	0.000	Poisson(log)	48	c.8
0.000	0.000	0.000	0.000	squadP	48	c.8

Table 80: Bias, and MSE of scenarios 106→108 (sample size of 1 million) for coefficents c.1 and c.2 under model misspecification

Bias		MSE				
bias	mcse	mse	mcse	model	event %	n.coef
0.001	0.000	0.000	0.000	BFGS	12	c.1
0.000	0.000	0.000	0.000	Fisher(glm)	12	c.1
-0.054	0.003	0.005	0.000	Nelder-Mead	12	c.1
0.000	0.000	0.000	0.000	Poisson(log)	12	c.1
0.012	0.000	0.000	0.000	squadP	12	c.1
0.001	0.000	0.000	0.000	BFGS	24	c.1
0.000	0.000	0.000	0.000	Fisher(glm)	24	c.1
-0.090	0.002	0.009	0.000	Nelder-Mead	24	c.1
0.000	0.000	0.000	0.000	Poisson(log)	24	c.1
0.000	0.000	0.000	0.000	squadP	24	c.1
0.001	0.000	0.000	0.000	BFGS	48	c.1
0.000	0.000	0.000	0.000	Fisher(glm)	48	c.1
-0.002	0.000	0.000	0.000	Nelder-Mead	48	c.1
0.000	0.000	0.000	0.000	Poisson(log)	48	c.1
0.000	0.000	0.000	0.000	squadP	48	c.1
-0.001	0.001	0.001	0.000	BFGS	12	c.2
-0.001	0.001	0.001	0.000	Fisher(glm)	12	c.2
0.003	0.014	0.071	0.005	Nelder-Mead	12	c.2
-0.001	0.001	0.001	0.000	Poisson(log)	12	c.2
0.050	0.001	0.003	0.000	squadP	12	c.2
0.001	0.001	0.000	0.000	BFGS	24	c.2
0.000	0.001	0.000	0.000	Fisher(glm)	24	c.2
-0.195	0.011	0.078	0.004	Nelder-Mead	24	c.2
0.000	0.001	0.000	0.000	Poisson(log)	24	c.2
0.000	0.001	0.000	0.000	squadP	24	c.2
0.004	0.001	0.000	0.000	BFGS	48	c.2
0.000	0.000	0.000	0.000	Fisher(glm)	48	c.2
0.007	0.001	0.000	0.000	Nelder-Mead	48	c.2
0.000	0.000	0.000	0.000	Poisson(log)	48	c.2
0.000	0.000	0.000	0.000	squadP	48	c.2

Table 81: Bias, and MSE of scenarios 106→108 (sample size of 1 million) for coefficents c.3 and c.4 under model misspecification

Bias		MSE				
bias	mcse	mse	mcse	model	event %	n.coef
0.000	0.000	0.000	0.000	BFGS	12	c.3
0.000	0.000	0.000	0.000	Fisher(glm)	12	c.3
0.023	0.003	0.003	0.000	Nelder-Mead	12	c.3
0.000	0.000	0.000	0.000	Poisson(log)	12	c.3
0.007	0.000	0.000	0.000	squadP	12	c.3
0.000	0.000	0.000	0.000	BFGS	24	c.3
0.000	0.000	0.000	0.000	Fisher(glm)	24	c.3
0.015	0.003	0.002	0.000	Nelder-Mead	24	c.3
0.000	0.000	0.000	0.000	Poisson(log)	24	c.3
0.000	0.000	0.000	0.000	squadP	24	c.3
0.000	0.000	0.000	0.000	BFGS	48	c.3
0.000	0.000	0.000	0.000	Fisher(glm)	48	c.3
0.003	0.000	0.000	0.000	Nelder-Mead	48	c.3
0.000	0.000	0.000	0.000	Poisson(log)	48	c.3
0.000	0.000	0.000	0.000	squadP	48	c.3
0.002	0.001	0.000	0.000	BFGS	12	c.4
0.001	0.001	0.000	0.000	Fisher(glm)	12	c.4
-0.078	0.005	0.016	0.001	Nelder-Mead	12	c.4
0.001	0.001	0.000	0.000	Poisson(log)	12	c.4
0.022	0.001	0.001	0.000	squadP	12	c.4
0.001	0.000	0.000	0.000	BFGS	24	c.4
0.000	0.000	0.000	0.000	Fisher(glm)	24	c.4
-0.130	0.003	0.020	0.001	Nelder-Mead	24	c.4
0.000	0.000	0.000	0.000	Poisson(log)	24	c.4
0.000	0.000	0.000	0.000	squadP	24	c.4
0.002	0.000	0.000	0.000	BFGS	48	c.4
0.000	0.000	0.000	0.000	Fisher(glm)	48	c.4
-0.001	0.000	0.000	0.000	Nelder-Mead	48	c.4
0.000	0.000	0.000	0.000	Poisson(log)	48	c.4
0.000	0.000	0.000	0.000	squadP	48	c.4

Table 82: Bias, and MSE of scenarios 106→108 (sample size of 1 million) for coefficents c.5 and c.6 under model misspecification

Bias		MSE				
bias	mcse	mse	mcse	model	event %	n.coef
0.000	0.000	0.000	0.000	BFGS	12	c.5
0.000	0.000	0.000	0.000	Fisher(glm)	12	c.5
0.007	0.001	0.001	0.000	Nelder-Mead	12	c.5
0.000	0.000	0.000	0.000	Poisson(log)	12	c.5
-0.002	0.000	0.000	0.000	squadP	12	c.5
0.000	0.000	0.000	0.000	BFGS	24	c.5
0.000	0.000	0.000	0.000	Fisher(glm)	24	c.5
0.003	0.002	0.001	0.000	Nelder-Mead	24	c.5
0.000	0.000	0.000	0.000	Poisson(log)	24	c.5
0.000	0.000	0.000	0.000	squadP	24	c.5
-0.001	0.000	0.000	0.000	BFGS	48	c.5
0.000	0.000	0.000	0.000	Fisher(glm)	48	c.5
0.000	0.000	0.000	0.000	Nelder-Mead	48	c.5
0.000	0.000	0.000	0.000	Poisson(log)	48	c.5
0.000	0.000	0.000	0.000	squadP	48	c.5
0.001	0.001	0.000	0.000	BFGS	12	c.6
0.001	0.001	0.000	0.000	Fisher(glm)	12	c.6
-0.052	0.006	0.013	0.001	Nelder-Mead	12	c.6
0.001	0.001	0.000	0.000	Poisson(log)	12	c.6
0.015	0.001	0.000	0.000	squadP	12	c.6
0.001	0.000	0.000	0.000	BFGS	24	c.6
0.000	0.000	0.000	0.000	Fisher(glm)	24	c.6
-0.201	0.003	0.043	0.001	Nelder-Mead	24	c.6
0.000	0.000	0.000	0.000	Poisson(log)	24	c.6
0.000	0.000	0.000	0.000	squadP	24	c.6
0.001	0.000	0.000	0.000	BFGS	48	c.6
0.000	0.000	0.000	0.000	Fisher(glm)	48	c.6
-0.003	0.000	0.000	0.000	Nelder-Mead	48	c.6
0.000	0.000	0.000	0.000	Poisson(log)	48	c.6
0.000	0.000	0.000	0.000	squadP	48	c.6

Table 83: Bias, and MSE of scenarios 106→108 (sample size of 1 million) for coefficents c.7 and c.8 under model misspecification

Bias		MSE				
bias	mcse	mse	mcse	model	event %	n.coef
0.003	0.000	0.000	0.000	BFGS	12	c.7
-0.001	0.000	0.000	0.000	Fisher(glm)	12	c.7
0.210	0.006	0.056	0.002	Nelder-Mead	12	c.7
-0.001	0.000	0.000	0.000	Poisson(log)	12	c.7
0.021	0.001	0.001	0.000	squadP	12	c.7
0.007	0.000	0.000	0.000	BFGS	24	c.7
0.000	0.000	0.000	0.000	Fisher(glm)	24	c.7
0.111	0.002	0.014	0.000	Nelder-Mead	24	c.7
0.000	0.000	0.000	0.000	Poisson(log)	24	c.7
0.000	0.000	0.000	0.000	squadP	24	c.7
0.011	0.000	0.000	0.000	BFGS	48	c.7
0.000	0.000	0.000	0.000	Fisher(glm)	48	c.7
0.009	0.000	0.000	0.000	Nelder-Mead	48	c.7
0.000	0.000	0.000	0.000	Poisson(log)	48	c.7
0.000	0.000	0.000	0.000	squadP	48	c.7
0.001	0.000	0.000	0.000	BFGS	12	c.8
0.000	0.000	0.000	0.000	Fisher(glm)	12	c.8
0.080	0.002	0.008	0.000	Nelder-Mead	12	c.8
0.000	0.000	0.000	0.000	Poisson(log)	12	c.8
0.008	0.000	0.000	0.000	squadP	12	c.8
0.002	0.000	0.000	0.000	BFGS	24	c.8
0.000	0.000	0.000	0.000	Fisher(glm)	24	c.8
0.055	0.001	0.003	0.000	Nelder-Mead	24	c.8
0.000	0.000	0.000	0.000	Poisson(log)	24	c.8
0.000	0.000	0.000	0.000	squadP	24	c.8
0.003	0.000	0.000	0.000	BFGS	48	c.8
0.000	0.000	0.000	0.000	Fisher(glm)	48	c.8
0.004	0.000	0.000	0.000	Nelder-Mead	48	c.8
0.000	0.000	0.000	0.000	Poisson(log)	48	c.8
0.000	0.000	0.000	0.000	squadP	48	c.8

9 Appendix B: The numerical analysis of RR estimation using squadP

squadP is implemented in R programming language for fitting a Log-Binomial Model, and in the latest update the name of the algorithm is changed into *BSW*.

9.0.1 The negative log-likelihood function of log-binomial model.

```
logll <- function(theta, y, x){
  p <- exp(x %*% theta)
  p[p >= 1] <- 1 - 1e-5
  LL <- sum(y*log(p) + (1-y)*log(1 - p))
  return(-LL)}
```

y is the outcome varible, and x is model matrix. *theta* is a numeric vector containing the initial values of the model parameters.

9.0.2 The score function (first derivative)

```
gradF <- function(theta, y, x) {
  p <- exp(x %*% theta)
  p[p >= 1] <- 1 - 1e-5
  s <- (y - p) / (1 - p)
  deriv1 <- as.vector(t(x) %*% s)
  return(deriv1)
}
```

*deriv*1 is a numeric vector containing the first derivatives of the log likelihood function.

9.0.3 Observed information matrix (second derivative)

```
hess <- function(theta, y, x) {
  p <- exp(x %*% theta)
  p[p >= 1] <- 1 - 1e-5
  s <- p * (y - 1) / (1 - p)^2
  im <- 0
  for(i in 1:nrow(x)){
    im <- im + x[i,] %*% t(x[i,]) * s[i]
  }
  colnames(im) <- names(theta)
  rownames(im) <- names(theta)
```

```
  return(-im)
}
```

$-im$ is a numeric matrix containing the second partial derivatives of the log likelihood function of the log-binomial model (Hessian matrix).

9.0.4 Linear inequality constraints

```
constr <- function(x) {
    colMax <- matrixStats::colMaxs(x)
    colMin <- matrixStats::colMins(x)
    const <- expand.grid(lapply(2:length(colMax), function(i) {
        c(colMin[i], colMax[i])
    }))
    Amat <- unname(as.matrix(cbind(rep(1, times = nrow(const)),
        const)))
    return(-Amat)
}
```

x is a model matrix. $-Amat$ is a matrix containing the linear inequality constraints

9.0.5 BSW algorithm

```
bsw <- function(formula, data, maxit = 200L) {
    call <- match.call()
    if (!inherits(x = formula, what = "formula")) {
        stop("\"formula\" must be of class \"formula\"")
    } else if (!is.integer(maxit)) {
        stop("\"maxit\" must be a positive integer")
    } else if (length(maxit) != 1L) {
        stop("single positive integer for \"maxit\" expected")
    } else {
        data <- stats::model.frame(formula = formula, data = data)
        y <- unname(stats::model.matrix(stats::as.formula(paste("~",
            all.vars(formula)[1])), data = data)[, -1])
        x <- stats::model.matrix(object = formula, data = data)
        theta <- c(log(mean(y)), rep(0, times = ncol(x) - 1))
        Amat <- constr(x)
        bvec <- rep(0, times = nrow(Amat))
        converged <- FALSE
```

```
        iter <- 0

        while (isFALSE(converged) & iter < maxit) {
            iter <- iter + 1
            Dmat <- Matrix::nearPD(hess(theta, y, x))$mat
            dvec <- gradF(theta, y, x) + t(theta) %*% Dmat
            fit <- quadprog::solve.QP(Dmat = Dmat, dvec = dvec,
                Amat = t(Amat), bvec = bvec)
            converged <- all(abs(fit$solution - theta) < 1e-04)
            theta <- fit$solution
            names(theta) <- colnames(x)
        }
        if (iter == maxit & converged == FALSE) {
            stop("Maximum number of iterations reached without convergence")
        }
        return(methods::new(Class = "bsw", call = call, formula = formula,
            coefficients = theta, iter = iter, converged = converged,
            y = y, x = x, data = data))
    }
}
```

coefficients is a numeric vector containing the estimated model parameters.
iter is a positive integer indicating the number of iterations.
converged is a logical constant that indicates whether the model has converged.

```
setMethod(f = "coef",
          signature = "bsw",
          definition = function(object) {
            return(object@coefficients)
          }
)
```

This function is for extracting the estimated model parameters of BSW.
The following function is for estimating confidence intervals of the estimated model parameters of BSW.

```
setMethod(f = "confint", signature = "bsw", definition = function(object,
    parm, level = 0.95, method = "wald", R = 1000L) {
    if (!is.numeric(level)) {
        stop("\"level\" must be a numeric value")
    } else if (length(level) != 1L) {
        stop("single numeric value for \"level\" expected")
    } else if (!is.character(method)) {
        stop("\"method\" must be a character string")
    } else if (length(method) != 1L) {
        stop("single character string for \"method\" expected")
    } else if (!(method %in% c("bca", "wald"))) {
        stop("\"method\" is misspecified. Currently available confidence interval
                   estimation procedures are: \"bca\" and \"wald\"")
    } else if (!is.integer(R)) {
        stop("\"R\" must be a positive integer")
    } else if (length(R) != 1L) {
        stop("single positive integer for \"R\" expected")
    } else if (R < 1000L) {
        stop("\"R\" must be a positive integer equal to or greater than 1000")
    } else {
        cf <- coef(object)
        pnames <- names(cf)
        if (missing(parm)) {
            parm <- pnames
        } else if (is.numeric(parm)) {
            parm <- pnames[parm]
        }
        alpha <- (1 - level)/2
        p <- c(alpha, 1 - alpha)
        ci <- array(data = NA, dim = c(length(parm), 2L), dimnames = list(parm,
            paste(x = format(x = 100 * p, digits = 3, scientific = FALSE,
                trim = TRUE), "%", sep = "")))

        if (method == "bca") {
            f <- function(formula, data, parm, indices) {
                dat <- data[indices, ]
                fit <- bsw(formula = formula, data = dat)
                return(coef(fit)[parm])
            }
```

```
            b <- boot::boot(data = object@data, statistic = f,
                R = R, formula = object@formula, parm = parm)
            ci[] <- matrix(unlist(lapply(1:ncol(b$t), function(i) {
                boot::boot.ci(b, conf = level, type = "bca",
                  index = i)$bca[4:5]
            })), ncol = 2, byrow = TRUE)
        }

        if (method == "wald") {
            se <- sqrt(diag(solve(hess(cf, object@y, object@x))))[parm]
            ci[] <- cf[parm] + se %o% stats::qnorm(p)
        }
        return(ci)
    }
})
```

At the end the estimated model parameters of BSW are summarized, furthermore the standered errors, and p-value are calculated using *summary* function.

```
setMethod(f = "summary", signature = "bsw", definition = function(object) {
    cf <- coef(object)
    ci <- confint(object)
    se <- sqrt(diag(solve(hess(cf, object@y, object@x))))
    z <- cf/se
    p <- 2 * stats::pnorm(abs(z), lower.tail = FALSE)
    coef.table <- cbind(as.matrix(cf), as.matrix(se), as.matrix(z),
        as.matrix(p), as.matrix(exp(cf)), exp(ci))
    colnames(coef.table) <- c("Estimate", "Std. Error", "z value",
        "Pr(>|z|)", "RR", colnames(ci))

    cat("Call:\n")
    print(object@call)
    cat("\nConvergence:", object@converged)
    cat("\nCoefficients:\n")
    print(coef.table)
    cat("\nIterations:", object@iter, "\n")
    return(invisible(list(coefficients = cf, std.err = se, z.value = z,
```

```
        p.value = p)))
})
```

9.0.6 Initial value optimization using Newton-type method

```
# the log-likelihood #c = n1*c + n0 * log(1-exp(c))
LL_init <- function(c) {
    return(-(n1 * c + n0 * log(1 - exp(c))))
}
LL_init(-0.2)
# solve 1st derivative of the logL. (gradient)
grad_init = function(c) {
    return((n0 * exp(c))/(1 - exp(c)) - n1)
}
grad_init(c)
# Hessian matrix
H_init = function(c) {
    return((n0 * exp(c))/(exp(c) - 1)^2)
}
H_init(c)
# newten method
convergence <- FALSE
while (!isTRUE(convergence)) {
    c <- c - solve(H_init(c)) %*% grad_init(c)
    convergence <- all(abs(c - c_new) < 1e-05)
    c_new <- c
    print(c_new)
}
```

10 Appendix C: The simulation study

10.0.1 Parallel computing function

The parallel function is generated and run on our local lunix server with 12 cores to fasten the process and reduce the computational cost.

```
sim <- function(f, n, m, betas, ncpus = parallel::detectCores(),
    seed) {
    cluster <- parallel::makePSOCKcluster(names = ncpus)
    parallel::clusterCall(cl = cluster, fun = lapply, X = c("logbin",
        "Matrix", "quadprog", "squadP"), FUN = require, character.only = TRUE)
    parallel::clusterExport(cl = cluster, varlist = c("f", "dg",
        "squadP", "fit.models", "fit.model_4", "sum_senarios"),
        envir = environment())
    parallel::clusterSetRNGStream(cl = cluster, iseed = seed)
    tmp1 <- pbapply::pblapply(cl = cluster, X = 1:m, FUN = f,
        n = n, betas = betas)
    parallel::stopCluster(cl = cluster)

    seeds <- as.matrix(x = data.table::rbindlist(l = lapply(X = 1:m,
        function(i) {
            as.list(x = tmp1[[i]][[1]])
        })))
    fisher <- as.matrix(x = data.table::rbindlist(l = lapply(X = 1:m,
        function(i) {
            as.list(x = tmp1[[i]][[2]])
        })))
    bfgs <- as.matrix(x = data.table::rbindlist(l = lapply(X = 1:m,
        function(i) {
            as.list(x = tmp1[[i]][[3]])
        })))
    poisson <- as.matrix(x = data.table::rbindlist(l = lapply(X = 1:m,
        function(i) {
            as.list(x = tmp1[[i]][[4]])
        })))
    nelderMead <- as.matrix(x = data.table::rbindlist(l = lapply(X = 1:m,
        function(i) {
            as.list(x = tmp1[[i]][[5]])
        })))
    squadP <- as.matrix(x = data.table::rbindlist(l = lapply(X = 1:m,
```

```
    function(i) {
        as.list(x = tmp1[[i]][[6]])
    })))
em <- as.matrix(x = data.table::rbindlist(l = lapply(X = 1:m,
    function(i) {
        as.list(x = tmp1[[i]][[7]])
    })))
# tmp7 <- data.table::as.data.table(x =
# matrix(unlist(tmp1[[i]][[6]]), ncol=8, byrow=T))

colnames(seeds) <- NULL
colnames9 <- c("intercept", "coef.1", "coef.2", "se.int",
    "se.1", "se.2", "s.size", "event", "model")
colnames13 <- c("intercept", "coef.1", "coef.2", "coef.3",
    "coef.4", "se.int", "se.1", "se.2", "se.3", "se.4", "s.size",
    "event", "model")
colnames21 <- c("intercept", "coef.1", "coef.2", "coef.3",
    "coef.4", "coef.5", "coef.6", "coef.7", "coef.8", "se.int",
    "se.1", "se.2", "se.3", "se.4", "se.5", "se.6", "se.7",
    "se.8", "s.size", "event", "model")
colnames25 <- c("intercept", "coef.1", "coef.2", "coef.3",
    "coef.4", "coef.5", "coef.6", "coef.7", "coef.8", "coef.9",
    "coef.10", "se.int", "se.1", "se.2", "se.3", "se.4",
    "se.5", "se.6", "se.7", "se.8", "se.9", "se.10", "s.size",
    "event", "model")

if (ncol(fisher) == 9) {
    colnames(fisher) <- colnames9
    colnames(bfgs) <- colnames9
    colnames(poisson) <- colnames9
    colnames(nelderMead) <- colnames9
    colnames(squadP) <- colnames9
    colnames(em) <- colnames9
} else if (ncol(fisher) == 13) {
    colnames(fisher) <- colnames13
    colnames(bfgs) <- colnames13
    colnames(poisson) <- colnames13
    colnames(nelderMead) <- colnames13
    colnames(squadP) <- colnames13
```

```
        colnames(em) <- colnames13
    } else if (ncol(fisher) == 21) {
        colnames(fisher) <- colnames21
        colnames(bfgs) <- colnames21
        colnames(poisson) <- colnames21
        colnames(nelderMead) <- colnames21
        colnames(squadP) <- colnames21
        colnames(em) <- colnames21
    } else {
        colnames(fisher) <- colnames25
        colnames(bfgs) <- colnames25
        colnames(poisson) <- colnames25
        colnames(nelderMead) <- colnames25
        colnames(squadP) <- colnames25
        colnames(em) <- colnames25
    }
    return(list(seeds = seeds, fisher = fisher, bfgs = bfgs,
        poisson = poisson, nelderMead = nelderMead, squadP = squadP,
        logbin_em = em))
}
```

10.0.2 The data generating function

This funtion is responsible for generating the random variables from marginal probability distributions. Origenally the function is created by Jakob Schöpe, However, it is modified for the use of the simulation being planned for this study.

```
dg <- function(i, param, dim, dispstr, margins, paramMargins,
    n, f, betas) {
    # Check passed arguments to smoothly run subsequent commands
    if (!is.integer(x = dim)) {
        stop("\"dim\" must be a positive integer")
    } else if (length(x = dim) != 1L) {
        stop("single positive integer for \"dim\" expected")
    } else if (!is.numeric(x = param)) {
        stop("\"param\" must be a real vector")
    } else if (length(x = param) != (dim * (dim - 1))/2) {
        stop("\"param\" must be a real vector of length ", (dim *
```

```
            (dim - 1))/2, " or \"dim\" has been misspecified")
    } else if (!is.character(x = dispstr)) {
        stop("\"dispstr\" must be a character string")
    } else if (length(x = dispstr) != 1L) {
        stop("single character string for \"dispstr\" expected")
    } else if (!(dispstr %in% c("ar1", "ex", "toep", "un"))) {
        stop("\"dispstr\" is misspecified. Currently available structures are:
          \"ar1\" for AR(1), \"ex\" for exchangeable, \"toep\" for Toeplitz
          or \"un\" for unstructured")
    } else if (!is.character(x = margins)) {
        stop("\"margins\" must be a character vector")
    } else if (length(x = margins) != dim) {
        stop("\"margins\" must be a character vector of length \"dim\"")
    } else if (!is.list(x = paramMargins)) {
        stop("\"paramMargins\" must be a list")
    } else if (length(x = paramMargins) != dim) {
        stop("\"paramMargins\" must be a list of length \"dim\"")
    } else if (!is.integer(x = n)) {
        stop("\"n\" must be a positive integer")
    } else if (length(x = n) != 1L) {
        stop("single positive integer for \"n\" expected")
    } else if (!inherits(x = f, what = "formula")) {
        stop("\"f\" must be of class \"formula\"")
    } else if (!is.numeric(x = betas)) {
        stop("\"betas\" must be a numeric vector")
    } else if (length(x = betas) != dim + 1) {
        stop("\"betas\" must be a numeric vector of length \"dim\"")
    } else if (!exists(x = ".Random.seed")) {
        stop("state for the pseudo-random number generator has not been set")
    } else {
        # Store the random number generator state
        seed <- .Random.seed
        # Predefine a normal copula
        copula_tmp <- copula::normalCopula(param = param, dim = dim,
            dispstr = dispstr)
        mvdc_tmp <- copula::mvdc(copula = copula_tmp, margins = margins,
            paramMargins = paramMargins)
        # Generate random variables from marginal probability
        # distributions
```

```
        data_tmp <- data.table::as.data.table(copula::rMvdc(n = n,
            mvdc = mvdc_tmp))
        # Predefine linear combinations
        b <- model.matrix(f, data = data_tmp) %*% betas
        # Compute individual probabilities for Y
        pr <- exp(b)
        # Generate a random variable from a binomial probability
        # distribution
        data_tmp$Y <- rbinom(n = n, size = 1, prob = pr)
        return(x = list(seed = seed, data = data_tmp))
    }
}
```

10.0.3 Regression models being evaluated and compared

This function *fit.models* returns the estimations of the six methods being studied and compared.

```
fit.models <- function(Y, V, events, sdata2, mdels_f,
    initial_val, n, sdata) {
    if (events > 0) {
        # log-likelihood function
        negll <- function(theta) {
            if (length(theta) != ncol(V)) {
                stop("The length of initial values doesn't fit with the number
                                     of columns")
            }
            eta <- exp(V %*% theta)
            llik <- dbinom(Y, size = 1, prob = eta,
                log = TRUE)
            .value <- sum(llik)
            return(-.value)
        }
        # GLM fisher scoring algorithm
        fit_fisher <- try(glm(formula = mdels_f, family = binomial(link = "log"),
            data = sdata2, start = initial_val))
        if (!is(fit_fisher, "try-error")) {
            if (!fit_fisher$converged) {
```

```
            coefs_fisher <- c(rep(NaN, length(initial_val) *
              2), n, events, 1)
        } else if (fit_fisher$converged) {
            coefs_fisher <- c(coef(fit_fisher),
              sqrt(diag(vcov(fit_fisher))), n,
              events, 1)
        }
    } else {
        coefs_fisher <- (c(rep(NA, length(initial_val) *
            2), n, events, 1))
    }

    # optim BFGS algorithm
    fit_opt_bfgs <- try(optim(initial_val, negll,
        NULL, method = "BFGS", hessian = TRUE,
        control = list(maxit = 2000)))
    if (!is(fit_opt_bfgs, "try-error")) {
        if (fit_opt_bfgs$convergence != 0) {
            coefs_bfgs <- c(rep(NaN, length(initial_val) *
              2), n, events, 2)
        } else if (fit_opt_bfgs$convergence == 0) {
            vcv.bfgs <- solve(fit_opt_bfgs$hessian)
            coefs_bfgs <- c(fit_opt_bfgs$par, sqrt(diag(vcv.bfgs)),
              n, events, 2)
        }
    } else {
        coefs_bfgs <- (c(rep(NA, length(initial_val) *
            2), n, events, 2))
    }

    # Poisson regresion model
    fit_poisson <- try(glm(formula = mdels_f, family = poisson(link = "log"),
        data = sdata2, control = list(maxit = 1000)))
    if (!is(fit_poisson, "try-error")) {
        if (!fit_poisson$converged) {
            coefs_poisson <- c(rep(NaN, length(initial_val) *
              2), n, events, 3)
        } else if (fit_poisson$converged) {
            coefs_poisson <- c(coef(fit_poisson),
```

```
          sqrt(diag(vcov(fit_poisson))), n,
          events, 3)
      }
    } else {
      coefs_poisson <- (c(rep(NA, length(initial_val) *
          2), n, events, 3))
    }

    # optim Nelder-Mead algorithm
    fit_nelderMead <- try(optim(initial_val, negll,
      NULL, method = "Nelder-Mead", hessian = TRUE,
      control = list(maxit = 20000)))
    if (!is(fit_nelderMead, "try-error")) {
      if (fit_nelderMead$convergence != 0) {
        coefs_nelderMead <- c(rep(NaN, length(initial_val) *
          2), n, events, 4)
      } else if (fit_nelderMead$convergence ==
        0) {
        vcv.nelderMead <- solve(fit_nelderMead$hessian)
        coefs_nelderMead <- c(fit_nelderMead$par,
          sqrt(diag(vcv.nelderMead)), n, events,
          4)
      }
    } else {
      coefs_nelderMead <- (c(rep(NA, length(initial_val) *
        2), n, events, 4))
    }

    # Modified Newton type algorithm squadP
    fit_modNewton <- try(squadP(frmla = mdels_f,
      data = sdata2))
    if (!is(fit_modNewton, "try-error")) {
      if (!fit_modNewton$Convergence) {
        coefs_modNewton <- c(rep(NaN, length(initial_val) *
          2), n, events, 5)
      } else if (fit_modNewton$Convergence) {
        vcv.modNewton <- try(solve(fit_modNewton$Hessian))
        if (!is(vcv.modNewton, "try-error")) {
          coefs_modNewton <- c(fit_modNewton$Constrained.solution,
```

```
                    sqrt(diag(vcv.modNewton)), n, events,
                    5)
                } else {
                  coefs_modNewton <- (c(rep(NA, length(initial_val) *
                    2), n, events, 5))
                }
            }
        } else {
            coefs_modNewton <- (c(rep(NA, length(initial_val) *
                2), n, events, 5))
        }

        # EM algorithm (logbin package)
        fit_em <- try(logbin(formula = mdels_f, method = "em",
            data = sdata2, start = initial_val, control = list(maxit = 1e+05)))
        if (!is(fit_em, "try-error")) {
            if (!fit_em$converged) {
                coefs_em <- c(rep(NaN, ncol(sdata2) *
                  2), n, events, 6)
            } else if (fit_em$converged) {
                coefs_em <- c(coef(fit_em), sqrt(diag(vcov(fit_em))),
                  n, events, 6)
            }
        } else {
            coefs_em <- (c(rep(NA, ncol(sdata2) * 2),
                n, events, 6))
        }

        return(list(seeds = sdata[[1]], fisher = coefs_fisher,
            bfgs = coefs_bfgs, poisson = coefs_poisson,
            nelderMead = coefs_nelderMead, squadP = coefs_modNewton,
            logbin_em = coefs_em))
    }
}
```

10.0.4 Estimation of all Senarios

sum_senarios function retrieve a list of estimations of different scenarios in a stuctured form.

```
sum_senarios <- function(f, n, m, betas) {
    a1 <- NULL
    listofdfs <- list()
    models <- list()
    for (i in n) {
        a1 <- sim(f, n = i, m, betas, seed = 1000001)
        listofdfs[[i]] <- a1
        models[[i]] <- rbind(listofdfs[[i]][[2]], listofdfs[[i]][[3]],
            listofdfs[[i]][[4]], listofdfs[[i]][[5]], listofdfs[[i]][[6]],
            listofdfs[[i]][[7]])
        models.est <- Filter(Negate(is.null), models)
        listofseeds <- unlist(listofdfs[[i]][[1]])
    }
    senar6 <- rbind(models.est[[1]], models.est[[2]], models.est[[3]],
        models.est[[4]], models.est[[5]], models.est[[6]], models.est[[7]],
        models.est[[8]], models.est[[9]])
    senar6 <- data.frame(senar6)
    senar6$model[senar6$model == 1] <- "Fisher(glm)"
    senar6$model[senar6$model == 2] <- "BFGS"
    senar6$model[senar6$model == 3] <- "Poisson(log)"
    senar6$model[senar6$model == 4] <- "Nelder-Mead"
    senar6$model[senar6$model == 5] <- "squadP"
    senar6$model[senar6$model == 6] <- "EM"
    return(list(seeds = listofseeds, models.est = senar6))
}
```

10.0.5 Fitting models function for scenarios with 2 covariates

```
fit.model_2 = function(i, n, betas) {
    margins <- c("norm", "norm")
    paramMargins <- list(list(mean = 0, sd = 1), list(mean = 20,
        sd = 4))  #list(size = 1, prob = 0.3)) #list(size = 1, prob = .1),
    f <- ~V1 + V2
```

```
    mdels_f = Y ~ V1 + V2
    initial_val <- c(-1.5, -0.2, -0.1)
    sdata <- dg(param = c(0.8), dim = 2L, dispstr = "un", margins = margins,
        paramMargins = paramMargins, n = n, f = f, betas = betas)
    sdata2 <- data.frame(sdata[[2]])
    Y <- sdata2[, 3]
    events <- sum(Y == 1)/n
    V <- model.matrix(f, data = sdata2)
    # call fit model function
    models_Fit <- fit.models(Y, V, events, sdata2, mdels_f, initial_val,
        n, sdata)
    return(models_Fit)
}
```

```
# Sample sizes (first 6 scenarios)
n45 = c(60L, 80L, 100L, 500L, 1000L, 50000L)
# Repeats for each senario
m = 1000L

# True values for 3% incidence rate
beta3 <- c(-3, -0.3, -0.02)
# True values for 6% incidence rate
beta6 <- c(-2.4, -0.3, -0.02)
# True values for 12% incidence rate
beta12 <- c(-1.5, -0.2, -0.03)
# True values for 24% incidence rate
beta24 <- c(-0.5, -0.08, -0.05)
# True values for 48% incidence rate
beta48 <- c(-0.35, -0.05, -0.02)


sumSen_1_9 <- sum_senarios(f = fit.model_2, n = n45, m = m, betas = beta3)
sumSen_10_18 <- sum_senarios(f = fit.model_2, n = n45, m = m,
    betas = beta6)
sumSen_19_27 <- sum_senarios(f = fit.model_2, n = n45, m = m,
    betas = beta12)
sumSen_28_36 <- sum_senarios(f = fit.model_2, n = n45, m = m,
    betas = beta24)
sumSen_37_45 <- sum_senarios(f = fit.model_2, n = n45, m = m,
```

```
    betas = beta48)
```

fit.model_2 function is repeated once with minor modification to fit for scenarions with 4 covariates and once more for scenarios with 8 covariates (not listed here).

11 Own Work

Abstracts and presentations

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- Sabau A, Daas L, Behkit A, Wagenpfeil S, Langenbucher A, Ardjomand N, Flockerzi E, Seitz B.: Efficacy, Safety and Predictability of Transepithelial Photorefractive Keratectomy - A meta-analysis, Journal of Cataract & Refractive Surgery: November 24,

2020 - Volume Publish Ahead of Print - Issue - doi: 10.1097/j.jcrs.0000000000000487.

- Heck S, Al-Shobash S, Rapp D, Le Duc-Dung, Omlor A, Bekhit A, Flaig M, Al-Kadah B, Herian W, Bals R, Wagenpfeil S, Dinh QT.: High probability of comorbidities in bronchial asthma in Germany. NJP Primary Care Respiratory Medicine. 2017;27:28.
- Comorbidities in Asthma: A Population-based Retrospective Cross-sectional Study; S Heck, S Al-Shobash, D Rapp, DD Le, A Omlor, A Bekhit, M Flaig, B Al-Kadah, W Herian, R Bals, S Wagenpfeil, QT Dinh; Pneumologie 2016; 70-V195.
- Comorbidities in Asthma: A Population-based Retrospective Cross-sectional Study; S Heck, F Kunze, D Rapp, DD Le, A Bekhit, M Flaig, B Al-Kadah, W Herian, R Bals, S Wagenpfeil, QT Dinh; Pneumologie 2016; 70-V200.
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- **The original BSW-algorithm of R cran package first implemented in this dissertation thesis was applied in:** Komajda M, Schöpe J, Wagenpfeil S, Tavazzi L, Böhm M, Ponikowski P, Anker SD, Filippatos GS, Cowie MR, QUALIFY investigators. Physicians' guideline adherence is associated with long-term heart failure mortality in outpatients with heart failure with reduced ejection fraction: the QUALIFY international registry. Eur J Heart Fail. 2019 Jul;21(7):921-929.